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Day and Thomson's Series.

A

TREATISE

OF

PLANE TRIGONOMETRY,

AND THE

MENSURATION OF HEIGHTS AND DISTANCES.

TO WHICH IS PREFIXED

A SUMMARY VIEW OF THE NATURE AND USE

OF

LOGARITHMS.

ADAPTED TO

THE METHOD OF INSTRUCTION IN SCHOOLS AND ACADEMIES

BY JEREMIAH DAY, D.D. LL.D.

LATE PRESIDENT OF YALE COLLEGE.

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PLANE TRIGONOMETRY.

Scarcely any department of Mathematics is more important, or more extensive in its applications, than Trigonometry. By it the mariner traces his path on the ocean; the geographer determines the latitude and longitude of places, the dimensions and positions of countries, the altitude of mountains, the courses of rivers, &c., and the astronomer calculates the distances and magnitudes of the heavenly bodies, predicts the eclipses of the sun and moon, and measures the progress of light from the stars.

The section on right angled triangles in this treatise, may perhaps be considered as needlessly minute. The solutions might, in all cases, be effected by the theorems which are given for oblique angled triangles. But the applications of rectangular trigonometry are so numerous, in navigation, surveying, astronomy, &c., that it was deemed important, to render familiar the various methods of stating the relations of the sides and angles; and especially to bring distinctly into view the principle on which most trigonometrical calculations are founded, the proportion between the parts of the given triangle, and a similar one formed from the sines, tangents, &c., in the tables.

As this treatise is intended to form a part of Day and Thomson's Course of Mathematics for the use of Schools and Academies, the references to Algebra are made to Thomson's Abridgment; and the references to Geometry, to Thomson's Legendre, as well as to Euclid's Elements.

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LOGARITHMS.

SECTION I.

NATURE OF LOGARITHMS.

Art. 1. The operations of Multiplication and Division, when they are to be often repeated, become so laborious, that it is an object of importance to substitute, in their stead, more simple methods of calculation, such as Addition and Subtraction. If these can be made to perform, in an expeditious manner, the office of multiplication and division, a great portion of the time and labor which the latter processes require, may be saved.

Now it has been shown, (Algebra, 189, 193,) that *powers* may be multiplied by adding their *exponents,* and divided, by subtracting their exponents. In the same manner, *roots* may be multiplied and divided, by adding and subtracting their fractional exponents. (Alg., 232, 239.) When these exponents are arranged in tables, and applied to the general purposes of calculation, they are called *Logarithms.*

2. LOGARITHMS, THEN, ARE THE EXPONENTS OF A SERIES OF POWERS AND ROOTS.

In forming a system of logarithms, some particular number is fixed upon, as the *base, radix,* or first power, whose logarithm is always 1. From this a series of powers is raised, and the exponents of these are arranged in tables for use. To explain this, let the number which is chosen for the first power be represented by a. Then taking a series of powers, both direct and reciprocal, as in Alg. 163;

$$a^4, a^3, a^2, a^1, a^0, a^{-1}, a^{-2}, a^{-3}, a^{-4}, \&c.$$

The logarithm of a^3 is 3, and the logarithm of a^{-1} is—1,

of a^1 is 1, of a^{-2} is—2,
of a^0 is 0, of a^{-3} is—3, &c.

Universally, the logarithm of a^x is x.

3. In the system of logarithms in common use, called *Briggs's* logarithms, the number which is taken for the radix or base is 10. The above series, then, by substituting 10 for a, becomes

$$10^4, 10^3, 10^2, 10^1, 10^0, 10^{-1}, 10^{-2}, 10^{-3}, \&c.$$

Or 10000, 1000, 100, 10, 1, $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$, &c.

Whose logarithms are

4, 3, 2, 1, 0, —1, —2, —3, &c.

4. The fractional exponents of *roots*, and of powers of roots, are converted into *decimals*, before they are inserted in the logarithmic tables. See Alg. 208.

The logarithm of $a^{\frac{1}{3}}$, or $a^{0.3333}$, is 0.3333,
of $a^{\frac{2}{3}}$, or $a^{0.6666}$, is 0.6666,
of $a^{\frac{3}{7}}$, or $a^{0.4285}$, is 0.4285,
of $a^{\frac{11}{3}}$, or $a^{3.6666}$, is 3.6666, &c.

These decimals are carried to a greater or less number of places, according to the degree of accuracy required.

5. In forming a system of logarithms, it is necessary to obtain the logarithm of each of the numbers in the natural series 1, 2, 3, 4, 5, &c.; so that the logarithm of any number may be found in the tables. For this purpose, the *radix* of the system must first be determined upon; and then every other number may be considered as some power or root of

this. If the radix is 10, as in the common system, every other number is to be considered as some power of 10.

If the exponent is a fraction, and the *numerator* be increased, the power will be increased; but if the *denominator* be increased, the power will be diminished.

6. To obtain then the logarithm of any number, according to Briggs's system, we have to find a power or root of 10 which shall be equal to the proposed number. The *exponent* of that power or root is the logarithm required. Thus

$$\left.\begin{array}{r} 7=10^{0.8451} \\ 20=10^{1.3010} \\ 30=10^{1.4771} \\ 400=10^{2.6020} \end{array}\right\} \begin{array}{c}\text{therefore the} \\ \text{logarithm}\end{array} \left\{\begin{array}{l} \text{of } 7 \text{ is } 0.8451 \\ \text{of } 20 \text{ is } 1.3010 \\ \text{of } 30 \text{ is } 1.4771 \\ \text{of } 400 \text{ is } 2.6020, \text{ \&c.} \end{array}\right.$$

7. A logarithm generally consists of two parts, an *integer* and a *decimal*. Thus the logarithm 2.60206, or, as it is sometimes written, 2+.60206, consists of the integer 2, and the decimal .60206. The integral part is called the *characteristic* or *index** of the logarithm; and is frequently omitted, in the common tables, because it can be easily supplied, whenever the logarithm is to be used in calculation.

By art. 3d, the logarithms of

	10000,	1000,	100,	10,	1,	.1,	.01,	.001, &c.
are	4,	3,	2,	1,	0,	—1,	—2,	—3, &c.

As the logarithms of 1 and of 10 are 0 and 1, it is evident, that, if any given number be between 1 and 10, its logarithm will be between 0 and 1, that is, it will be greater than 0, but less than 1. It will therefore have 0 for its index, with a decimal annexed.

Thus, the logarithm of 5 is 0.69897.

---

* The term *index*, as it is used here, may possibly lead to some confusion in the mind of the learner. For the logarithm itself is the index or exponent of a power. The characteristic, therefore, is the index of an index.

For the same reason, if the given number be between

10 and 100,	the log.	1 and 2, i.e.	$1+$the dec. part.
100 and 1000,	will be	2 and 3,	$2+$the dec. part.
1000 and 10000,	between	3 and 4,	$3+$the dec. part.

We have, therefore, when the logarithm of an integer or mixed number is to be found, this general rule:

8. *The index of the logarithm is always one less, than the number of integral figures, in the natural number whose logarithm is sought:* or, the index shows how far the first figure of the natural number is removed from the place of units.

Thus, the logarithm of 37 is 1.56820.

Here, the number of figures being *two*, the index of the logarithm is 1.

The logarithm of 253 is 2.40312.

Here the proposed number 253 consists of *three* figures, the first of which is in the second place from the unit figure. The index of the logarithm is therefore 2.

The logarithm of 62.8 is 1.79796.

Here it is evident that the mixed number 62.8 is between 10 and 100. The index of its logarithm must, therefore, be 1.

9. As the logarithm of 1 is 0, the logarithm of a number *less* than 1, that is, of any proper *fraction*, must be *negative*.

Thus, by art. 3d,

The logarithm of $\frac{1}{10}$ or .1 is -1,
of $\frac{1}{100}$ or .01 is -2,
of $\frac{1}{1000}$ or .001 is -3, &c.

10. If the proposed number is *between* $\frac{1}{100}$ and $\frac{1}{1000}$, its logarithm must be between -2 and -3. To obtain the logarithm, therefore, we must either *subtract* a certain fractional part from -2, or *add* a fractional part to -3; that

is, we must either annex a *negative decimal* to -2, or a *positive one* to -3.

Thus, the logarithm
of .008 is either $-2-.09691$, or $-3+90309$.*

The latter is generally most convenient in practice, and is more commonly written $\bar{3}.90309$. The line over the index denotes, that *that* is negative, while the *decimal* part of the logarithm is positive.

$$\text{The logarithm}\left\{\begin{array}{ll} \text{of } 0.3, & \text{is } \bar{1}.47712, \\ \text{of } 0.06, & \text{is } \bar{2}.77815, \\ \text{of } 0.009, & \text{is } \bar{3}.95424, \end{array}\right.$$

And universally,

11. *The negative index of a logarithm shows how far the first significant figure of the natural number, is removed from the place of units, on the right;* in the same manner as a *positive* index shows how far the first figure of the natural number is removed from the place of units on the left. (Art. 8.) Thus, in the examples in the last article,

The decimal 3 is in the *first* place from that of units,
6 is in the *second* place,
9 is in the *third* place;

And the indices of the logarithms are $\bar{1}$, $\bar{2}$, and $\bar{3}$.

12. It is often more convenient, however to make the *index* of the logarithm positive, as well as the decimal part. This is done by adding 10 to the index.

Thus, for -1, 9 is written, for -2, 8, &c.
Because $-1+10=9$, $-2+10=8$, &c.

---

* That these two expressions are of the same value will be evident, if we subtract the same quantity, $+.90309$ from each. The remainders will be equal, and therefore the quantities from which the subtraction is made must be equal.

With this alteration,

$$\text{The logarithm} \left\{ \begin{array}{l} \bar{1}.90309 \\ \bar{2}.90309 \\ \bar{3}.90309 \end{array} \right\} \text{becomes} \left\{ \begin{array}{l} 9.90309, \\ 8.90309, \\ 7.90309, \text{ \&c.} \end{array} \right.$$

This is making the index of the logarithm 10 too great. But with proper caution, it will lead to no error in practice.

13. The *sum* of the logarithms of two numbers, is the logarithm of the *product* of those numbers; and the *difference* of the logarithms of two numbers, is the logarithm of the *quotient* of one of the numbers divided by the other. (Art. 2.) In Briggs's system, the logarithm of 10 is 1. (Art. 3.) If therefore any number be multiplied or divided by 10, its logarithm will be increased or diminished by 1: and as this is an integer, it will only change the *index* of the logarithm, without affecting the decimal part.

Thus, the logarithm of 4730 is 3.67486
And the logarithm of 10 is 1.
The logarithm of the product 47300 is 4.67486
And the logarithm of the quotient 473 is 2.67486

Here the *index* only is altered, while the decimal part remains the same. We have then this important property,

14. The DECIMAL PART *of the logarithm of any number is the same, as that of the number multiplied or divided by* 10, 100, 1000, &c.

Thus the log. of 45670, is 4.65963,
4567, 3.65963,
456.7, 2.65963,
45.67, 1.65963,
4.567, 0.65963,
.4567, $\bar{1}.65963$, or 9.65963,
.04567, $\bar{2}.65963$, 8.65963,
.004567, $\bar{3}.65963$, 7.65963

This property, which is peculiar to Briggs's system, is of great use in abridging the logarithmic tables. For when we have the logarithm of any number, we have only to change the index, to obtain the logarithm of every other number, whether integral, fractional, or mixed, consisting of the same significant figures. The decimal part of the logarithm of a fraction found in this way, is always *positive*. For it is the same as the decimal part of the logarithm of a whole number.

17. *If a series of numbers be in* GEOMETRICAL *progression, their logarithms will be in* ARITHMETICAL *progression.* For, in a geometrical series ascending, the quantities increase by a common *multiplier;* (Alg. 359.) That is, each succeeding term is the *product* of the preceding term into the ratio. But the *logarithm* of this product is the *sum* of the logarithms of the preceding term and the ratio; that is, the logarithms increase by a common *addition,* and are, therefore, in arithmetical progression. (Alg. 326.) In a geometrical progression *descending,* the terms decrease by a common *divisor,* and their logarithms, by a common *difference.**

Thus, the numbers 1, 10, 100, 1000, 10000, &c., are in geometrical progression.

And their logarithms 0, 1, 2, 3, 4, &c., are in arithmetical progression.

---

* See Note A.

SECTION II.

DIRECTIONS FOR TAKING LOGARITHMS AND THEIR NUMBERS FROM THE TABLES.*

Art. 24. The purpose which logarithms are intended to answer, is to enable us to perform arithmetical operations with *greater expedition,* than by the common methods. Before any one can avail himself of this advantage, he must become so familiar with the tables, that he can readily find the logarithm of any number; and, on the other hand, the number to which any logarithm belongs.

In the common tables, the *indices* to the logarithms of the first 100 numbers are inserted. But, for all other numbers, the *decimal* part only of the logarithm is given; while the index is left to be supplied, according to the principles in Arts. 8 and 11.

25. *To find the logarithm of any number between* 1 *and* 100:

Look for the proposed number, on the left; and against it, in the next column, will be the logarithm, with its index Thus,

The log. of 18 is 1.25527. The log. of 73 is 1.86332.

26. *To find the logarithm of any number between* 100 *and* 1000; or of any number consisting of not more than three significant figures, with ciphers annexed.

---

* The best English Tables are Hutton's in 8vo. and Taylor's in 4to. In these, the logarithms are carried to seven places of decimals, and proportional parts are placed in the margin. The smaller tables are numerous; and, when accurately printed, are sufficient for common calculations.

In the smaller tables, the three first figures of each number, are generally placed in the left hand column; and the fourth figure is placed at the head of the other columns.

Any number, therefore, between 100 and 1000, may be found on the left hand; and directly opposite, in the next column, is the decimal part of its logarithm. To this the *index* must be prefixed, according to the rule in Art. 8.

The log. of 458 is 2.66087, The log. of 935 is 2.97081,
of 796 2.90091, of 386 2.58659.

If there are *ciphers* annexed to the significant figures, the logarithm may be found in a similar manner. For, by Art. 14, the *decimal* part of the logarithm of any number is the same, as that of the number multiplied into 10, 100, &c. All the difference will be in the *index*; and this may be supplied by the same general rule.

The log. of 4580 is 3.66087, The log. of 326000 is 5.51322,
of 79600 4.90091, of 8010000 6.90363.

27. *To find the logarithm of any number consisting of* FOUR *figures, either with, or without, ciphers annexed.*

Look for the three first figures, on the left hand, and for the fourth figure, at the head of one of the columns. The logarithm will be found, opposite the three first figures, and in the column which, at the head, is marked with the fourth figure.*

The log. of 6234 is 3.79477, The log. of 783400 is 5.89398,
of 5231 3.71858, of 6281000 6.79803.

28. *To find the logarithm of a number containing* MORE *than* FOUR *significant figures.*

By turning to the tables, it will be seen, that if the *differences* between several numbers be small, in comparison with the numbers themselves; the differences of the *logarithms*

---

* In Taylor's, Hutton's, and other tables, *four* figures are placed in the left hand column, and the *fifth* at the top of the page.

will be nearly proportioned to the differences of the *numbers*. **Thus,**

The log. of 1000 is 3.00000,
of 1001 3.00043,
of 1002 3.00087,
of 1003 3.00130,
of 1004 3.00173, &c.

Here the differences in the numbers are, 1, 2, 3, 4, &c., and the corresponding differences in the logarithms, are 43, 87, 130, 173, &c.

Now 43 is nearly half of 87, one-third of 130, one-fourth of 173, &c

Upon this principle, we may find the logarithm of a number which is between two other numbers whose logarithms are given by the tables. Thus, the logarithm of 21716 is not to be found in those tables which give the numbers to four places of figures only.

But by the table, the log. of 21720 is 4.33686
and the log. of 21710 is 4.33666

The difference of the two numbers is 10; and that of the logarithms 20.

Also, the difference between 21710, and the proposed number 21716, is 6.

If, then, a difference of 10 in the numbers
make a difference of 20 in the logarithms:
A difference of 6 in the numbers will
make a difference of 12 in the logarithms.

That is, 10 : 20 : : 6 : 12.

If, therefore, 12 be added to 4.33666, the log. of 21710;
The sum will be 4.33678, the log. of 21716.
We have, then, this

RULE.

To find the logarithm of a number consisting of more than four figures:

Take out the logarithm of two numbers, one greater, and the other less, than the number proposed: Find the difference of the two numbers, and the difference of their logarithms: Take also the difference between the least of the two numbers, and the proposed number. Then say,

As the difference of the two numbers,
To the difference of their logarithms;
So is the difference between the least of the two numbers, and the proposed number,
To the proportional part to be added to the least of the two logarithms.

It will generally be expedient to make the *first four figures*, in the least of the two numbers, the same as in the proposed number, substituting ciphers, for the remaining figures; and to make the greater number the same as the less, with the addition of a unit to the last significant figure. Thus,

For 36843, take 36840, and 36850,
For 792674, 792600, 792700,
For 6537825, 6537000, 6538000, &c.

The first term of the proportion will then be 10, or 100, or 1000, &c.

Ex. 1. Required the logarithm of 362572.

The logarithm of 362600 is 5.55943
of 362500 5.55931
The differences are 100, and 12.

Then 100 : 12 : : 72 : 8.64, or 9 nearly.
And the log. 5.55931+9=5.55940, the log. required.

Ex. 2. The log. of 78264 is 4.89356
3. The log. of 143542 is 5.15698
4. The log. of 1129535 is 6.05290.

By a little practice, such a facility in abridging these calculations may be acquired, that the logarithms may be taken out, in a very short time. When great accuracy is not required, it will be easy to make an allowance sufficiently near, without formally stating a proportion. In the larger tables, the proportional parts which are to be added to the logarithms, are already prepared, and placed in the margin.

29. *To find the logarithm of a* DECIMAL FRACTION.

The logarithm of a decimal is the same as that of a whole number, excepting the *index*. (Art. 14.) To find then the logarithm of a decimal, take out that of a whole number consisting of the same figures; *observing to make the negative index equal to the distance of the first significant figure of the fraction from the place of units.* (Art. 11.)

The log. of 0.07643, is $\bar{2}.88326$, or 8.88326, (Art. 12.)
of 0.00259, $\bar{3}.41330$, or 7.41330,
of 0.0006278, $\bar{4}.79782$, or 6.79782.

30. *To find the logarithm of a* MIXED *decimal number.*

Find the logarithm, in the same manner as if *all* the figures were integers; and then prefix the index which belongs to the *integral* part, according to Art. 8.

The logarithm of 26.34 is 1.42062.

The index here is 1, because 1 is the index of the logarithm of every number greater than 10, and less than 100. (Art. 7.)

The log. of 2.36 is 0.37291, The log. of 364.2 is 2.56134,
of 27.8 1.44404, of 69.42 1.84148.

31. *To find the logarithm of a* VULGAR FRACTION.

From the nature of a vulgar fraction, the numerator may be considered as a *dividend,* and the denominator as a *divisor;* in other words, the value of the fraction is equal to the quotient of the numerator divided by the denominator (Alg. 110.) But in logarithms, division is performed by *subtraction;* that is, the *difference* of the logarithms of two numbers, is the logarithm of the *quotient* of those numbers.

(Art. 1.) To find then the logarithm of a vulgar fraction, *subtract the logarithm of the denominator* from *that of the numerator*. The difference will be the logarithm of the fraction. Or the logarithm may be found, by first reducing the vulgar fraction to a *decimal*. If the numerator is less than the denominator, the index of the logarithm must be *negative*, because the value of the fraction is less than a unit. (Art. 9.)

Required the logarithm of $\frac{34}{87}$.

The log. of the numerator is 1.53148
of the denominator 1.93952

of the fraction 1.59196, or 9.59196.
The logarithm of $\frac{362}{7854}$ is $\bar{2}.66362$, or 8.66362.
of $\frac{7}{6329}$ $\bar{3}.04376$, or 7.04376.

32. If the logarithm of a *mixed number* is required, reduce it to an improper fraction, and then proceed as before.

The logarithm of $3\frac{7}{9}=\frac{34}{9}$ is 0.57724.

33. *To find the* NATURAL NUMBER *belonging to any logarithm.*

In computing by logarithms, it is necessary, in the first place, to take from the tables the logarithms of the numbers which enter into the calculation; and, on the other hand, at the close of the operation, to find the number belonging to the logarithm obtained in the result. This is evidently done by *reversing* the methods in the preceding articles.

Where great accuracy is not required, look in the tables for the logarithm which is *nearest* to the given one; and directly opposite on the left hand, will be found the *three first* figures, and at the top, over the logarithm, the *fourth* figure of the number required. This number, by pointing off decimals, or by adding ciphers, if necessary, must be made to correspond with the *index* of the given logarithm, according to Arts. 8 and 11.

The natural number belonging

to 3.86493 is 7327, to 1.62572 is 42.24,
to 2.90141 796.9, to $\bar{2}.89115$ 0.07783.

In the last example, the index requires that the first significant figure should be in the *second* place from units, and therefore a cipher must be prefixed. In other instances, it is necessary to annex ciphers on *the right,* so as to make the number of figures exceed the index by 1.

The natural number belonging

to 6.71567 is 5196000, to $\bar{3}.65677$ is 0.004537,
to 4.67062 46840, to $\bar{4}.59802$ 0.0003963.

34. When great accuracy is required, and the given logarithm is not exactly, or very nearly, found in the tables, it will be necessary to reverse the rule in Art. 28.

Take from the tables two logarithms, one the next greater, the other the next less than the given logarithm. Find the difference of the two logarithms, and the difference of their natural numbers; also the difference between the least of the two logarithms, and the given logarithm. Then say,

As the difference of the two logarithms,
To the difference of their numbers;
So is the difference between the given
logarithm and the least of the other two,
To the proportional part to be added to
the least of the two numbers.

Required the number belonging to the logarithm 2.67325.

Next great. log.	2.67330.	Its numb.	471.3.	Given log.	2.67325.
Next less	2.67321.	Its numb.	471.2.	Next less	2.67321.
Differences	9		0.1		4

Then, 9 : 0.1 : : 4 : 0.044, which is to be added
to the number 471.2
The number required is 471.244.

The natural number belonging
to 4.37627 is 23783.45, to 1.73698 is 54.57357,
to 3.69479 4952.08, to $\bar{1}.09214$ 0.123635.

35. *Correction of the Tables.*—The tables of logarithms have been so carefully and so repeatedly calculated, by the ablest computers, that there is no room left to question their general correctness. They are not, however, exempt from the common imperfections of the press. But an error of this kind is easily corrected, by comparing the logarithm with any two others to whose *sum* or *difference* it ought to be equal. (Art. 1.)

Thus $48=24\times2=16\times3=12\times4=8\times6$. Therefore, the logarithm of 48 is equal to the *sum* of the logarithms of 24 and 2, of 16 and 3, &c.

And, $3=\frac{6}{2}=\frac{12}{4}=\frac{15}{5}=\frac{18}{6}=\frac{21}{7}$, &c. Therefore, the logarithm of 3 is equal to the *difference* of the logarithms of 6 and 2, of 12 and 4, &c.

SECTION III.

METHODS OF CALCULATING BY LOGARITHMS.

Art. 36. The arithmetical operations for which logarithms were originally contrived, and on which their great utility depends, are chiefly multiplication, division, involution, evolution, and finding the term required in single and compound proportion. The principle on which all these calculations are conducted, is this:

If the logarithms of two numbers be added, the SUM *will be the logarithm of the* PRODUCT *of the numbers;* and,

If the logarithm of one number be subtracted from that of another, the DIFFERENCE *will be the logarithm of the* QUOTIENT *of one of the numbers divided by the other.*

In proof of this, we have only to call to mind, that logarithms are the EXPONENTS *of a series of powers and roots.* (Arts. 2, 5.) And it has been shown, that powers and roots are *multiplied* by *adding* their exponents; and *divided,* by *subtracting* their exponents. (Alg. 189, 193, 232, 239.)

MULTIPLICATION BY LOGARITHMS.

37. ADD THE LOGARITHMS OF THE FACTORS: THE SUM WILL BE THE LOGARITHM OF THE PRODUCT.

In making the addition, 1 is to be carried for every 10, from the decimal part of the logarithm, to the index. (Art. 7.)

	Numbers.	Logarithms.		Numbers.	Logarithms.
Mult.	36.2 (Art. 30.)	1.55871	Mult.	640	2.80618
Into	7.84	0.89432	Into	2.316	0.36474
Prod.	283.8	2.45303	Prod.	1482	3.17092

The logarithms of the two factors are taken from the

tables. The product is obtained, by finding, in the **tables,** **the natural** number belonging to the sum. (Art. 33.)

Mult.	89.24	1.95056	Mult.	134.	2.12710
Into	3.687	0.56667	Into	25.6	1.40824
Prod.	329.	2.51723	Prod.	3430	3.53534

38. When any or all of the indices of the logarithms are *negative,* they are to be added according to the rules for the addition of positive and negative quantities in algebra. But it must be kept in mind, that the decimal part of the logarithm is *positive.* (Art. 10.) Therefore, that which is carried from the decimal part to the index, must be considered positive also.

Mult.	62.84	1.79824	Mult.	0.0294	$\bar{2}.46835$
Into	0.682	$\bar{1}.83378$	Into	0.8372	$\bar{1}.92283$
Prod.	42.86	1.63202	Prod.	0.0246	$\bar{2}.39118$

In each of these examples, +1 is to be carried from the decimal part of the logarithm. This, added to —1, the lower index, makes it 0; so that there is nothing to be added to the upper index.

If any perplexity is occasioned, by the addition of positive and negative quantities, it may be avoided, by borrowing 10 to the index. (Art. 12.)

Mult.	62.84	1.79824	Mult.	0.0294	8.46835
Into	0.682	9.83378	Into	0.8372	9.92283
Prod.	42.86	1.63202	Prod.	0.0246	8.39118

Here 10 is added to the negative indices, and afterwards rejected from the index of the sum of the logarithms.

Multiply	26.83	1.42862		1.42862
Into	0.00069	$\bar{4}.83885$	or	6.83885
Product	0.01851	$\bar{2}.26747$		8.26747

Here +1 carried to —4 makes it —3, which added to the upper index +1, gives —2 for the index of the sum.

Multiply	.00845	$\bar{3}.92686$	or	7.92686
Into	1068.	3.02857		3.02857
Product	9.0246	0.95543		0.95543

The product of 0.0362 into 25.38 is 0.9188
of 0.00467 into 348.1 is 1.626
of 0.0861 into 0.00843 is 0.0007258

39. *Any number of factors* may be multiplied together, by adding their logarithms. If there are several *positive*, and several *negative* indices, these are to be reduced to one, as in algebra, by taking the difference between the sum of those which are negative, and the sum of those which are positive, increased by what is carried from the decimal part of the logarithms. (Alg. 53.)

Multiply	6832	3.83455		3.83455
Into	0.00863	$\bar{3}.93601$	or	7.93601
And	0.651	$\bar{1}.81358$		9.81358
And	0.0231	$\bar{2}.36361$	or	8.36361
And	62.87	1.79844		1.79844
Prod.	55.74	1.74619		1.74619

Ex. 2. The prod. of 36.4×7.82×68.91×0.3846 is 7544.

3. The prod. of 0.00629×2.647×0.082×278.8×0.00063 is 0.0002398.

40. *Negative* quantities are multiplied, by means of logarithms, in the same manner as those which are positive. (Art. 16.) But, after the operation is ended, the proper sign must be applied to the natural number expressing the product, according to the rules for the multiplication of positive and negative quantities in algebra. The negative index of a *logarithm,* must not be confounded with the sign which denotes that the *natural number* is negative. That which the index

of the logarithm is intended to show, is not whether the natural number is *positive or negative,* but whether it is *greater or less than a unit.* (Art. 16.)

Mult.	+36.42	1.56134	Mult.	—2.681	0.42830
Into	—67.31	1.82808	Into	+37.24	1.57101
Prod.	— 2451	3.38942	Prod.	—99.84	1.99931

In these examples, the logarithms are taken from the tables, and added, in the same manner, as if both factors were positive. But after the product is found, the negative sign is prefixed to it, because + is multiplied into —. (Alg. 82.)

Mult.	0.263	$\bar{1}.41996$	Mult.	0.065	$\bar{2}.81291$
Into	0.00894	$\bar{3}.95134$	Into	0.693	$\bar{1}.84073$
Prod.	0.002351	$\bar{3}.37130$	Prod.	0.04504	$\bar{2}.65364$

Here the indices of the logarithms are negative, but the product is positive, because the factors are both positive.

Mult.	—62.59	1.79650	Mult.	—68.3	1.83442
Into	—0.00863	$\bar{3}.93601$	Into	—0.0096	$\bar{3}.98227$
Prod.	+0.5402	$\bar{1}.73251$	Prod.	+0.6557	$\bar{1}.81669$

DIVISION BY LOGARITHMS.

41. FROM THE LOGARITHM OF THE DIVIDEND, SUBTRACT THE LOGARITHM OF THE DIVISOR; THE DIFFERENCE WILL BE THE LOGARITHM OF THE QUOTIENT. (Art. 36.)

	Numbers.	Logarithms.		Numbers.	Logarithms.
Divide	6238	3.79505	Divide	896.3	2.95245
By	2982	3.47451	By	9.847	0.99330
Quot.	2.092	0.32054	Quot.	91.02	1.95915

42. The *decimal* part of the logarithm may be subtracted as in common arithmetic. But for the *indices*, when either of them is negative, or the lower one is greater than the upper one, it will be necessary to make use of the general rule for subtraction in algebra; that is, to change the signs of the subtrahend, and then proceed as in addition. (Alg. 60.) When 1 is carried from the decimal part, this is to be considered affirmative, and applied to the index, before the sign is changed.

Divide	0.8697	$\bar{1}$.93937	or	9.93937
By	98.65	1.99410		1.99410
Quot.	0.008816	$\bar{3}$.94527		7.94527

In this example, the upper logarithm being less than the lower one, it is necessary to borrow 10, as in other cases of subtraction; and therefore to carry one to the lower index, which then becomes +2. This changed to —2, and added to —1 above it, makes the index of the difference of the logarithms —3.

Divide	29.76	1.47363		1.47363
By	6254	3.79616		3.79616
Quot.	0.00476	$\bar{3}$.67747	or	7.67747

Here, 1 carried to the lower index, makes it +4. This changed to —4, and added to 1 above it, gives —3 for the index of the difference of the logarithms.

Divide	6.832	0.83455	Divide	0.00634	$\bar{3}$.80209
By	.0362	$\bar{2}$.55871	By	62.18	1.79365
Quot.	188.73	2.27584	Quot.	0.000102	$\bar{4}$.00844

The quotient of 0.0985 divided by 0.007241, is 13.6

The quotient of 0.0621 divided by 3.68, is 0.01687

43. To divide *negative* quantities, proceed in the same manner as if they were positive, (Art. 40.) and prefix to the

quotient, the sign which is required by the rules for division in algebra.

Divide	+3642	3.56134	Divide	—0.657	$\bar{1}.81757$
By	—23.68	1.37438	By	+0.0793	$\bar{2}.89927$
Quot.	—153.8	2.18696	Quot.	—8.285	0.91830

In these examples, the sign of the divisor being different from that of the dividend, the sign of the quotient must be negative. (Alg. 100.)

Divide	—0.364	$\bar{1}.56110$	Divide	—68.5	1.83569
By	—2.56	0.40824	By	+0.094	$\bar{2}.97313$
Quot.	+0.1422	$\bar{1}.15286$	Quot.	—728.7	2.86256

INVOLUTION BY LOGARITHMS.

44. Involving a quantity is multiplying it into itself. By means of logarithms, multiplication is performed by addition. If, then, the logarithm of any quantity be *added to itself*, the logarithm of a *power* of that quantity will be obtained. But adding a logarithm, or any other quantity, to itself, is *multiplication*. The involution of quantities, by means of logarithms, is therefore performed, by multiplying the logarithms.

Thus the logarithm
of 100 is 2
of 100×100, that is, of $\overline{100^2}$ is $2+2 = 2\times2$
of $100\times100\times100$, $\overline{100^3}$ is $2+2+2 = 2\times3$
of $100\times100\times100\times100$, $\overline{100^4}$ is $2+2+2+2 = 2\times4$

On the same principle, the logarithm of $\overline{100^n}$ is $2\times n$.

And the logarithm of x^n, is $(\text{log. } x)\times n$. Hence,

45. To involve a quantity by logarithms, **MULTIPLY THE LOGARITHM OF THE QUANTITY, BY THE INDEX OF THE POWER REQUIRED.**

The reason of the rule is also evident, from the consideration, that logarithms are the exponents of powers and roots, and a power or root is involved, by *multiplying* its index into the index of the power required. (Alg. 170, 242.)

Ex. 1.	What is the cube of		6.296 ?
Root	6.296,	its log.	0.79906
	Index of the power		3
Power	249.6		2.39718

2.	Required the 4th power of		21.32
Root	21.32	log.	1.32879
		Index	4
Power	206614		5.31516

3.	Required the 6th power of		1.689
Root	1.689	log.	0.22763
		Index	6
Power	23.215		1.36578

4.	Required the 144th power of		1.003
Root	1.003	log.	0.00130
		Index	144
Power	1.539		0.18720

46. It must be observed, as in the case of multiplication, (Art. 38.) that what is carried from the *decimal* part of the logarithm is *positive*, whether the index itself is positive or negative. Or, if 10 be added to a negative index, to render it positive, (Art, 12.) this will be multiplied, as well as the other figures, so that the logarithm of the square, will be 20 too great; of the cube, 30 too great, &c.

Ex. 1.	Required the cube of		0.0649		
Root	0.0649	log.	$\bar{2}.81224$	or	8.81224
		Index	3		3
Power	0.0002733		$\bar{4}.43672$		6.43672

2. Required the 4th power of 0.1234

Root	0.1234	log.	$\bar{1}\ 09132$	or	9.09132
		Index	4		4
Power	0.0002319		$\bar{4}.36528$		6.36528

3. Required the 6th power of 0.9977

Root	0.9977	log.	$\bar{1}.99900$	or	9.99900
		Index	6		6
Power	0.9863		$\bar{1}.99400$		9.99400

4. Required the cube of 0.08762

Root	0.08762	log.	$\bar{2}.94260$	or	8.94260
		Index	3		3
Power	0.0006727		$\bar{4}.82780$		6.82780

5. The 7th power of 0.9061 is 0.5015.

6. The 5th power of 0.9344 is 0.7123.

EVOLUTION BY LOGARITHMS.

47. Evolution is the opposite of involution. Therefore, as quantities are involved, by the *multiplication* of logarithms, roots are extracted by the *division* of logarithms ; that is,

To extract the root of a quantity by logarithms, DIVIDE *THE LOGARITHM OF THE QUANTITY,* BY THE NUMBER EXPRESSING THE ROOT REQUIRED.

The reason of the rule is evident also, from the fact, that logarithms are the exponents of powers and roots, and evolution is performed, by dividing the exponent, by the number expressing the root required. (Alg. 210.)

1. Required the square root of 648.3

	Numbers.	Logarithms.
Power	648.3	2)2.81178
Root	25.46	1.40589

2. Required the cube root of 897.1

Power	897.1	3)2.95284
Root	9.645	0.98428

In the first of these examples, the logarithm of the given number is divided by 2 ; in the other, by 3.

3. Required the 10th root of 6948.

Power	6948	10)3.84186
Root	2.422	0.38418

4. Required the 100dth root of 983.

Power	983	100)2.99255
Root	1.071	0.02992

The division is performed here, as in other cases of decimals, by removing the decimal point to the left.

5. What is the ten thousandth root of 49680000 ?

Power	49680000	10000)7.69618
Root	1.00179	0.00077

We have, here, an example of the great rapidity with which arithmetical operations are performed by logarithms.

48. If the index of the logarithm is *negative,* and is *not divisible* by the given divisor, without a remainder, a difficulty will occur, unless the index be altered.

Suppose the cube root of 0.0000892 is required. The logarithm of this is $\bar{5}.95036$. If we divide the index by 3, the quotient will be —1, with —2 remainder. This remainder, if it were positive, might, as in other cases of division, be prefixed to the next figure. But the remainder is *negative,* while the decimal part of the logarithm is positive ; so that, when the former is prefixed to the latter, it will make neither +2.9 nor —2.9, but —2+.9. This embarrassing intermixture of positives and negatives may be avoided, by adding to the index another negative number, to make it exactly divisible by the divisor. Thus, if to the index —5 there be added —1, the sum —6 will be divisible by 3. But this addition of a negative number must be *compensated*, by the addition of an equal positive number, which may be prefixed to the decimal part of the logarithm. The division may then be continued, without difficulty, through the whole.

Thus, if the logarithm $\bar{5}.95036$ be altered to $\bar{6}+1.95036$ it may be divided by 3, and the quotient will be $\bar{2}.65012$. We have then this rule,

49. *Add to the index*, if necessary, *such a negative number as will make it exactly divisible by the divisor, and prefix an equal positive number to the decimal part of the logarithm.*

1. Required the 5th root of 0.009642
Power 0.009642 log. $\bar{3}.98417$
or $\bar{5}+2.98417$
Root 0.3952 $\bar{1}.59683$

2. Required the 7th root of 0.0004935.
Power 0.0004935 log. $\bar{4}.69329$
or $7)\bar{7}+3.69329$
Root 0.337 $\bar{1}.52761$

50. If, for the sake of performing the division conveniently, the negative index be rendered *positive*, it will be expedient to borrow as many tens, as there are units in the number denoting the root.

What is the fourth root of 0.03698?

Power 0.03698 $4)\bar{2}.56797$ or $4)38.56797$
Root 0.4385 $\bar{1}.64199$ 9.64199

Here the index, by borrowing, is made 40 too great, that is, +38 instead of —2. When, therefore, it is divided by 4, it is still 10 too great, +9 instead of —1.

What is the 5th root of 0.008926?

Power	0.008926	5)$\bar{3}$.95066	or 5)47.95066
Root	0.38916	$\bar{1}$.59013	9.59013

51. A *power of a root* may be found by first *multiplying* the logarithm of the given quantity into the index of the power, (Art. 45.) and then *dividing* the product by the number expressing the root. (Art. 47.)

1. What is the value of $(53)^{\frac{6}{7}}$, that is, the 6th power of the 7th root of 53?

Given number 53	log.	1.72428
Multiplying by		6
Dividing by		7)10.34568
Power required 30.06		1.47795

2. What is the 8th power of the 9th root of 654?

PROPORTION BY LOGARITHMS.

52. In a proportion, when three terms are given, the fourth is found in common arithmetic, by multiplying together the second and third, and dividing by the first. But when logarithms are used, *addition* takes the place of multiplication, and *subtraction*, of division.

To find, then, by logarithms, the fourth term in a proportion, ADD THE LOGARITHMS OF THE SECOND AND THIRD TERMS, AND *from the sum* SUBTRACT THE LOGARITHM OF THE FIRST TERM. The remainder will be the logarithm of the term required.

Ex. 1. Find a fourth proportional to 7964, 378, and 27960.

	Numbers.	Logarithms.
Second term	378	2.57749
Third term	27960	4.44654
		7.02403
First term	7964	3.90113
Fourth term	1327	3.12290

2. Find a 4th proportional to 768, 381, and 9780.

Second term	381	2.58092
Third term	9780	3.99034
		6.57126
First term	768	2.88536
Fourth term	4852	3.68590

ARITHMETICAL COMPLEMENT.

53. When one number is to be subtracted from another, it is often convenient, first to subtract it from 10, then to *add the difference* to the other number, and afterwards to reject the 10.

Thus, instead of $a-b$, we may put $10-b+a-10$.

In the first of these expressions, b is subtracted from a. In the other, b is subtracted from 10, the difference is added to a, and 10 is afterwards taken from the sum. The two expressions are equivalent, because they consist of the same terms, with the addition, in one of them, of $10-10=0$. The alteration is, in fact, nothing more than borrowing 10, for the sake of convenience, and then rejecting it in the result.

Instead of 10, we may borrow, as occasion requires, 100, 1000, &c.

Thus, $a-b=100-b+a-100=1000-b+a-1000$, &c.

54. The DIFFERENCE *between a given number and* 10, *or* 100, *or* 1000, *&c., is called the* ARITHMETICAL COMPLEMENT *of that number.*

The arithmetical complement of a number consisting of *one* integral figure, either with or without decimals, is found, by subtracting the number from 10. If there are *two* integral figures, they are subtracted from 100; if *three*, from 1000, &c.

Thus, the arithmetical compl't of 3.46 is $10-3.46=6.54$
of 34.6 is $100-34.6=65.4$
of 346. is $1000-346.=654.$ &c.

According to the rule for subtraction in arithmetic, any number is subtracted from 10, 100, 1000, &c. by beginning on the right hand, and taking each figure from 10, after *increasing* all except the first, by *carrying* 1.

Thus, if from 10.00000
We subtract 7.63125

The difference, or arith'l compl't is 2.36875, which is obtained by taking 5 from 10, 3 from 10, 2 from 10, 4 from 10, 7 from 10, and 8 from 10. But, instead of taking each figure, *increased by* 1 from 10; we may take it *without being increased*, from 9.

Thus, 2 from 9 is the same as 3 from 10,
3 from 9 the same as 4 from 10, &c. Hence,

55. *To obtain the* ARITHMETICAL COMPLEMENT *of a number, subtract the right hand significant figure from* 10, *and each of the other figures from* 9. If, however, there are *ciphers* on the right hand of all the significant figures, they are to be set down without alteration.

In taking the arithmetical complement of a logarithm, if the index is *negative,* it must be *added* to 9; for adding a negative quantity is the same as subtracting a positive one. (Alg. 81.) The difference between —3 and +9, is not 6, but 12.

The arithmetical complement

of 6.24897	is 3.75103	of $\bar{2}$.70649	is 11.29351
of 2.98643	7.01357	of 3.64200	6.35800
of 0.62430	9.37570	of 9.35001	0.64999

56. The principal use of the arithmetical complement, is in working proportions by logarithms; where some of the terms are to be *added,* and one or more to be *subtracted.* In the Rule of Three or simple proportion, two terms are to be added, and from the sum, the first term is to be subtracted. But if, instead of the logarithm of the first term,

we substitute its arithmetical complement, this may be *added* to the sum of the other two, or more simply all three may be added together, by one operation. After the index is diminished by 10, the result will be the same as by the common method. For subtracting a number is the same, as adding its arithmetical complement, and then rejecting 10, 100, or 1000, from the sum. (Art. 53.)

It will generally be expedient, to place the terms in the same order, in which they are arranged in the statement of the proportion.

1.	As	6273	*a. c.* 6.20252	2.	As	253	*a. c.* 7.59688
	Is to	769.4	2.88615		Is to	672.5	2.82769
	So is	37.61	1.57530		So is	497	2.69636
	To	4.613	0.66397		To	1321.1	3.12093
3.	As	46.34	*a. c.* 8.33404	4.	As	9.85	*a. c.* 9.00656
	Is to	892.1	2.95041		Is to	643	2.80821
	So is	7.638	0.88298		So is	76.3	1.88252
	To	147	2.16743		To	4981	3.69729

COMPOUND PROPORTION.

57. In compound, as in single proportion, the term required may be found by logarithms, if we substitute addition for multiplication, and subtraction for division.

Ex. 1. If the interest of $365, for 3 years and 9 months, be $82.13; what will be the interest of $8940, for 2 years and 6 months?

In common arithmetic, the statement of the question is made in this manner.

$$\left.\begin{array}{ll}365 & \text{dollars}\\ 3.75 & \text{years}\end{array}\right\} : \left\{\begin{array}{ll}8940 & \text{dollars}\\ 2.5 & \text{years}\end{array}\right\} :: 82.13 \text{ dollars} :$$

And the method of calculation is, to *divide* the *product* of the third, fourth, and fifth terms, by the *product* of the first two.* This, if logarithms are used, will be to *subtract* the *sum* of the logarithms of the first two terms, from the *sum* of the logarithms of the other three.

First two terms	365 log.	2.56229
	3.75	0.57403
Sum of the logarithms		3.13632
Third and fourth terms	8940	3.95134
	2.5	0.39794
Fifth term	82.13	1.91450
Sum of the logs. of the 3rd, 4th, and 5th,		6.26378
Do.	1st and 2nd,	3.13632
Term required	1341	3.12746

58. The calculation will be more simple, if, instead of *subtracting* the logarithms of the first two terms, we *add* their *arithmetical complements.* But, it must be observed, that *each* arithmetical complement increases the index of the logarithm by 10. If the arithmetical complement be introduced into *two* of the terms, the index of the sum of the logarithms will be 20 too great; if it be in *three* terms, the index will be 30 too great, &c.

First two terms	365	*a. c.*	7.43771
	3.75	*a. c.*	9.42597
Third and fourth terms	8940		3.95134
	2.5		0.39794
Fifth term	82.13		1.91450
Term required	1341		23.12746

The result is the same as before, except that the index of the logarithm is 20 too great.

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* See Arithmetic.

Ex. 2. If the wages of 53 men for 42 days be 2200 dollars; what will be the wages of 87 men for 34 days?

$$\left.\begin{array}{l}53 \text{ men}\\ 42 \text{ days}\end{array}\right\} : \left\{\begin{array}{l}87 \text{ men}\\ 34 \text{ days}\end{array}\right\} :: 2200 :$$

First two terms	53. *a. c.*	8.27572
	42. *a. c.*	8.37675
Third and fourth terms	87	1.93952
	34	1.53148
Fifth term	2200	3.34242
Term required	2923.5	3.46589

59. In the same manner, if the product of *any number* of quantities, is to be divided, by the product of several others; we may add together the logarithms of the quantities to be divided, and the arithmetical complements of the logarithms of the divisors.

Ex. If 29.67×346.2 be divided by $69.24\times7.862\times497$; what will be the quotient?

Numbers to be divided	29.67	1.47232
	346.2	2.53933
Divisors	69.24 *a. c.*	8.15964
	7.862 *a. c.*	9.10447
	497 *a. c.*	7.30364
Quotient	0.03797	8.5794

In this way, the calculations in *Conjoined Proportion* may be expeditiously performed.

COMPOUND INTEREST.

60. In calculating compound interest, the amount for the first year, is made the principal for the second year; the amount for the second year, the principal for the third

year, &c. Now the amount at the end of each year, must be proportioned to the principal at the beginning of the year. If the principal for the first year be 1 dollar, and if the amount of 1 dollar for 1 year$=a$; then, (Alg. 341.)

$$1 : a :: \begin{cases} a : a^2 = \text{the amount for the 2d year, or the principal for the 3d;} \\ a^2 : a^3 = \text{the amount for the third year, or the principal for the 4th;} \\ a^3 : a^4 = \text{the amount for the 4th year, or the principal for the 5th.} \end{cases}$$

That is, the amount of 1 dollar for any number of years is obtained by finding the amount for 1 year, and involving this to a power whose index is equal to the number of years. And the amount of any other principal, for the given time, is found by multiplying the amount of 1 dollar, into the number of dollars, or the fractional part of a dollar.

If logarithms are used, the *multiplication* required here may be performed by *addition;* and the *involution* by *multiplication.* (Art. 45.) Hence,

61. To calculate Compound Interest, *Find the amount of* 1 *dollar for* 1 *year; multiply its logarithm by the number of years; and to the product, add the logarithm of the principal.* The sum will be the logarithm of the *amount* for the given time. From the amount subtract the principal, and the remainder will be the *interest.*

If the interest becomes due *half yearly* or *quarterly;* find the amount of one dollar, for the half year or quarter, and multiply the logarithm by the number of half years or quarters in the given time.

If P=the principal,
a=the amount of 1 dollar for 1 year,
n=any number of years, and
A=the amount of the given principal for n years; then,

$$A = a^n \times P.$$

Taking the logarithms of both sides of the equation, and reducing it, so as to give the value of each of the four quantities, in terms of the others, we have

1. $\text{Log. A}=n\times\text{log. }a+\text{log. P.}$
2. $\text{Log. P}=\text{log. A}-n\times\text{log. }a.$
3. $\text{Log. }a=\dfrac{\text{log. A}-\text{log. P.}}{n}$
4. $n=\dfrac{\text{log. A}-\text{log. P.}}{\text{log }a.}$

Any three of these quantities being given, the fourth may be found.

Ex. 1. What is the amount of 20 dollars, at 6 per cent. compound interest, for 100 years?

Amount of 1 dollar for 1 year	1.06	log.	0.0253059
Multiplying by			100
			2.53059
Given principal	20		1.30103
Amount required	$6786		3.83162

2. What is the amount of 1 cent at 6 per cent. compound interest, in 500 years?

Amount of 1 dollar for 1 year	1.06	log.	0.0253059
Multiplying by			500
			12.65295
Given principal	0.01		-2.00000
Amount	$44,973,000,000		10.65295

More exact answers may be obtained, by using logarithms of a greater number of decimal places.

3. What is the amount of 1000 dollars, at 6 per cent. compound interest, for 10 years? Ans. 1790.80.

4. What principal, at 4 per cent. interest, will amount to 1643 dollars in 21 years? Ans. 721.

5. What principal, at 6 per cent., will amount to 202 dollars in 4 years? Ans. 160.

6. At what rate of interest, will 400 dollars amount to 569⅓, in 9 years? Ans. 4 per cent.

7. In how many years will 500 dollars amount to 900, at 5 per cent. compound interest? Ans. 12 years.

8. In what time will 10,000 dollars amount to 16,288, at 5 per cent compound interest? Ans. 10 years.

9. At what rate of interest, will 11,106 dollars amount to 20.000 in 15 years? Ans. 4 per cent.

10. What principal, at 6 per cent. compound interest, will amount to 3188 dollars in 8 years? Ans. $2000.

11. What will be the amount of 1200 dollars, at 6 per cent compound interest, in 10 years, if the interest is converted into principal every *half year?* Ans. 2167.3 dolls.

12. In what time will a sum of money double, at 6 per cent compound interest? Ans. 11.9 years.

13. What is the amount of 5000 dollars, at 6 per cent. compound interest, for 28¼ years? Ans. 25.942 dollars.

INCREASE OF POPULATION.

62. The natural increase of population in a country, may be calculated in the same manner as compound interest; on the supposition, that the yearly rate of increase is regularly proportioned to the actual number of inhabitants. From the population at the beginning of the year, the *rate* of increase being given, may be computed the whole increase during the year. This, *added* to the number at the beginning, will give the amount, on which the increase of the *second* year is to be calculated, in the same manner as the

first year's interest on a sum of money, added to the sum itself, gives the amount on which the interest for the second year is to be calculated.

If P=the population at the beginning of the year,

a=1+the fraction which expresses the rate of increase,

n=any number of years; and

A=the amount of the population at the end of n years;

then, as in the preceding article,

$$A=a^n \times P, \text{ and}$$

1. $\text{Log. } A = n \times \text{log. } a + \text{log. } P.$
2. $\text{Log. } P = \text{log. } A - n \times \text{log. } a.$
3. $\text{Log. } a = \dfrac{\text{log. } A - \text{log. } P.}{n}$
4. $n = \dfrac{\text{log. } A - \text{log. } P.}{\text{log. } a.}$

Ex. 1. The population of the United States in 1840 was (in round numbers) 17,070,000.* Supposing the yearly rate

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* For some very interesting views of the progress of population, &c., in the United States, see Prof. George Tucker's elaborate essays, first published in the Merchant's Magazine, 1842—3, and subsequently in a separate volume.

The following tables show the official census of the United States from 1790 to 1840 with the decennial rate of increase.

POPULATION.					
1790.	1800.	1810.	1820.	1830.	1840.
3,929,827	5,305,925	7,239,814	9,638,131	12,866,020	17,069,453

DECENNIAL INCREASE.				
1800.	1810.	1820.	1830.	1840.
35.02	36.45	33.35	33.26	33.67

of increase to be $\frac{1}{34}$ part of the whole, what will be the population in 1850?

Here P=17,070,000. n=10. $a=1+\frac{1}{34}=\frac{35}{34}$.

And log. A=10×log. $\frac{35}{34}$+log. (17,070,000,)
Therefore, A=22,810,000, the population in 1850.

2. If the number of inhabitants in a country be five millions at the beginning of a century; and if the yearly rate of increase be $\frac{1}{30}$; what will be the number at the end of 50 years? and what at the end of the century?

Ans. 25,763,000, and 132,750,000.

3. If the population of a country, at the end of a century, is found to be 45,860,000; and if the yearly rate of increase has been $\frac{1}{120}$; what was the population at the commencement of the century? Ans. 20 millions.

4. The population of the United States in 1810 was 7,240,000; in 1820, 9,625,000. What was the annual rate of increase between these two periods, supposing the increase each year to be proportioned to the population at the beginning of the year?

$$\text{Here log. } a=\frac{\text{log. } 9{,}625{,}000-\text{log. } 7{,}240{,}000}{10}$$

Therefore, $a=1.029$; and $\frac{29}{1000}$, or 2.9 per cent. is the rate of increase.

5. The population of the United States on the 1st August, 1820, was 9,638,000—in 1830, the time of taking the census was changed to the 1st June, and at that time the population was 12,866,000.—What was the annual rate of increase? And what would have been the amount of population to be added for the subsequent two months?

6. In how many years, will the population of a country advance from two millions to five millions; supposing the yearly rate of increase to be $\frac{7}{360}$? Ans. 47½ years.

7. If the population of a country, at a given time, be seven millions; and if the yearly rate of increase be $\frac{1}{25}$th; what will be the population at the end of 35 years?

8. The population of the United States in 1800 was 5,306,000. What was it in 1780, supposing the yearly rate of increase to be $\frac{1}{28}$?

9. In what time will the population of a country advance, from four millions to seven millions, if the ratio of increase be $\frac{3}{100}$?

10. What must be the rate of increase, that the population of a place may change from nine thousand to fifteen thousand, in 12 years?

If the population of a country is not affected by immigration or emigration, the rate of increase will be equal to the difference between the ratio of the *births*, and the ratio of the *deaths*, when compared with the whole population.

Ex. 11. If the population of a country, at any given time, be ten millions; and the ratio of the annual number of births to the whole population be $\frac{1}{24}$, and the ratio of deaths $\frac{1}{45}$, what will be the number of inhabitants, at the end of 60 years?

Here the yearly rate of increase $=\frac{1}{24}-\frac{1}{45}=\frac{7}{360}$.

And the population, at the end of 60 years $=31,750,000$.

The rate of increase or decrease from *immigration* or *emigration*, will be equal to the difference between the ratio of immigration and the ratio of emigration; and if this differ- be added to, or subtracted from, the difference between the ratio of the births and that of the deaths, the whole rate of increase will be obtained.

Ex. 12. If in a country, the ratio of births be $\frac{1}{30}$,
the ratio of deaths $\frac{1}{40}$,
the ratio of immigration $\frac{1}{50}$,
the ratio of emigration $\frac{1}{60}$,

and if the population this year be 10 millions, what will it be 20 years hence?

The rate of the natural increase $=\frac{1}{30}-\frac{1}{40}=\frac{1}{120}$;
That of increase from immigration $=\frac{1}{50}-\frac{1}{60}=\frac{1}{300}$;
The sum of the two is $\frac{7}{600}$;
And the population at the end of 20 years, is 12,611,000.

13. If the ratio of the births be $\frac{1}{20}$,
of the deaths $\frac{1}{30}$,
of immigration $\frac{1}{40}$,
of emigration $\frac{1}{50}$,
in what time will three millions increase to four and a half millions?

If the period in which the population will *double* be given; the numbers for several successive periods, will evidently be in a geometrical progression, of which the ratio is 2; and as the number of periods will be one less than the number of terms;

If P=the first term,
A=the last term,
n=the number of periods;
Then will $A=P\times 2^n$, (Alg. 439.)
Or log. A=log. $P+n\times$log. 2.

Ex 1. If the descendants of a single pair double once in 25 years, what will be their number at the end of one thousand years?

The number of periods here is 40.
And $A=2\times 2^{40}=2,199,200,000,000$.

2. If the descendants of Noah, beginning with his three sons and their wives, doubled once in 20 years for 300 years, what was their number, at the end of this time?

Ans. 196,608.

3. The population of the United States in 1820 being

9,638,000; what must it be in the year 2020, supposing it to double once in 25 years? Ans. 2,467,333,000.

4. Supposing the descendants of the first human pair to double once in 50 years, for 1650 years, to the time of the deluge, what was the population of the world, at that time?

EXPONENTIAL EQUATIONS.

62. An EXPONENTIAL equation is one in which the letter expressing the unknown quantity is an *exponent.*

Thus $a^x=b$, and $x^x=bc$, are exponential equations. These are most easily solved by logarithms. As the two members of an equation are equal, their logarithms must also be equal. If the logarithm of each side be taken, the equation may then be reduced, by the rules given in algebra.

Ex. What is the value of x in the equation $3^x=243$?

Taking the logarithms of both sides, log. $3^x=$log. 243.

But the logarithm of a *power* is equal to the logarithm of the root, multiplied into the index of the power. (Art. 45.)

Therefore (log. 3)$\times x=$log. 243; and dividing by log. 3.

$$x=\frac{\text{log. }243}{\text{log }3.}=\frac{2.38561}{0.47712}=5.\quad\text{So that } 3^5=243.$$

64. The exponent of a power may be itself a power, as in the equation

$$a^{m^x}=b\,;$$

where x is the exponent of the power m^x, which is the exponent of the power a^{m^x}.

Ex. 4. Find the value of x, in the equation $9^{3^x}=1000$.

$$3^x\times(\text{log. }9)=\text{log. }1000.\quad\text{Therefore, } 3^x=\frac{\text{log. }1000.}{\text{log. }9}=3.14.$$

Then, as $3^x=3.14$. $x(\text{log. }3)=\text{log. }3.14$

Therefore, $x=\dfrac{\text{log. }3.14}{\text{log. }3}=\frac{4969296}{4771213}=1.04$.

In cases like this, where the factors, divisors, &c. are logarithms, the calculation may be facilitated, by taking the *logarithms of the logarithms.* Thus the value of the fraction $\frac{4969296}{4771213}$ is most easily found, by subtracting the logarithm of the logarithm which constitutes the denominator, from the logarithm of that which forms the numerator.

5. Find the value of x, in the equation $\dfrac{ba^x+d}{c}=m$

Ans. $x=\dfrac{\text{log. }(cm-d)-\text{log. }b.}{\text{log. }a.}$

TRIGONOMETRY.

SECTION I.

SINES, TANGENTS, SECANTS, &C.

ART. 71. TRIGONOMETRY *treats of the relations of the sides and angles of* TRIANGLES. Its first object is to determine the length of the sides, and the quantity of the angles. In addition to this, from its principles are derived many interesting methods of investigation in the higher branches of analysis, particularly in physical astronomy.

72. Trigonometry is either *plane* or *spherical.* The former treats of triangles bounded by *right lines;* the latter, of triangles bounded by *arcs of circles.*

Divisions of the Circle.

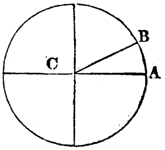
73. In a triangle there are two classes of quantities which are the subjects of inquiry, the *sides* and the *angles.* For the purpose of measuring the latter, a *circle* is introduced.

The periphery of every circle, whether great or small, is supposed to be divided into 360 equal parts called *degrees,* each degree into 60 *minutes,* each minute into 60 *seconds,* each second into 60 *thirds,* &c., marked with the characters $^\circ$, $'$, $''$, $'''$, &c. Thus, $32^\circ\ 24'\ 13''\ 22'''$ is 32 degrees, 24 minutes, 13 seconds, 22 thirds.

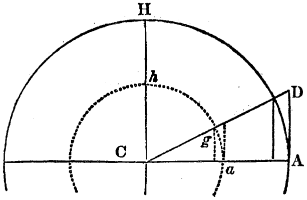
A degree, then, is not a magnitude of a given *length;* but a certain *portion* of the whole circumference of any circle. It is evident that the 360th part of a large circle is greater than the same part of a small one. On the other hand, the *number* of degrees in a small circle, is the same as in a large one.

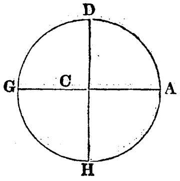
The fourth part of a circle is called a *quadrant*, and contains 90 degrees.

74. To *measure* an angle, a circle is so described that its center shall be the angular point, and its periphery shall cut the two lines which include the angle. The *arc* between the two lines is considered a *measure of the angle*, because, by Euc. 33. 6, angles at the center of a given circle, have the same ratio to each other, as the arcs on which they stand. Thus the arc AB, is a measure of the angle ACB.



It is immaterial what is the size of the circle, provided it cuts the lines which include the angle. Thus, the angle ACD is measured by either of the arcs AG, *ag*. For ACD is to ACH, as AG to AH, or as *ag* to *ah*. (Euc. 33. 6.)





75. In the circle ADGH, let the two diameters AG and DH be perpendicular to each other. The angles ACD, DCG, GCH, and HCA, will be right angles; and the periphery of the circle will be divided into four equal parts, each containing 90 degrees. As a right angle is subtended by an arc of 90°, the angle itself is said to contain 90°. Hence, in two

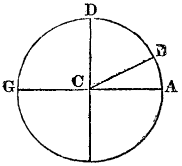
right angles, there are 180°; in four right angles, 360°; and in any other angle, as many degrees, as in the arc by which it is subtended.

76. The sum of the three angles of any triangle being equal to two right angles, (Euc. 32. 1.*) is equal to 180°. Hence, there can never be more than one obtuse angle in a triangle. For the sum of two obtuse angles is more than 180°.

77. *The* COMPLEMENT *of an arc or an angle, is the difference between the arc or angle and* 90 *degrees.*

The complement of the arc AB is DB; and the complement of the angle ACB is DCB. The complement of the arc BDG is also DB.

The complement of 10° is 80°, of 60° is 30°,
of 20° is 70°, of 120° is 30°,
of 50° is 40°, of 170° is 80°, &c.



Hence, an acute angle and its complement are always equal to 90°. The angles ACB and DCB are together equal to a right angle. The two acute angles of a right angled triangle are equal to 90°: therefore each is the complement of the other.

78. *The* SUPPLEMENT *of an arc or an angle is the difference between the arc or angle and* 180 *degrees.*

The supplement of the arc BDG is AB; and the supplement of the angle BCG is BCA.

The supplement of 10° is 170°, of 120° is 60°,
of 80° is 100°, of 150° is 30°, &c.

Hence an angle and its supplement are always equal to

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* Thomson's Geometry, 28, 1.

180°. The angles BCA and BCG are together equal to two right angles.

79. Cor. As the three angles of a plane triangle are equal to two right angles, that is, to 180° (Euc. 32. 1.) the sum of any two of them is the supplement of the other. So that the third angle may be found, by subtracting the sum of the other two from 180°. Or the sum of any two may be found, by subtracting the third from 180°.

80. A straight line drawn from the centre of a circle to any part of the periphery, is called a *radius* of the circle. In many calculations, it is convenient to consider the *radius*, whatever be its length, as *a unit.* (Alg. 510.) To this must be referred the numbers expressing the lengths of other lines. Thus, 20 will be twenty times the radius, and 0.75, three-fourths of the radius.

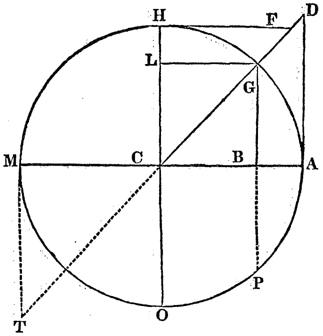
Definitions of Sines, Tangents, Secants, &c.

81. To facilitate the calculations in Trigonometry, there are drawn, within and about the circle, a number of straight lines, called *Sines, Tangents, Secants, &c.* With these the learner should make himself perfectly familiar. The direct and proper measure of an angle is an *arc* of a circle. (Art. 74.) But trigonometrical solutions are commonly made with the aid of certain *straight* lines, which have known relations to the arcs to which they belong.

82. *The* SINE *of an arc is a straight line drawn from one end of the arc, perpendicular to a diameter which passes through the other end.*

Thus, BG is the sine of the arc AG. For BG is a line drawn from the end G of the arc, perpendicular to the diameter AM which passes through the other end A of the arc.

Cor. The sine is *half the chord* of *double the arc.* The sine BG is half PG, which is the chord of the arc PAG, double the arc AG.



83. *The* VERSED SINE *of an arc is that part of the diameter which is between the sine and the arc.*

Thus, BA is the versed sine of the arc AG.

84. *The* TANGENT *of an arc, is a straight line drawn perpendicularly from the extremity of the diameter which passes through one end of the arc, and extended till it meets a line drawn from the centre through the other end.*

Thus, AD is the tangent of the arc AG.

85. *The* SECANT *of an arc is a straight line drawn from the centre, through one end of the arc, and extended to the tangent which is drawn from the other end.*

Thus CD is the secant of the arc AG.

86. In Trigonometry, the terms *tangent* and *secant* have a more limited meaning, than in Geometry. In both, indeed, the tangent *touches* the circle, and the secant *cuts* it. But in Geometry, these lines are of no determinate length; whereas, in Trigonometry, they extend from the diameter to the point in which they intersect each other.

87. The lines just defined are sines, tangents, and secants of *arcs.* BG is the sine of the arc AG. But this arc subtends the *angle* GCA. BG is then the sine of the arc which subtends the *angle* GCA. This is more concisely expressed, by saying that BG is the sine of the angle GCA. And universally, the sine, tangent, and secant of an *arc,* are said to be the sine, tangent, and secant of the *angle* which stands at the centre of the circle, and is subtended by the arc. Whenever, therefore, the sine, tangent, or secant of an angle is spoken of; we are to suppose a circle to be

drawn whose centre is the angular point; and that the lines mentioned belong to that arc of the periphery which subtends the angle.

88. The *sine* and *tangent* of an acute angle, are *opposite* to the angle. But the *secant* is one of the lines which *include* the angle. Thus, the sine BG, and the tangent AD, are opposite to the angle DCA. But the secant CD is one of the lines which include the angle.

89. *The sine complement or* COSINE *of an angle, is the sine of the* COMPLEMENT *of that angle.* Thus, if the diameter HO be perpendicular to MA, the angle HCG is the complement of ACG; (Art. 77.) and LG, or its equal CB, is the sine of HCG. (Art. 82.) It is, therefore, the *cosine* of GCA. On the other hand, GB is the sine of GCA, and the cosine of GCH.

So also the *cotangent* of an angle is the tangent of the *complement* of the angle. Thus, HF is the cotangent of GCA. And the *cosecant* of an angle is the secant of the *complement* of the angle. Thus, CF is the cosecant of GCA.

Hence, as in a right angled triangle, one of the acute angles is the complement of the other; (Art. 77.) the sine, tangent, and secant of one of these angles, are the cosine, cotangent, and cosecant of the other.

90. The sine, tangent, and secant of the *supplement* of an angle, are each equal to the sine, tangent, and secant of the angle itself. It will be seen, by applying the definition. (Art 82.) to the figure,. that the sine of the obtuse angle GCM is BG, which is also the sine of the acute angle GCA. It should be observed, however, that the sine of an acute angle is *opposite* to it; while the sine of an obtuse angle *falls without* the angle, and is opposite to its supplement. Thus BG, the sine of the angle MCG, is not opposite to MCG, but to its supplement ACG.

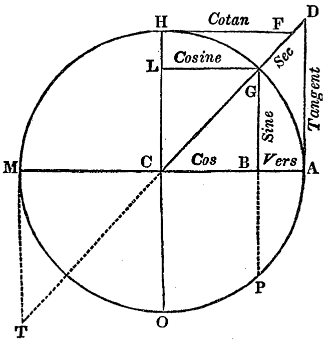
The *tangent* of the obtuse angle MCG is MT, or its equal

AD, which is also the tangent of ACG. And the *secant* of MCG is CD, which is also the secant of ACG.

91. But the *versed sine* of an angle *is not the same* as that of its *supplement.* The versed sine of an *acute* angle is equal to the *difference* between the cosine and radius. But the versed sine of an *obtuse* angle is equal to the *sum* of the cosine and radius. Thus, the versed sine of ACG is $AB=AC-BC$. (Art. 83.) But the versed sine of MCG is $MB=MC+BC$.

Relations of Sines, Tangents, Secants, &c., to each other.

92. The relations of the sine, tangent, secant, cosine, &c., to each other, are easily derived from the proportions of the sides of similar triangles. (Euc. 4. 6.*) In the quadrant ACH, these lines form three similar triangles, viz. ACD, BCG or LCG, and HCF. For, in each of these, there is one right angle, because the sines and tangents are, by definition, perpendicular to AC; as the cosine and cotangent are to CH. The lines CH, BG, and AD, are parallel, because CA makes a right angle with each. (Euc. 27. 1.†) For the same reason, CA, LG, and HF, are parallel. The alternate angles GCL, BGC, and the opposite angle CDA, are equal; (Euc. 29. 1.‡) as are also the angles GCB, LGC, and HFC. The triangles ACD, BCG, and HCF, are therefore similar.



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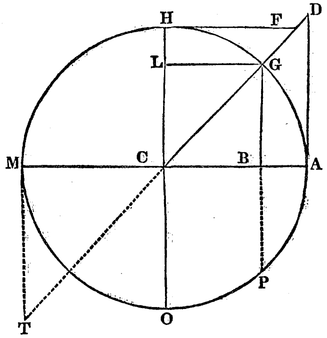
* Thomson, 18. 4. † Ibid. 18. 1. ‡ Ibid. 24. 1.

It should also be observed, that the line BC, between the sine and the centre of the circle, is parallel and equal to the cosine; and that LC, between the cosine and centre, is parallel and equal to the sine; (Euc. 34. 1.*) so that one may be taken for the other in any calculation.

93. From these similar triangles, are derived the following proportions; in which R is put for radius,



sin for sine,
cos for cosine,
tan for tangent,
cot for cotangent,
sec for secant,
cosec for cosecant.

By comparing the triangles CBG and CAD,

1. AC : BC : : AD : BG, that is, R : cos : : tan : sin.
2. CG : CD : : BG : AD R : sec : : sin : tan.
3. CB : CA : : CG : CD cos : R : : R : sec.

Therefore $R^2 = \cos \times \sec$.

By comparing the triangles CLG and CHF,

4. CH : CL : : HF : LG, that is, R : sin : : cot : cos.
5. CG : CF : : LG : HF, R : cosec : : cos : cot.
6. CL : CH : : CG : CF sin : R : : R : cosec.

Therefore $R^2 = \sin \times \mathrm{cosec}$.

By comparing the triangles CAD and CHF,

7. CH : AD : : CF : CD, that is, R : tan : : cosec : sec.
8. CA : HF : : CD : CF R : cot : : sec : cosec.
9. AD : AC : : CH : HF tan : R : : R : cot.

Therefore $R^2 = \tan \times \cot$.

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* Thomson, 26. 1.

It will not be necessary for the learner to commit these proportions to memory. But he ought to make himself so familiar with the manner of stating them from the figure, as to be able to explain them, whenever they are referred to.

94. Other relations of the sine, tangent, &c., may be derived from the proposition, that the square of the hypothenuse is equal to the sum of the squares of the perpendicular sides. (Euc. 47. 1.—Thomson 11. 4.)

In the right angled triangles CBG, CAD, and CHF,

1. $\overline{CG}^2=\overline{CB}^2+\overline{BG}^2$, that is, $R^2=\cos^2+\sin^2$,*
2. $\overline{CD}^2=\overline{CA}^2+\overline{AD}^2$ $\qquad \sec^2=R^2+\tan^2$,
3. $\overline{CF}^2=\overline{CH}^2+\overline{HF}^2$ $\qquad \text{cosec}^2=R^2+\cot^2$,

And, extracting the root of both sides, (Alg. 296.)

$$R=\sqrt{\cos^2+\sin^2}=\sqrt{\sec^2-\tan^2}=\sqrt{\text{cosec}^2-\cot^2}$$

Hence, if R=1, (Alg 385.)

$$\text{Sin}=\sqrt{1-\cos^2} \qquad \text{Sec}=\sqrt{1+\tan^2}$$
$$\text{Cos}=\sqrt{1-\sin^2} \qquad \text{Cosec}=\sqrt{1+\cot^2}$$

95. *The sine of* 90°, *The chord of* 60°, *And the tangent of* 45° } *are,* in any circle, *each equal to the radius,* and therefore equal to each other.

Demonstration.

1. In the quadrant ACH, (figure on the next page,) the arc AH is 90°. The sine of this, according to the definition, (Art. 82.) is CH, the radius of the circle.

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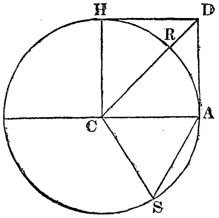
* Sin² is here put for the square of the sine, cos² for the square of the cosine, &c.



2. Let AS be an arc of 60°. Then the angle ACS, being measured by this arc, will also contain 60°; (Art. 75.) and the triangle ACS will be equilateral. For the sum of the three angles is equal to 180°. (Art. 76.) From this, taking the angle ACS, which is 60°, the sum of the remaining two is 120°. But these two are *equal,* because they are subtended by the equal sides, CA and CS, both radii of the circle. Each, therefore, is equal to *half* 120°, that is, to 60°. All the angles being equal, the sides are equal, and therefore AS, the chord of 60°, is equal to CS, the radius.

3. Let AR be an arc of 45°. AD will be its tangent, and the angle ACD subtended by the arc, will contain 45°. The angle CAD is a right angle, because the tangent is, by definition, perpendicular to the radius AC. (Art. 84.) Subtracting ACD, which is 45°, from 90°, (Art. 77.) the other acute angle ADC will be 45° also. Therefore the two legs of the triangle ACD are equal, because they are subtended by equal angles; (Euc. 6. 1.) that is, AD the tangent of 45°, is equal to AC the radius.

Cor. The *cotangent* of 45° is also equal to radius. For the complement of 45° is itself 45°. Thus, HD, the cotangent of ACD, is equal to AC the radius.

96. The sine of 30° is equal to *half radius.* For the sine of 30° is equal to half the chord of 60°. (Art. 82. cor.) But by the preceding article, the chord of 60° is equal to radius. Its half, therefore, which is the sine of 30°, is equal to half radius.

Cor. 1. The *cosine* of 60° is equal to half radius. For the cosine of 60° is the sine of 30°. (Art. 89.)

Cor. 2. The cosine of $30° = \frac{1}{2}\sqrt{3}$. For

$$\text{Cos}^2\,30° = R^2 - \sin^2 30° = 1 - \tfrac{1}{4} = \tfrac{3}{4}.$$

Therefore,

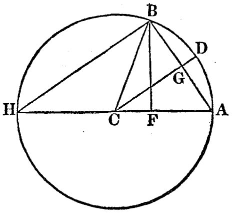
$$\text{Cos } 30^\circ = \sqrt{\tfrac{3}{4}} = \tfrac{1}{2}\sqrt{3}.$$

96. *b*. The sine of $45^\circ = \frac{1}{\sqrt{2}}$. For

$$R^2 = 1 = \sin^2 45^\circ + \cos^2 45 = 2 \sin^2 45^\circ$$

Therefore, $\text{Sin } 45^\circ = \sqrt{\tfrac{1}{2}} = \frac{1}{\sqrt{2}}$.

97. The *chord* of any arc is a *mean proportional*, between the *diameter* of the circle, and the *versed sine* of the arc.



Let ADB, be an arc, of which AB is the chord, BF the sine, and AF the versed sine. The angle ABH is a right angle, (Euc. 31. 3.*) and the triangles ABH, and ABF, are similar. (Euc. 8. 6.†) Therefore,

$$AH : AB :: AB : AF.$$

That is, the diameter is to the chord, as the chord to the versed sine.

Let the arc AD$=a$, and ADB$=2a$. Draw BF perpendicular to AH. This will divide the right angled triangle ABH into two similar triangles. (Euc. 8. 6.) The angles ACD and AHB are equal. (Euc. 20. 3.‡) Therefore the four triangles ACG, AHB, FHB, and FAB are similar; and the line BH is twice CG, because BH : CG : : HA : CA.

The sides of the four triangles are,

AG=sin a,	CG=cos a.	HF=vers. sup. $2a$,
AB=2 sin a,	BH=2 cos a.	AC=the radius,
BF=sin $2a$,	AF=vers $2a$,	AH=the diameter.

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[*] Thomson, 13. 2. Cor. 2. [†] Ibid. 22. 4. [‡] Ibid. 13. 2.

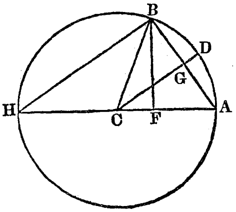
A variety of proportions may be stated, between the homologous sides of these triangles: For instance,

By comparing the triangles ACG and ABF,

$AC : AG :: AB : AF$, that is, $R : \sin a :: 2 \sin a : \text{vers } 2a$
$AC : CG :: AB : BF$, $\quad R : \cos a :: 2 \sin a : \sin 2a$
$AG : CG :: AF : BF$, $\quad \text{Sin } a : \cos a :: \text{vers } 2a : \sin 2a$

Therefore,

$$R \times \text{vers } 2a = 2\sin^2 a$$
$$R \times \sin 2a = 2\sin a \times \cos a$$
$$\text{Sin } a \times \sin 2a = \text{vers } 2a \times \cos a$$



By comparing the triangles ACG and BFH,

$AC : CG :: BH : HF$, that is, $R : \cos a :: 2\cos a : \text{vers. sup. } 2a$
$AG : CG :: BF : HF$, $\quad \text{Sin } a : \cos a :: \sin 2a : \text{vers. sup. } 2a$

Therefore,

$$R \times \text{vers. sup. } 2a = 2 \cos^2 a$$
$$\text{Sin } a \times \text{vers. sup. } 2a = \cos a \times \sin 2a$$

&c. &c.

That is, the product of radius into the versed sine of the supplement of twice a given arc, is equal to twice the square of the cosine of the arc.

And the product of the sine of an arc, into the versed sine of the supplement of twice the arc, is equal to the product of the cosine of the arc, into the sine of twice the arc, &c., &c.

SECTION II.

THE TRIGONOMETRICAL TABLES.

Art. 98. To facilitate the operations in trigonometry, the sine, tangent, secant, &c., have been calculated for every degree and minute, and in some instances, for every second, of a quadrant, and arranged in tables. These constitute what is called the *Trigonometrical Canon.* It is not necessary to extend these tables beyond 90°; because the sines, tangents, and secants, are of the same magnitude, in one of the quadrants of a circle, as in the others. Thus the sine of 30° is equal to that of 150°. (Art. 90.)

99. And in any instance, if we have occasion for the sine, tangent, or secant of an *obtuse angle*, we may obtain it, by looking for its equal, the sine, tangent, or secant of the *supplementary acute angle.*

100. The tables are calculated for a circle whose radius is supposed to be a *unit.* It may be an inch, a yard, a mile, or any other denomination of length. But the *sines, tangents, &c.*, must always be understood to be of the same denomination as the radius.

101. All the *sines*, except that of 90°, are *less than radius*, (Art. 82.) and are expressed in the tables by decimals.

Thus the sine of 20° is 0.34202, of 60° is 0.86603,
of 40° is 0.64279, of 89° is 0.99985, &c.

When the tables are intended to be very exact, the decimal is carried to a greater number of places.

The *tangents* of all angles less than 45° are also less than radius. (Art. 95.) But the tangents of angles greater than 45°, are *greater* than radius, and are expressed by a whole number and a decimal. It is evident that all the *secants* also must be greater than radius, as they extend from the centre, to a point without the circle.

102. The numbers in the table here spoken of, are called *natural* sines, tangents, &c. They express the lengths of the several lines which have been defined in Arts. 82, 83, &c. By means of them, the angles and sides of triangles may be accurately determined. But the calculations must be made by the tedious processes of multiplication and division. To avoid this inconvenience, another set of tables has been provided, in which are inserted the *logarithms* of the natural sines, tangents, &c. By the use of these, addition and subtraction are made to perform the office of multiplication and division. On this account, the tables of logarithmic, or as they are sometimes called, *artificial* sines, tangents, &c., are much more valuable, for practical purposes, than the *natural* sines, &c. Still it must be remembered that the former are derived from the latter. The artificial sine of an angle, is the logarithm of the natural sine of that angle. The artificial tangent is the logarithm of the natural tangent, &c.

103. One circumstance, however, is to be attended to, in comparing the two sets of tables. The radius to which the *natural* sines, &c., are calculated, is *unity*. (Art. 100.) The secants, and a part of the tangents are, therefore, *greater* than a unit; while the sines, and another part of the tangents, are *less* than a unit. When the logarithms of these are taken, some of the indices will be *positive*, and others *negative*; (Art. 9.) and the throwing of them together in the same table, if it does not lead to error, will at least be attended with inconvenience. To remedy this, 10 is added to each of the indices. (Art. 12.) They are then all positive. Thus the natural sine of 20° is 0.34202. The logarithm of this is $\bar{1}.53405$. But the index, by the addition of 10, be-

comes 10—1=9. The logarithmic sine in the tables is therefore 9.53405.*

Directions for taking Sines, Cosines, &c., from the tables.

104. The *cosine, cotangent,* and *cosecant* of an angle, are the sine, tangent, and secant of the *complement* of the angle. (Art. 89.) As the complement of an angle is the difference between the angle and 90°, and as 45 is the half of 90; if any given angle within the quadrant is greater than 45°, its complement is less; and, on the other hand, if the angle is *less* than 45°, its complement is greater. Hence, every cosine, cotangent, and cosecant of an angle greater than 45°, has its equal among the sines, tangents, and secants of angles less than 45°, and *v. v.*

Now, to bring the trigonometrical tables within a small compass, the same column is made to answer for the *sines* of a number of angles *above* 45°, and for the *cosines* of an equal number *below* 45°.

Thus 9.23967 is the log. *sine* of 10°, and the *cosine* of 80°,
9.53405 the *sine* of 20°, and the *cosine* of 70°, &c.

The tangents and secants are arranged in a similar manner. Hence,

105. *To find the Sine, Cosine, Tangent, &c., of any number of degrees and minutes.*

If the given angle is *less* than 45°, look for the degrees at the *top* of the table, and the minutes on the *left;* then, opposite to the minutes, and under the word sine at the head of the column, will be found the sine; under the word tangent, will be found the tangent, &c.

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* Or the tables may be supposed to be calculated to the radius 10000000000, whose logarithm is 10.

The log. sin	of 43° 25′ is	9.83715	The tan	of 17° 20′ is	9.49430
	of 17° 20′	9.47411		of 8° 46′	9.18812
The cos	of 17° 20′	9.97982	The cot	of 17° 20′	10.50570
	of 8° 46′	9.99490		of 8° 46′	10.81188

The first figure is the index; and the other figures are the decimal part of the logarithm.

106. If the given angle is between 45° and 90°; look for the degrees at the *bottom* of the table, and the minutes on the *right;* then, opposite to the minutes, and *over* the word sine at the foot of the column, will be found the sine; over the word tangent, will be found the tangent, &c.

Particular care must be taken, when the angle is less than 45°, to look for the title of the column, at the *top,* and for the minutes on the *left;* but when the angle is between 45° and 90°, to look for the title of the column at the *bottom,* and for the minutes, on the *right.*

The log. sine	of 81° 21′ is	9.99503
The cosine	of 72° 10′	9.48607
The tangent	of 54° 40′	10.14941
The cotangent	of 63° 22′	9.70026

107. If the given angle is *greater* than 90°, look for the sine, tangent, &c., of its *supplement.* (Art. 98, 99.)

The log. sine of	96° 44′ is	9.99699
The cosine of	171° 16′	9.99494
The tangent of	130° 26′	10.06952
The cotangent of	156° 22′	10.35894

108. *To find the sine, cosine, tangent, &c., of any number of degrees, minutes, and* SECONDS.

In the common tables, the sine, tangent, &c., are given only to every *minute* of a degree.[*] But they may be found to *seconds,* by taking *proportional parts* of the difference of the numbers as they stand in the tables. For, within a single minute, the variations in the sine, tangent, &c., are nearly proportional to the variations in the angle. Hence,

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[*] In the very valuable tables of Michael Taylor, the sines and tangents are given to *every second.*

To find the sine, tangent, &c., to seconds: Take out the number corresponding to the given degree and minute; and also that corresponding to the next greater minute, and find their difference. Then state this proportion;

As 60, to the given number of seconds;

So is the difference found, to the correction for the seconds.

This correction, in the case of sines, tangents, and secants, is to be *added* to the number answering to the given degree and minute; but for cosines, cotangents, and cosecants, the correction is to be *subtracted*;

For, as the sines *increase*, the cosines *decrease*.

Ex. 1. What is the logarithmic sine of 14° 43′ 10″?

The sine of 14° 43′ is	9.40490
of 14° 44′	9.40538
Difference	48

Here it is evident that the sine of the required angle is greater than that of 14° 43′, but less than that of 14° 44′. And as the difference corresponding to a whole minute or 60″ is 48; the difference for 10″ must be a proportional part of 48. That is,

$$60'' : 10'' :: 48 : 8$$

the correction to be *added* to the sine of 14° 43′.

Therefore the sine of 14° 43′ 10″ is 9.40498.

2. What is the logarithmic cosine of 32° 16′ 45″?

The cosine of 32° 16′ is	9.92715
of 32° 17′	9.92707
Difference	8

Then, $60'' : 45'' :: 8 : 6$ the correction to be *subtracted* from the cosine of 32° 16′.

Therefore the cosine of 32° 16′ 45″ is 9.92709.

The tangent of	24° 15′ 18″	is	9.65376
The cotangent of	31° 50′ 5″	is	10.20700
The sine of	58° 14′ 32″	is	9.92956
The cosine of	55° 10′ 26″	is	9.75670

If the given number of seconds be any even part of 60, as $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, &c., the correction may be found, by taking a like part of the difference of the numbers in the tables, without stating a proportion in form.

109. *To find the degrees and minutes belonging to any given sine, tangent, &c.*

This is reversing the method of finding the sine, tangent, &c,. (Art. 105, 6, 7.)

Look in the column of the same name, for the sine, tangent, &c., which is *nearest* to the given one; and if the title be at the *head* of the column, take the degrees at the *top* of the table, and the minutes on the *left;* but if the title be at the *foot* of the column, take the degrees at the *bottom,* and the minutes on the *right.*

Ex. 1. What is the number of degrees and minutes belonging to the logarithmic sine 9.62863?

The nearest sine in the tables is 9.62865. The title of sine is at the head of the column in which these numbers are found. The degrees at the top of the page are 25, and the minutes on the left are 10. The angle required is, therefore 25° 10′.

The angle belonging to

the sine	9.87993 is	49° 20′	the cos	9.97351	is	19° 48′
the tan	9.97955	43° 39′	the cotan	9.75791		60° 12′
the sec	10.65396	77° 11′	the cosec	10.49066		18° 51′

110. *To find the degrees, minutes, and* SECONDS, *belonging to any given sine, tangent, &c.*

This is reversing the method of finding the sine, tangent, &c., to seconds. (Art. 108.)

First find the difference between the sine, tangent, &c., next greater than the given one, and that which is next less; then the difference between this less number and the given one; then

As the difference first found, is to the other difference;

So are 60 seconds, to the number of seconds, which, in the case of sines, tangents, and secants, are to be *added* to the degrees and minutes belonging to the least of the two numbers taken from the tables; but for cosines, cotangents, and cosecants are to be *subtracted.*

Ex. 1. What are the degrees, minutes, and seconds, belonging to the logarithmic sine 9.40498?

Sine next greater	14° 44′	9.40538	Given sine	9.40498
Next less	14° 43′	9.40490	Next less	9.40490
Difference		48	Difference	8

Then, 48 : 8 : : 60″ : 10″, which added to 14° 43′, gives 14° 43′ 10″ for the answer.

2. What is the angle belonging to the cosine 9.09773?

Cosine next greater	82° 48′	9.09807	Given cosine	9.09773
Next less	82° 49′	9.09707	Next less	9.09707
Difference		100	Difference	66

Then, 100 : 66 : : 60″ : 40″, which subtracted from 82° 49′, gives 82° 48′ 20″ for the answer.

It must be observed here, as in all other cases, that of the two angles, the less has the greater cosine.

The angle belonging to

the sin 9.20621 is	9° 15′ 6″	the tan 10.43434 is	69° 48′ 16″
the cos 9.98157	16° 34′ 30″	the cot 10.33554	24° 47′ 16″

Method of Supplying the Secants and Cosecants.

111. In some trigonometrical tables, the secants and cosecants are not inserted. But they may be easily obtained from the sines and cosines. For, by Art. 93, proportion 3d,

$$\cos \times \sec = R^2.$$

That is, the product of the cosine and secant, is equal to the square of radius. But, in logarithms, addition takes the place of multiplication; and, in the tables of logarithmic sines, tangents, &c., the radius is 10. (Art. 103.) Therefore, in these tables,

$$\cos + \sec = 20. \quad \text{Or } \sec = 20 - \cos.$$

Again, by Art 93, proportion 6,

$$\sin \times \operatorname{cosec} = R^2.$$

Therefore, in the tables,

$$\sin + \operatorname{cosec} = 20. \quad \text{Or, } \operatorname{cosec} = 20 - \sin. \quad \text{Hence,}$$

112. To obtain the *secant*, subtract the cosine from 20; and to obtain the *cosecant*, subtract the sine from 20.

These subtractions are most easily performed, by taking the right hand figure from 10, and the others from 9, as in finding the arithmetical complement of a logarithm; (Art. 55.) observing, however, to add 10 to the index of the secant or cosecant. In fact the secant is the arithmetical complement of the cosine, with 10 added to the index.

For the secant $= 20 - \cos$.
And the arith. comp. of cos $= 10 - \cos$. (Art. 54.)

So also the cosecant is the arithmetical complement of the sine, with 10 added to the index. The tables of secants and cosecants are, therefore, of use, in furnishing the arithmetical complement of the sine and cosine, in the following simple manner:

113. For the arithmetical complement of the *sine*, subtract 10 from the index of the cosecant; and for the arithmetical complement of the *cosine*, subtract 10 from the index of the secant.

By this, we may save the trouble of taking each of the figures from 9.

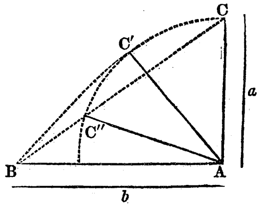
SECTION III.

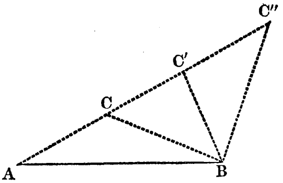
SOLUTIONS OF RIGHT ANGLED TRIANGLES.

ART. 114. In a triangle there are *six parts*, three sides, and three angles. In every trigonometrical calculation, it is necessary that some of these should be known, to enable us to find the others. *The number of parts which must be given, is* THREE, *one of which must be a* SIDE.

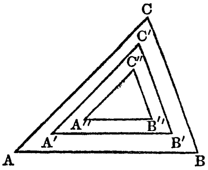
If only two parts be given, they will be either two sides, a side and an angle, or two angles; neither of which will limit the triangle to a particular form and size.

If *two sides* only be given, they may make any angle with each other; and may, therefore, be the sides of a thousand different triangles. Thus, the two lines a and b may belong either to the triangle ABC, or ABC′, or ABC″. So that it will be impossible, from knowing two of the sides of a triangle, to determine the other parts.





Or, if a *side and an angle* only be given, the triangle will be indeterminate. Thus, if the side AB and the angle at A be given; they may be parts either of the triangle ABC, or ABC′, or ABC″.

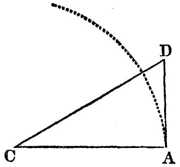


Lastly, if two *angles,* or even if *all* the angles be given, they will not determine the length of the sides. For the triangles ABC, A′B′C′, A″B″C″, and a hundred others which might be drawn, with sides parallel to these, will all have the same angles. So that one of the parts given must always be a side. If this and any other two parts, either sides or angles, be known, the other three may be found, as will be shown, in this and the following section.

115. Triangles are either *right angled* or *oblique angled.* The calculations of the former are the most simple, and those which we have the most frequent occasion to make. A great portion of the problems in the mensuration of heights and distances, in surveying, navigation and astronomy, are solved by rectangular trigonometry. Any triangle whatever may be divided into two right angled triangles, by drawing a perpendicular from one of the angles to the opposite side.

116. One of the six parts in a right angled triangle, is always given, viz. the right angle. This is a *constant* quantity; while the other angles and the sides are variable. It is also to be observed, that, if one of the *acute* angles is given, the other is known of course. For one is the complement of the other. (Art. 76, 77.) So that, *in a right angled triangle, subtracting one of the acute angles from* 90° *gives the*

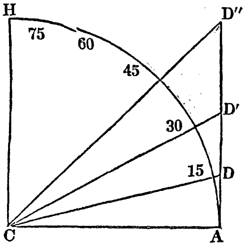
other. There remain, then, only *four* parts, one of the acute angles, and the three sides, to be sought by calculation. If any *two* of these be given, with the right angle, the others may be found.



117. To illustrate the method of calculation, let a case be supposed in which a right angled triangle CAD, has one of its sides equal to the radius to which the trigonometrical tables are adapted.

In the first place, let the *base* of the triangle be equal to the tabular radius. Then, if a circle be described, with this radius, about the angle C as a centre, DA will be the *tangent*, and DC the *secant* of that angle. (Art. 84, 85.) So that the radius, the tangent, and the secant of the angle at C, constitute the three sides of the triangle. The *tangent*, taken from the tables of natural sines, tangents, &c., will be the length of the *perpendicular ;* and the *secant* will be the length of the *hypothenuse.* If the tables used be logarithmic, they will give the *logarithms* of the lengths of the two sides.

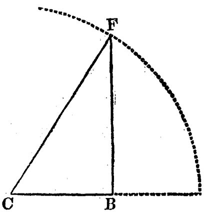
11

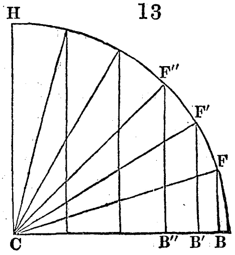


In the same manner, *any* right angled triangle whatever, whose base is equal to the radius of the tables, will have its other two sides found among the tangents and secants. Thus, if the quadrant AH, be divided into portions of 15° each; then, in the triangle

CAD, AD will be the tan, and CD the sec of 15°,
In CAD′, AD′ will be the tan, and CD′ the sec of 30°,
In CAD″, AD″ will be the tan, and CD″ the sec of 45°, &c.

118. In the next place, let the *hypothenuse* of a right angled triangle CBF, be equal to the radius of the tables. Then, if a circle be described, with the given radius, and about the angle C as a centre; BF will be the *sine*, and BC the *cosine* of that angle. (Art. 82, 89.) Therefore the sine of the angle at C, taken from the tables, will be the length of the *perpendicular*, and the cosine will be the length of the *base*.



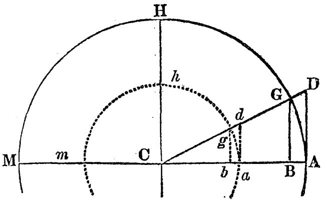


And any right angled triangle whatever, whose hypothenuse is equal to the tabular radius, will have its other two sides found among the sines and cosines. Thus, if the quadrant AH, be divided into portions of 15° each in the points F, F′, F″, &c.; then, in the triangle,

CBF, FB will be the sin, and CB the cos, of 15°,
In CB′F′, F′B′ will be the sin, and CB′ the cos, of 30°,
In CB″F″, F″B″ will be the sin, and CB″ the cos, of 45°, &c.

119. By merely *turning to the tables*, then, we may find the parts of any right angled triangle which has one of its sides equal to the radius of the tables. But for determining the parts of triangles which have *not* any of their sides equal to the tabular radius, the following proportion is used;

As the radius of one circle,
To the radius of any other;
So is a sine, tangent, or secant, in one,
To the sine, tangent, or secant, of the same number of degrees, in the other.



In the two concentric circles AHM, *ahm*, the arcs AG and *ag*, contain the same number of degrees. (Art. 74.) The sines of these arcs are BG and *bg*, the tangents AD and *ad*, and the secants CD and C*d*. The four triangles, CAD, CBG, C*ad*, and C*bg*, are similar. For each of them, from the nature of sines and tangents, contains one right angle; the angle at C is common to them all; and the other acute angle in each is the complement of that at C. (Art. 77.) We have, then, the following proportions. (Euc. 4. 6.*)

1. CG : C*g* : : BG : *bg*.

That is, one radius is to the other, as one *sine* to the other.

2. CA : C*a* : : DA : *da*.

That is, one radius is to the other, as one *tangent* to the other.

3. CA : C*a* : : CD : C*d*.

That is, one radius is to the other, as one *secant* to the other.

Cor. BG : *bg* : : DA : *da* : : CD : C*d*.

That is, as the sine in one circle, to the sine in the other; so is the tangent in one, to the tangent in the other; and so is the secant in one, to the secant in the other.

This is a general principle, which may be applied to most trigonometrical calculations. If one of the sides of the proposed triangle be made radius, each of the other sides will be the sine, tangent, or secant, of an arc described by this radius. Proportions are then stated, between these lines, and the *tabular* radius, sine, tangent, &c.

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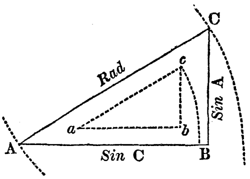
* Thomson 18. 4.

120. A line is said to be *made radius,* when a circle is described, or supposed to be described, whose semi-diameter is equal to the line, and whose centre is at one end of it.

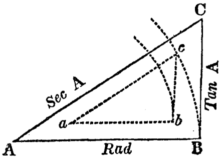
121. In any right angled triangle, *if the* HYPOTHENUSE *be made radius, one of the legs will be a* SINE *of its opposite angle, and the other leg a* COSINE *of the same angle.*

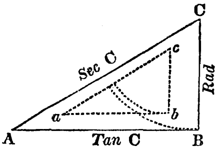


Thus, if to the triangle ABC a circle be applied whose radius is AC, and whose centre is A, then BC will be the *sine,* and BA the *cosine,* of the angle at A. (Art. 82, 89.)

If, while the same line is radius, the other end C be made the centre, then BA will be the *sine,* and BC the *cosine,* of the angle at C.

122. *If either of the* LEGS *be made radius, the other leg will be a* TANGENT *of its opposite angle, and the hypothenuse will be a* SECANT *of the same angle ;* that is, of the angle between the secant and the radius.





Thus, if the *base* AB (Fig. 15.) be made radius, the centre being at A, BC will be the *tangent,* and AC the *secant,* of the angle at A. (Art. 84. 85.)

But, if the *perpendicular* BC, (Fig. 16.) be made radius, with the centre at C, then AB will be the *tangent,* and AC the *secant,* of the angle at C.

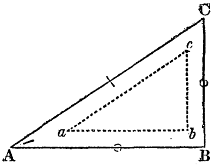
123. As the side which is the sine, tangent, or secant of one of the acute angles, is the cosine, cotangent, or cose-

cant of the other; (Art. 89.) the *perpendicular* BC (Fig. 14.) is the *sine* of the angle A, and the *cosine* of the angle C; while the *base* AB, is the *sine* of the angle C, and the *cosine* of the angle A.

If the base is made radius, as in Fig 15, the *perpendicular* BC is the *tangent* of the angle A, and the *cotangent* of the angle C; while the *hypothenuse* is the *secant* of the angle A, and the *cosecant* of the angle C.

If the perpendicular is made radius, as in Fig. 16, the *base* AB is the *tangent* of the angle C, and the *cotangent* of the angle A; while the *hypothenuse* is the *secant* of the angle C, and the *cosecant* of the angle A.

124. Whenever a right angled triangle is proposed, whose sides or angles are required; a *similar* triangle may be formed, from the sines, tangents, &c., of the *tables*. (Art. 117, 118.) The parts required are then found, by stating proportions between the similar sides of the two triangles. If the triangle proposed be ABC, (Fig. 17.) another *abc* may be formed, having the same angles with the first, but differing from it in the length of its sides, so as to correspond with the numbers in the tables. If similar sides be made radius in both, the remaining similar sides will be lines of *the same name;* that is, if the perpendicular in one of the triangles be a *sine*, the perpendicular in the other will be a sine; if the base in one be a *cosine*, the base in the other will be a cosine, &c.



If the *hypothenuse* in each triangle be made radius, as in Fig. 14, the perpendicular *bc*, will be the *tabular sine* of the angle at *a;* and the perpendicular BC, will be a sine of the equal angle A, in a circle of which AC is radius.

If the *base* in each triangle be made radius, as in Fig. 15, then the perpendicular *bc*, will be the *tabular tangent* of the angle at *a*; and BC will be a tangent of the equal angle A, in a circle of which AB, is radius, &c.

125. From the relations of the similar sides of these triangles, are derived the two following *theorems*, which are sufficient for calculating the parts of any right angled triangle whatever, when the requisite data are furnished. One is used, when a *side* is to be found; the other, when an *angle* is to be found.

THEOREM I.

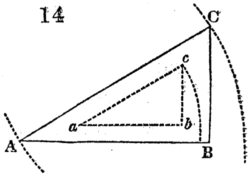
126. When a *side* is required;

> AS THE TABULAR SINE, TANGENT, &C., OF THE SAME NAME WITH THE GIVEN SIDE,
> TO THE GIVEN SIDE;
> SO IS THE TABULAR SINE, TANGENT, &C., OF THE SAME NAME WITH THE REQUIRED SIDE,
> TO THE REQUIRED SIDE.

It will be readily seen, that this is nothing more than a statement in general terms, of the proportions between the similar sides of two triangles, one proposed for solution, and the other formed from the numbers in the tables.

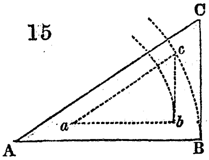
Thus, if the hypothenuse be *given*, and the base or perpendicular be *required*; then in Fig. 14, where *ac* is the tabular radius, *bc* the tabular sine of *a*, or its equal A, and *ab* the tabular sine of C; (Art. 124.)

14



$ac : AC :: bc : BC$, that is, $R : AC :: \sin A : BC$.
$ac : AC :: ab : AB$, $R : AC :: \sin C : AB$.

15



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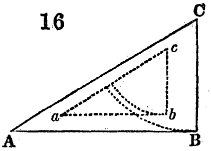


In Fig. 15, where *ab* is the tabular radius, *ac* the tabular secant of A, and *bc* the tabular tangent of A;

ac : AC : : *bc* : BC, that is, sec A : AC : : tan A : BC.
ac : AC : : *ab* : AB, sec A : AC : : R : AB.

In Fig. 16, where *bc* is the tabular radius, *ac* the tabular secant of C, and *ab* the tabular tangent of C;

ac : AC : : *bc* : BC, that is, sec C : AC : : R : BC.
ac : AC : : *ab* : AB, sec C : AC : : tan C : AB.

THEOREM II.

127. When an *angle* is required;

AS THE GIVEN SIDE MADE RADIUS,
TO THE TABULAR RADIUS;
SO IS ANOTHER GIVEN SIDE,
TO THE TABULAR SINE, TANGENT, &C., OF THE SAME NAME.

Thus, if the side made radius, and one other side be given, then, in Fig. 14,

AC : *ac* : : BC : *bc*, that is, AC : R : : BC : sin A.
AC : *ac* : : AB : *ab*, AC : R : : AB : sin C.

In Fig. 15,

AB : *ab* : : BC : *bc*, that is, AB : R : : BC : tan A.
AB : *ab* : : AC : *ac*, AB : R : : AC : sec A.

In Fig. 16,

BC : *bc* : : AB : *ab*, that is, BC : R : : AB : tan C.
BC : *bc* : : AC : *ac*, BC : R : : AC : sec C.

It will be observed that in these theorems, *angles* **are not** introduced, though they are among the quantities which are either given or required, in the calculation of triangles. But the tabular sines, tangents, &c., may be considered the *representatives* of angles, as one may be found from the other, by merely turning to the tables.

128. In the theorem for finding a *side,* the first term of the proportion is a *tabular number.* But, in the theorem for finding an *angle,* the first term is a side. Hence, in applying the proportions to particular cases, this rule is to be observed;

To find a SIDE, *begin with a tabular number,*
To find an ANGLE, *begin with a side.*

Radius is to be reckoned among the tabular numbers.

129. In the theorem for finding an *angle,* the first term is a *side made radius.* As in every proportion, the three first terms must be given to enable us to find the fourth, it is evident, that where this theorem is applied, the side made radius must be a *given* one. But, in the theorem for finding a *side,* it is not necessary that either of the terms should be radius. Hence,

130. *To find a* SIDE, ANY *side may be made radius,*
To find an ANGLE, *a* GIVEN *side must be made radius.*

It will generally be expedient, in both cases, to make radius one of the terms in the proportion; because, in the tables of natural sines, tangents, &c., radius is 1, and in the logarithmic tables it is 10. (Art. 103.)

131. The proportions in Trigonometry are of the same nature as other simple proportions. The fourth term is found, therefore, as in the Rule of Three in Arithmetic, by *multiplying together the second and third terms, and dividing their product by the first term.* This is the mode of calculation, when the tables of *natural* sines, tangents, &c., are used. But the operation by logarithms is so much more expeditious,

that it has almost entirely superseded the other method. In logarithmic calculations, addition takes the place of multiplication; and subtraction the place of division.

The logarithms expressing the lengths of the *sides* of a triangle, are to be taken from the tables of common logarithms. The logarithms of the *sines, tangents, &c.*, are found in the tables of artificial sines, &c. The calculation is then made *by adding the second and third terms, and subtracting the first.* (Art. 52.)

132. The logarithmic radius 10, or, as it is written in the tables, 10.00000, is so easily added and subtracted, that the three terms of which it is one, may be considered as, in effect, reduced to two. Thus, if the tabular radius is in the *first* term, we have only to add the other two terms, and then take 10 from the index; for this is subtracting the first term. If radius occurs in the *second* term, the first is to be subtracted from the third, after its index is increased by 10. In the same manner, if radius is in the *third* term, the first is to be subtracted from the second.

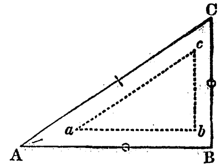
133. Every species of right angled triangles may be solved upon the principle, that the sides of similar triangles are proportional, according to the two theorems mentioned above. There will be some advantages, however, in giving the examples in distinct classes.

There must be given, in a right angled triangle, *two* of the parts, besides the right angle. (Art. 116.) These may be;

1. The hypothenuse and an angle; or
2. The hypothenuse and a leg; or
3. A leg and an angle; or
4. The two legs.

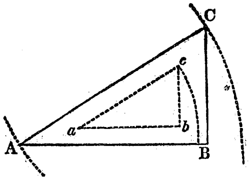
CASE I.

$$\textbf{134. Given}\left\{\begin{array}{l}\text{The hypothenuse,}\\ \text{And an angle,}\end{array}\right\}\text{to find}\left\{\begin{array}{l}\text{The base and}\\ \text{Perpendicular.}\end{array}\right.$$



Ex. 1. If the hypothenuse AC,* be 45 miles, and the angle at A 32° 20′, what is the length of the base AB, and the perpendicular BC?

In this case, as *sides* only are required, *any* side may be made radius. (Art. 130.)



If the hypothenuse be made radius, BC will be the sine of A, and AB the sine of C, or the cosine of A. (Art. 121.) And if *abc* be a similar triangle, whose hypothenuse is equal to the *tabular* radius, *bc* will be the tabular sine of A, and *ab* the tabular sine of C. (Art. 124.)

To find the *perpendicular*, then, by Theorem I, we have this proportion;

ac : AC : : *bc* : BC.
Or R : AC : : Sin A : BC.

Whenever the terms Radius, Sine, Tangent, &c., occur in a proportion like this, the *tabular* Radius, &c., is to be understood, as in Arts. 126, 127.

The numerical calculation, to find the length of BC, may be made, either by *natural* sines, or by *logarithms*. See Art. 131.

By natural Sines.

1 : 45 : : 0.53484 : 24.068=BC.

---

* The parts which are *given* are distinguished by a mark across the line, or at the opening of the angle, and the parts *required* by a cipher.

Computation by Logarithms.

As radius		10.00000
To the hypothenuse	45	1.65321
So is the Sine of A	32° 20′	9.72823
To the perpendicular	24.068	1.38144

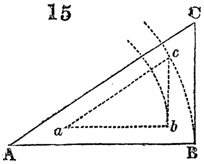
Here the logarithms of the second and third terms are added, and from the sum, the first term 10 is subtracted. (Art. 132.) The remainder is the logarithm of 24.068=BC.

Subtracting the angle at A from 90°, we have the angle at C=57° 40′. (Art. 116.) Then to find the *base* AB;

$$ac : AC :: ab : AB$$
$$\text{Or } R : AC :: \text{Sin } C : AB = 38.023.$$

Both the sides required are now found, by making the hypothenuse radius. The results here obtained may be verified, by making either of the other sides radius.

15



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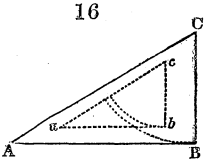


If the *base* be made radius, as in Fig. 15, the perpendicular will be the *tangent*, and the hypothenuse the *secant* of the angle at A. (Art. 122.) Then,

$$\text{Sec } A : AC :: R : AB$$
$$R : AB :: \text{Tan } A : BC$$

By making the arithmetical calculations, in these two proportions, the values of AB and BC, will be found the same as before.

If the *perpendicular* be made radius, as in Fig. 16, AB will be the *tangent*, and AC the *secant* of the angle at C. Then,

$$\text{Sec } C : AC :: R : BC$$
$$R : BC :: \text{Tan } C : AB$$

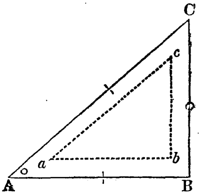
Ex. 2. If the hypothenuse of a right angled triangle be 250 rods, and the angle at the base 46° 30′; what is the length of the base and perpendicular?

Ans. The base is 172.1 rods, and the perpendic. 181.35.

CASE II.

135. Given $\left\{\begin{array}{l}\text{The hypothenuse,}\\ \text{And one leg.}\end{array}\right\}$ to find $\left\{\begin{array}{l}\text{The angles and}\\ \text{The other leg.}\end{array}\right.$

18



Ex. 1. If the hypothenuse be 35 leagues, and the base 26; what is the length of the perpendicular, and the quantity of each of the acute angles?

To find the angles it is necessary that one of the *given* sides be made radius. (Art. 130.)

If the *hypothenuse* be radius, the base and perpendicular will be sines of their opposite angles. Then,

$$AC : R :: AB : \text{Sin } C = 47^\circ\ 58\tfrac{1}{2}'$$

And to find the *perpendicular* by Theorem I;

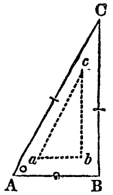
$$R : AC :: \text{Sin } A : BC = 23.43$$

If the *base* be radius, the perpendicular will be *tangent*, and the hypothenuse *secant* of the angle at A. Then,

$$AB : R :: AC : \text{Sec } A$$
$$R : AB :: \text{Tan } A : BC$$

In this example, where the hypothenuse and base are given, the angles cannot be found by making the *perpendicular* radius. For to find an angle, a *given* side must be made radius. (Art. 130.)

136. Ex. 2. If the hypothenuse be 54 miles, and the perpendicular 48 miles, what are the angles, and the base?



Making the *hypothenuse* radius.

AC : R :: BC : Sin A
R : AC :: Sin C : AB

The numerical calculation will give A=62° 44′ and AB =24.74.

Making the *perpendicular* radius.

BC : R :: AC : Sec C
R : BC :: Tan C : AB

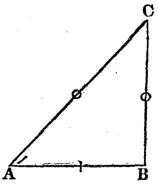
The angles cannot be found by making the *base* radius, when its length is not given.

Case III.

137. Given $\left\{\begin{array}{l}\text{The angles,}\\ \text{and one leg.}\end{array}\right\}$ to find $\left\{\begin{array}{l}\text{The hypothenuse,}\\ \text{and the other leg.}\end{array}\right.$

Ex. 1. If the base be 60, and the angle at the base 47° 12′, what is the length of the hypothenuse and the perpendicular?

In this case, as *sides* only are required, *any* side may be radius.



Making the *hypothenuse* radius.

Sin C : AB :: R : AC=88.31
R : AC :: Sin A : BC=64.8

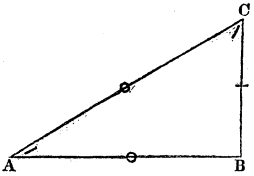
Making the *base* radius. (Fig. 20.)

R : AB : : Sec A : AC
R : AB : : Tan A : BC

Making the *perpendicular* radius.

Tan C : AB : : R : BC
R : BC : : Sec C : AC

138. Ex. 2. If the perpendicular be 74, and the angle C 61° 27′, what is the length of the base and the hypothenuse ?



Making the *hypothenuse* radius.

Sin A : BC : : R : AC
R : AC : : sin C : AB

Making the *base* radius.

Tan A : BC : : R : AB
R : AB : : sec A : AC

Making the *perpendicular* radius.

R : BC : : sec C : AC
R : BC : : tan C : AB

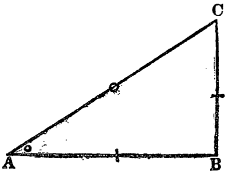
The hypothenuse is 154.83 and the base 136.

CASE IV.

139. Given $\left\{\begin{array}{l}\text{The base, and}\\ \text{Perpendicular,}\end{array}\right\}$ to find $\left\{\begin{array}{l}\text{The hypothenuse,}\\ \text{And the angles.}\end{array}\right.$

Ex. 1. If the base be 284, and the perpendicular 192, what are the angles, and the hypothenuse?

In this case, one of the legs



must be made radius, to find an angle; because the hypothenuse is not given.

Making the *base* radius.

$$AB : R :: BC : \tan A = 34^\circ\ 4'$$
$$R : AB :: \sec A : AC = 342.84$$

Making the *perpendicular* radius.

$$BC : R :: AB : \tan C$$
$$R : BC :: \sec C : AC$$

Ex. 2. If the base be 640, and the perpendicular 480, what are the angles and hypothenuse?

Ans. The hypothenuse is 800, and the angle at the base 36° 52′ 12″.

Examples for Practice.

1. Given the hypothenuse 68, and the angle at the base 39° 17′; to find the base and perpendicular.
2. Given the hypothenuse 850, and the base 594, to find the angles, and the perpendicular.
3. Given the hypothenuse 78, and perpendicular 57, to find the base, and the angles.
4. Given the base 723, and the angle at the base 64° 18′, to find the hypothenuse and perpendicular.
5. Given the perpendicular 632, and the angle at the base 81° 36′, to find the hypothenuse and the base.
6. Given the base 32, and the perpendicular 24, to find the hypothenuse, and the angles.

140. The preceding solutions are all effected, by means of the tabular sines, tangents, and secants. But, when any *two sides* of a right angled triangle are given, the third side may be found, without the aid of the trigonometrical tables, by the proposition, that *the square of the hypothenuse is equal*

to the sum of the squares of the two perpendicular sides. (Euc. 47. 1.)

If the legs be given, extracting the square root of the *sum* of their squares, will give the hypothenuse. Or, if the hypothenuse and one leg be given, extracting the square root of the *difference* of the squares, will give the other leg.

$$\left.\begin{array}{l}\text{Let } h=\text{the hypothenuse}\\ \quad\;\; p=\text{the perpendicular}\\ \quad\;\; b=\text{the base}\end{array}\right\}\text{ of a right angled triangle.}$$

Then $h^2=b^2+p^2$, or (Alg. 248.) $h=\sqrt{b^2+p^2}$

By trans. $b^2=h^2-p^2$, or $b=\sqrt{h^2-p^2}$

And $p^2=h^2-b^2$, or $p=\sqrt{h^2-b^2}$

Ex. 1. If the base is 32, and the perpendicular 24, what is the hypothenuse? Ans. 40.

2. If the hypothenuse is 100, and the base 80, what is the perpendicular? Ans. 60.

3. If the hypothenuse is 300, and the perpendicular 220, what is the base?

Ans. $\overline{300}^2-\overline{220}^2=41600$, the root of which is 204 nearly.

141. It is generally most convenient to find the difference of the squares by *logarithms.* But this is not to be done by *subtraction.* For subtraction, in logarithms, performs the office of *division.* (Art. 41.) If we subtract the logarithm of b^2 from the logarithm of h^2, we shall have the logarithm, not of the *difference* of the squares, but of their *quotient.* There is, however, an indirect, though very simple method, by which the difference of the squares may be obtained by logarithms. It depends on the principle, that the *difference of the squares of two quantities is equal to the product of the sum and difference of the quantities.* (Alg. 191.) Thus,

$$h^2-b^2=(h+b)\times(h-b)$$

as will be seen at once, by performing the multiplication. The two factors may be multiplied by *adding* their logarithms. Hence,

142. *To obtain the difference of the squares of two quantities, add the logarithm of the sum of the quantities to the logarithm of their difference.* After the logarithm of the difference of the squares is found; the *square root* of this difference is obtained, by dividing the logarithm by 2. (Art. 47.)

Ex. 1. If the hypothenuse be 75 inches, and the base 45, what is the length of the perpendicular?

Sum of the given sides	120	log. 2.07918
Difference of do.	30	1.47712
Dividing by		2)3.55630
Side required	60	1.77815

2. If the hypothenuse is 135, and the perpendicular 108, what is the length of the base? Ans. 81.

SECTION IV.

SOLUTIONS OF OBLIQUE ANGLED TRIANGLES.

Art. 143. The sides and angles of oblique angled triangles may be calculated by the following theorems.

Theorem I.

In any plane triangle, THE SINES OF THE ANGLES ARE AS THEIR OPPOSITE SIDES.

Let the angles be denoted by the letters A, B, C, and their opposite sides by a, b, c, as in Fig. 23 and 24. From one of the angles, let the line p be drawn perpendicular to the opposite side. This will fall either within or without the triangle.

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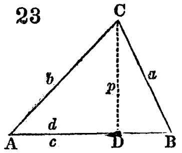

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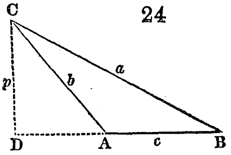

1. Let it fall *within* as in Fig. 23. Then, in the right angled triangles ACD, and BCD, according to Art. 126,

$$R : b :: \sin A : p$$
$$R : a :: \sin B : p$$

Here, the two *extremes* are the same in both proportions. The other four terms are, therefore, *reciprocally* proportional :* that is,

$$a : b :: \sin A : \sin B.$$

2. Let the perpendicular p fall *without* the triangle, as in Fig. 24. Then in the right angled triangles ACD and BCD;

$$R : b :: \sin A : p$$
$$R : a :: \sin B : p$$

Therefore, as before,

$$a : b :: \sin A : \sin B.$$

Sin A is here put both for the sine of DAC, and for that of BAC. For, as one of these angles is the *supplement* of the other, they have the same sine. (Art. 90.)

The sines which are mentioned here, and which are used in calculation are *tabular* sines. But the proportion will be the same, if the sines be adapted to any other radius. (Art. 119.)

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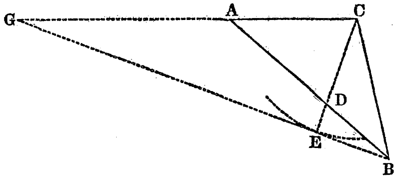
* Euclid, 23. 5.

THEOREM II.

144. In a plane triangle,

As THE SUM OF ANY TWO OF THE SIDES,
To THEIR DIFFERENCE;
So IS THE TANGENT OF HALF THE SUM OF THE OPPOSITE ANGLES;
To THE TANGENT OF HALF THEIR DIFFERENCE.

Thus, the sum of AB and AC, is to their difference; as the tangent of half the sum of the angles ACB and ABC, to the tangent of half their difference.



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Demonstration.

Extend CA to G, making AG equal to AB; then CG is *the sum of the two sides* AB and AC. On AB, set off AD, equal to AC; then BD *is the difference of the sides* AB and AC.

The sum of the two angles ACB and ABC, is equal to the sum of ACD and ADC; because each of these sums is the supplement of CAD. (Art. 79.) But as AC=AD by construction, the angle ADC=ACD (Euc. 5 1.[*]) Therefore ACD is *half the sum* of ACB and ABC. As AB=AG, the angle AGB=ABG, or DBE. Also, GCE, or ACD=ADC=BDE. (Euc. 15. 1.[†]) Therefore in the triangles

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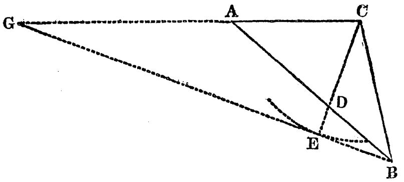
[*] Thomson's Legendre, 11. 1.

[†] Ibid. 4. 1.



GCE, and DBE, the two remaining angles DEB, and CEG, are equal; (Art. 79.) So that CE is perpendicular to BG. (Euc. Def. 7. 1.*) If then CE is made radius, GE is the tangent of GCE, (Art. 84.) that is, *the tangent of half the sum of the angles opposite to* AB and AC.

If from the greater of the two angles ACB and ABC, there be taken ACD their half sum; the remaining angle ECB will be their half difference. The tangent of this angle, CE being radius, is EB, that is, *the tangent of half the difference of the angles opposite to* AB and AC. We have then,

CG=the sum of the sides AB and AC;
DB=their difference;
GE=the tangent of half the sum of the opposite angles;
EB=the tangent of half their difference.

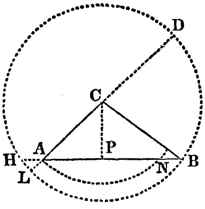
But by similar triangles,

CG : DB : : GE : EB. Q. E. D.

THEOREM III.

145. If upon the longest side of a triangle, a perpendicular be drawn from the opposite angle;

AS THE LONGEST SIDE,
TO THE SUM OF THE TWO OTHERS;
SO IS THE DIFFERENCE OF THE LATTER,
TO THE DIFFERENCE OF THE SEGMENTS MADE BY THE PERPENDICULAR.



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* Thomson's Legendre, Def. 12, 1.

In the triangle ABC, if a perpendicular be drawn from C upon AB;

$$AB : CB+CA :: CB-CA : BP-PA.*$$

Demonstration.

Describe a circle on the centre C, and with the radius BC. Through A and C, draw the diameter LD, and extend BA to H. Then by (Euc. 35. 3.†)

$$AB\times AH=AL\times AD$$

Therefore,

$$AB : AD :: AL : AH$$

But $AD=CD+CA=CB+CA$
And $AL=CL-CA=CB-CA$
And $AH=HP-PA=BP-PA$ (Euc. 3. 3.—Thom. 6. 2.)

If, then, for the three last terms in the proportion, we substitute their equals, we have,

$$AB : CB+CA :: CB-CA : PB-PA.$$

146. It is to be observed, that the greater segment is next the greater side. If BC is greater than AC, PB is greater than AP. With the radius AC, describe the arc AN. The segment NP=AP. (Euc. 3. 3.) But BP is greater than NP.

147. The two segments are to each other, as the tangents of the opposite angles, or the cotangents of the adjacent angles. For, in the right angled triangles ACP, and BCP, if CP be made radius, (Art. 126.)

$$R : PC :: \text{Tan } ACP : AP$$
$$R : PC :: \text{Tan } BCP : BP$$

Therefore, by equality of ratios, (Alg. 346.‡)

$$\text{Tan } ACP : AP :: \text{Tan } BCP : BP$$

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* See note B. † Thomson's Legendre, 28. 4. Cor. ‡ **Euc. 11. 5.**

That is, the segments are as the tangents of the opposite angles. And the tangents of these are the *cotangents* of the adjacent angles A and B. (Art. 89.)

Cor. The greater segment is opposite to the greater angle. And of the angles at the base, the less is next the greater side. If BP is greater than AP, the angle BCP is greater than ACP ; and B is less than A. (Art. 77.)

148. To enable us to find the sides and angles of an oblique angled triangle, *three* of them must be *given.* (Art. 114.)

These may be, either

1. Two angles and a side, or
2. Two sides and an angle *opposite* one of them, or
3. Two sides and the *included* angle, or
4. The three sides.

The two first of these cases are solved by Theorem I, (Art 143.) the third by Theorem II, (Art. 144.) and the fourth by Theorem III. (Art. 145.)

149 In making the calculations, it must be kept in mind, that the greater side is always opposite to the greater angle, (Euc 18, 19. 1.*) that there can be only one *obtuse* angle in a triangle, (Art. 76.) and therefore, that the angles opposite to the two least sides must be *acute.*

CASE I.

150. Given,

Two angles, and A side,	to find	The remaining angle, and The other two sides.

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* Thomson's Legendre, 13. 1.

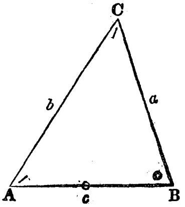
The third angle is found by merely subtracting the sum of the two which are given from 180°. (Art. 79.)

The sides are found, by stating, according to Theorem I, the following proportion;

As the sine of the angle opposite the *given* side,
To the length of the given side;
So is the sine of the angle opposite the *required* side
To the length of the required side.

As a *side* is to be found, it is necessary to begin with a *tabular number*.

Ex. 1. In the triangle ABC, the side b is given 32 rods, the angle A 56° 20′, and the angle C 49° 10′, to find the angle B, and the sides a and c.



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The sum of the two given angles $56° 20' + 49° 10' = 105° 30'$; which subtracted from 180°, leaves 74° 30′ the angle B. Then,

$$\text{Sin B} : b :: \begin{cases} \text{Sin A} : a \\ \text{Sin C} : c \end{cases}$$

Calculation by logarithms.

As the sine of B	74° 30′	*a. c.*	0.01609
To the side b	32		1.50515
So is the sine of A	56° 20′		9.92027
To the side a	27.64		1.44151
As the sine of B	74° 30′	*a. c.*	0.01609
To the side b	32		1.50515
So is the sine of C	49° 10′		9.87887
To the side c	25.13		1.40011

The *arithmetical complement* used in the first term here, may be found in the usual way, or by taking out the *cosecant* of the given angle, and rejecting 10 from the index. (Art. 113.)

Ex. 2. Given the side b 71, the angle A 107° 6′, and the angle C 27° 40′; to find the angle B, and the sides a and c. The angle B is 45° 14′. Then,

$$\text{Sin B} : b :: \begin{cases} \text{Sin A} : a = 95.58 \\ \text{Sin C} : c = 46.43 \end{cases}$$

When one of the given angles is *obtuse*, as in this example, the sine of its *supplement* is to be taken from the tables. (Art. 99.)

CASE II.

151. Given,

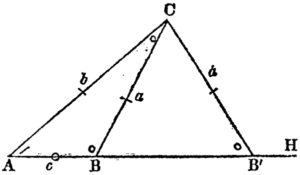
$$\left.\begin{array}{l}\text{Two sides, and}\\ \text{An opposite angle,}\end{array}\right\} \text{ to find } \left\{\begin{array}{l}\text{The remaining side and}\\ \text{The other two angles.}\end{array}\right.$$

One of the required angles is found, by beginning with a side, and, according to Theorem I, stating the proportion,

As the side opposite the given angle,
To the sine of that angle;
So is the side opposite the required angle,
To the sine of that angle.

The third angle is found, by subtracting the sum of the other two from 180°; and the remaining side is found, by the proportion in the preceding article.

152. In this second case, if the side opposite to the given angle be shorter than the other given side the solution will be *ambiguous*. Two different triangles may be formed, each of which will satisfy the conditions of the problem.



Let the side *b*, the angle A, and the length of the side opposite this angle be given. With the latter for radius, (if it be shorter than *b*,) describe an arc, cutting the line AH in the points B and B′. The lines BC and B′C, will be equal. So that, with the same data, there may be formed two different triangles, ABC and AB′C.

There will be the same ambiguity in the numerical calculation. The answer found by the proportion will be the *sine* of an angle. But this may be the sine either of the *acute* angle AB′C, or of the *obtuse* angle ABC. For, BC being equal to B′C, the angle CB′B is equal to CBB′. Therefore ABC, which is the supplement of CBB′, is also the supplement of CB′B. But the sine of an angle is the same, as the sine of its supplement. (Art. 90.) The result of the calculation will, therefore, be ambiguous. In practice, however, there will generally be some circumstances which will determine whether the angle required is acute or obtuse.

If the side opposite the given angle be *longer* than the other given side, the angle which is subtended by the latter, will necessarily be acute. For there can be but one obtuse angle in a triangle, and this is always subtended by the longest side. (Art. 149.)

If the *given* angle be obtuse, the other two will, of course, be acute. There can, therefore, be no ambiguity in the solution.

Ex. 1. Given the angle A, 35° 20′, the opposite side *a* 50, and the side *b* 70; to find the remaining side, and the other two angles.

To find the angle opposite to *b*, (Art. 151.)

$$a : \sin A :: b : \sin B$$

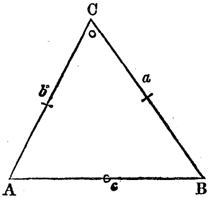
The calculation here gives the acute angle AB′C 54° 3 50″, and the obtuse angle ABC 125° 56′ 10″. If the latter be added to the angle at A 35° 20′, the sum will be 161° 16′ 10″, the supplement of which, 18° 43′ 50″, is the angle ACB. Then in the triangle ABC, to find the side c=AB,

$$\mathrm{Sin}\ A : a :: \sin C : c = 27.76$$

If the *acute* angle AB′C 54° 3′ 50″ be added to the angle at A 35° 20′, the sum will be 89° 23′ 50″, the supplement of which, 90° 36′ 10″, is the angle ACB′. Then, in the triangle AB′C,

$$\mathrm{Sin}\ A : CB' :: \sin C : AB' = 86.45.$$

Ex. 2. Given the angle at A, 63° 35′, the side b 64, and the side a 72; to find the side c, and the angles B and C.



$$a : \sin A :: b : \sin B = 52° 45' 25''$$
$$\mathrm{Sin}\ A : a :: \sin C : c = 72.05$$

The sum of the angles A and B, is 116° 20′ 25″, the supplement of which, 63° 39′ 35″, is the angle C.

In this example the solution is *not ambiguous*, because the side opposite the given angle is longer than the other given side.

Ex. 3. In a triangle of which the angles are A, B, and C, and the opposite sides a, b, and c, as before; if the angle A be 121° 40′, the opposite side a 68 rods, and the side b 47 rods; what are the angles B and C, and what is the length of the side c? Ans. B is 36° 2′ 4″, C 22° 17′ 56″, and c 30.3.

In this example also, the solution is not ambiguous, because the *given* angle is obtuse.

Case III.

153. Given,

Two sides, and The included angle,	to find	The remaining side, and The other two angles.

In this case, the angles are found by Theorem II. (Art. 144.) The required side may be found by Theorem I.

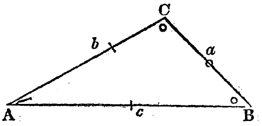
In making the solutions, it will be necessary to observe, that by subtracting the given angle from 180°, the *sum* of the other two angles is found; (Art. 79.) and, that *adding half the difference of two quantities to their half sum gives the greater quantity, and subtracting the half difference from the half sum gives the less.* The latter proposition may be geometrically demonstrated thus;

Let AE, be the greater of two magnitudes, and BE the less. Bisect AB in D, and make AC equal to BE. Then,

AB is the *sum* of the two magnitudes;
CE their *difference*;
DA or DB *half* their *sum*;
DE or DC *half* their *difference*;

But $DA+DE=AE$ the *greater* magnitude;
And $DE-DE=BE$ the *less*.

Ex. 1. In the triangle ABC, the angle A is given 26° 14′, the side b 39, and the side c 53; to find the angles B and C, and the side a.



The *sum* of the sides b and c is $53+39=92$
And their *difference* $53-39=14$
The sum of the angles B and C=180°—26° 14′=153° 46′
And *half the sum* of B and C is 76° 53′

Then, by Theorem II, (Fig. 30.)

$$(b+c) : (b-c) :: \tan \tfrac{1}{2}(B+C) : \tan \tfrac{1}{2}(B\sim C)$$

To and from the half sum	76°	53′	
Adding and subtracting the half difference	33	8	50
We have the greater angle	110	1	50
And the less angle	43	44	10

As the greater of the two given sides is c, the greater angle is C, and the less angle B. (Art. 149.)

To find the side a, by Theorem I.

$$\sin B : b :: \sin A : a = 24.94.$$

Ex. 2. Given the angle A 101° 30′, the side b 76, and the side c 109; to find the angles B and C, and the side a.

B is 30° 57½′, C 47° 32½′, and a 144.8

Case IV.

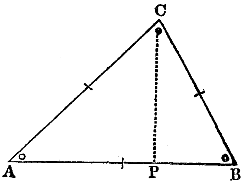
154. Given the three sides, to find the angles.

In this case, the solutions may be made, by drawing a perpendicular to the longest side, from the opposite angle. This will divide the given triangle into two *right angled* triangles. The two segments may be found by Theorem III. (Art. 145.)

There will then be given, in each of the right angled triangles, the hypothenuse and one of the legs, from which the angles may be determined, by rectangular trigonometry. (Art. 135.)

Ex. 1. In the triangle ABC, the side AB is 39, AC 35, and BC 27. What are the angles?

Let a perpendicular be drawn from C, dividing the longest side AB into the two segments AP and BP. Then by Theorem III,



$$AB : AC+BC :: AC-BC : AP-BP.$$

As the longest side	39 *a. c.*	8.40894
To the sum of the two others	62	1.79239
So is the difference of the latter	8	0.90309
To the difference of the segments	12.72	1.10442

The greater of the two segments is AP, because it is next the side AC, which is greater than BC. (Art. 146.)

To and from half the sum of the segments		19.5
Adding and subtracting half their difference, (Art. 153.)		6.36
We have the greater segment	AP	25.86
And the less	BP	13.14

Then, in each of the right angled triangles APC and BPC, we have given the hypothenuse and base; and by Art. 135.

$$AC : R :: AP : \cos A = 42° \; 21' \; 57''$$
$$BC : R :: BP : \cos B = 60° \; 52' \; 42''$$

And subtracting the sum of the angles A and B from 180°, we have the remaining angle ACB=76° 45′ 21″.

Ex. 2. If the three sides of a triangle are 78, 96, and 104; what are the angles?

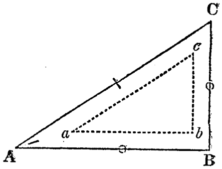
Ans. 45° 41′ 48″, 61° 43′ 27″, and 72° 34′ 45″.

Examples for Practice.

1. Given the angle A 54° 30′, the angle B 63° 10′, and the side *a* 164 rods; to find the angle C, and the sides *b* and *c*.
2. Given the angle A 45° 6′, the opposite side *a* 93, and the side *b* 108; to find the angles B and C, and the side *c*.
3. Given the angle A 67° 24′, the opposite side *a* 62, and the side *b* 46; to find the angles B and C, and the side *c*.
4. Given the angle A 127° 42′, the opposite side *a* 381, and the side *b* 184; to find the angles B and C, and the side *c*.

5. Given the side b 58, the side c 67, and the included angle $A=36°$; to find the angles B and C, and the side a.
6. Given the three sides, 631, 268, and 546; to find the angles.

155. The three theorems demonstrated in this section, have been here applied to *oblique angled* triangles only. But they are equally applicable to *right angled* triangles.

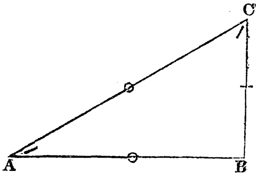


Thus, in the triangle ABC, according to Theorem I, (Art. 143.)

Sin B : AC : : sin A : BC

This is the same proportion as one stated in Art 134, except that, in the first term here, the *sine of* B is substituted for *radius*. But, as B is a right angle, its sine is *equal* to *radius*. (Art. 95.)

Again, in the triangle ABC, by the same theorem;



Sin A : BC : : sin C : AB

This is also one of the proportions in rectangular trigonometry, when the hypothenuse is made radius.

The other two theorems might be applied to the solution of right angled triangles. But, when one of the angles is *known* to be a right angle, the methods explained in the preceding section, are much more simple in practice.

SECTION V.

GEOMETRICAL CONSTRUCTION OF TRIANGLES, BY THE PLANE SCALE.

Art. 156. To facilitate the construction of geometrical figures, a number of graduated lines are put upon the common two feet scale; one side of which is called the *Plane Scale,* and the other side, *Gunter's Scale.* The most important of these are the scales of *equal parts,* and the line of *chords.* In forming a given triangle, or any other right lined figure, the parts which must be made to agree with the conditions proposed, are the *lines* and the *angles.* For the former, a scale of equal parts is used; for the latter, a line of chords.

157. The line on the upper side of the plane scale, is divided into *inches* and *tenths* of an inch. Beneath this, on the left hand, are two *diagonal* scales of equal parts,* divided into inches and half inches, by perpendicular lines. On the larger scale, one of the inches is divided into tenths, by lines which pass *obliquely* across, so as to intersect the parallel lines which run from right to left. The use of the oblique lines is to measure *hundredths* of an inch, by inclining more and more to the right, as they cross each of the parallels.

To take off, for instance, an extent of 3 inches, 4 tenths, and 6 hundredths;

Place one foot of the dividers at the intersection of the perpendicular line marked 3 with the parallel line marked 6, and the other foot at the intersection of the latter with the oblique line marked 4.

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* These lines are not represented by a figure, as the learner is supposed to have the scale before him.

The other diagonal scale is of the same nature. The divisions are smaller, and are numbered from left to right.

158. In geometrical constructions, what is often required, is to make a figure, not *equal* to a given one, but only *similar*. Now figures are similar which have equal angles, and the sides about the equal angles *proportional*. (Euc. Def. 1. 6.*) Thus a land surveyor, in plotting a field, makes the several lines in his plan to have the same proportion to each other, as the sides of the field. For this purpose a scale of equal parts may be used, of any dimensions whatever. If the sides of the field are 2, 5, 7, and 10 *rods*, and the lines in the plan are 2, 5, 7, and 10 *inches*, and if the angles are the same in each, the figures are similar. One is a copy of the other, upon a smaller scale.

So any two right lined figures are similar, if the angles are the same in both, and if the number of smaller parts in each side of one, is equal to the number of larger parts in the corresponding sides of the other. The several divisions on the scale of equal parts may, therefore, be considered as representing any measures of length, as feet, rods, miles, &c. All that is necessary is, that the scale be not changed, in the construction of the same figure; and that the several divisions and subdivisions be properly proportioned to each other. If the larger divisions, on the diagonal scale, are units, the smaller ones are tenths and *hundredths*. If the larger are tens, the smaller are units and tenths.

159. In laying down an *angle*, of a given number of degrees, it is necessary to *measure* it. Now the proper measure of an angle is an arc of a circle. (Art. 74.) And the measure of an arc, where the radius is given, is its *chord*. For the chord is the distance, in a straight line, from one

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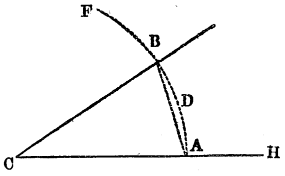
* **Thomson's Legendre, Def. 3. 4.**



end of the arc to the other. Thus the chord AB, is a measure of the arc ADB, and of the angle ACB.

To form the *line of chords*, a circle is described, and the length of its chords determined for every degree of the quadrant. These measures are put on the plane scale, on the line marked CHO.

160. The chord of 60° is equal to *radius*. (Art. 95.) In laying down or measuring an angle, therefore, an arc must be drawn, with a radius which is equal to the extent from 0 to 60 on the line of chords. There are generally on the scale, two lines of chords. Either of these may be used; but the angle must be measured by the same line from which the radius is taken.

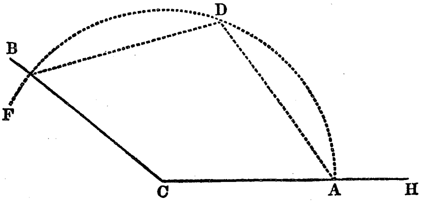
161. To *make an angle*, then, of a given number of degrees; from one end of a straight line as a centre, and with a radius equal to the chord of 60° on the line of chords, describe an arc of a circle cutting a straight line. From the point of intersection, extend the chord of the given number of degrees, applying the other extremity to the arc; and through the place of meeting, draw the other line from the angular point.

If the given angle is *obtuse*, take from the scale the chord of *half* the number of degrees, and apply it *twice* to the arc. Or make use of the chords of any two arcs whose *sum* is equal to the given number of degrees.

A *right angle* may be constructed, by drawing a perpendicular without using the line of chords.

Ex. 1. To make an angle of 32 degrees. With the point C, in the line CH, for a centre, and with the chord of 60° for radius, describe the arc ADF. Extend the chord of 32° from A to B; and through B, draw the line BC. Then is ACB an angle of 32 degrees.

2. To make an angle of 140 degrees. On the line CH, with the chord of 60°, describe the arc ADF; and extend



the chord of 70° from A to D, and from D to B. The arc ADB=70°×2=140°.

On the other hand:

162. To *measure an angle;* On the angular point as a centre, and with the chord of 60° for radius, describe an arc to cut the two lines which include the angle, The distance between the points of intersection, applied to the line of chords, will give the measure of the angle in degrees. If the angle be *obtuse,* divide the arc into two parts.

Ex. 1. To measure the angle ACB. (Fig. 33, page 101.) Describe the arc ADF, cutting the lines CH and CB. The distance AB, will extend 32° on the line of chords.

2. To measure the angle ACB. (Fig. 34.) Divide the arc ADB into two parts, either equal or unequal, and measure each part, by applying its chord to the scale. The sum of the two will be 140°.

163. Besides the lines of chords, and of equal parts, on the plane scale; there are also lines of natural *sines, tangents,* and *secants,* marked Sin., Tan., and Sec.; of *semitangents,* marked S. T.; of *longitude,* marked Lon. or M. L.; of *rhumbs,* marked Rhu. or Rum., &c. These are not necessary in trigonometrical construction. Some of them are used in Navigation; and some of them in the projections of the Sphere.

164. In Navigation, the quadrant, instead of being graduated in the usual manner, is divided into *eight* portions, called *Rhumbs.* The *Rhumb line,* on the scale, is a line of chords, divided into rhumbs and quarter-rhumbs, instead of degrees.

165. The line of *Longitude* is intended to show the number of geographical miles in a degree of longitude, at different distances from the equator. It is placed over the line of chords, with the numbers in an inverted order: so that the figure above shows the length of a degree of longitude, in any latitude denoted by the figure below.* Thus, at the equator, where the latitude is 0, a degree of longitude is 60 geographical miles. In latitude 40, it is 46 miles; in latitude 60, 30 miles, &c.

166. The graduation on the line of *secants* begins where the line of sines ends. For the greater sine is only equal to radius; but the secant of the least arc is greater than radius.

167. The *semitangents* are the tangents of *half* the given arcs. Thus, the semitangent of 20° is the tangent of 10°. The line of semitangents is used in one of the projections of the sphere.

168. In the construction of *triangles,* the sides and angles which are *given,* are laid down according to the directions in Arts. 158, 161. The parts *required* are then measured, according to Arts. 158, 162. The following problems correspond with the four cases of oblique angled triangles; (Art. 148.) but are equally adapted to right angled triangles.

169. Prob. I. *The angles and one side* of a triangle being given; to find, by construction, the other two sides.

Draw the given side. From the ends of it, lay off two of the given angles. Extend the other sides till they intersect; and then measure their lengths on a scale of equal parts.

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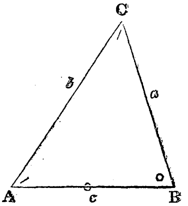
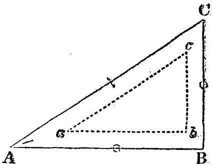
* Sometimes the line of longitude is placed *under* the line of **chords.**



Ex. 1. Given the side *b* 32 rods, the angle A 56° 20′, and the angle C 49° 10′; to construct the triangle, and find the lengths of the sides *a* and *c*.

Their lengths will be 25 and 27½.



2. In a right angled triangle, given the hypothenuse 90, and the angle A 32° 20′, to find the base and perpendicular.

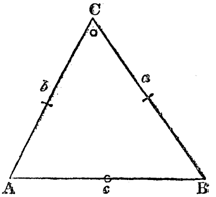
The length of AB will be 76, and of BC 48.

3. Given the side AC 68, the angle A 124°, and the angle C 37°: to construct the triangle.

170. Prob. II. *Two sides and an opposite angle* being given, to find the remaining side, and the other two angles.

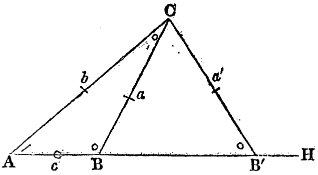
Draw one of the given sides; from one end of it, lay off the given angle; and extend a line indefinitely for the required side. From the other end of the first side, with the remaining given side for radius, describe an arc cutting the indefinite line. The point of intersection will be the end of the required side.

If the side opposite the given angle be less than the other given side, the case will be *ambiguous*. (Art. 152.)



Ex. 1. Given the angle A 63° 35′, the side *b* 32, and the side *a* 36.

The side AB will be 36 nearly, the angle B 52° 45½′, and C 63° 39½′.



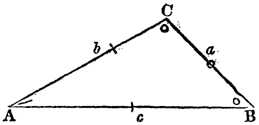
2. Given the angle A 35° 20′, the opposite side *a* 25, and the side *b* 35.

Draw the side *b* 35, make the angle A 35° 20′, and extend AH indefinitely. From C with radius 25, describe an arc cutting AH in B and B′. Draw CB and CB′, and two triangles will be formed, ABC and AB′C, each corresponding with the conditions of the problem.

3. Given the angle A 116°, the opposite side *a* 38, and the side *b* 26; to construct the triangle.

171. PROB. III. *Two sides and the included angle* being given; to find the other side and angles.

Draw one of the given sides. From one end of it lay off the given angle, and draw the other given side. Then connect the extremities of this and the first line.



Ex. 1. Given the angle A 26° 14′, the side *b* 78, and the side *c* 106; to find B, C, and *a*.

The side *a* will be 50, the angle B 43° 44′, and C 110° 2′.

30

2. Given A 86°, *b* 65, and *c* 83; to find B, C, and *a*.

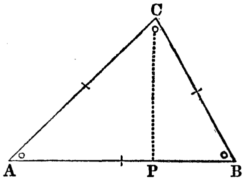
172. PROB. IV. The *three sides* being given; to find the angles.

Draw one of the sides, and from one end of it, with an extent equal to the second side, describe an arc. From the other end, with an extent equal to the third side, describe a second arc cutting the first; and from the point of intersection draw the two sides. (Euc. 22. 1.)

Ex. 1. Given AB 78, AC 70, and BC 54, to find the angles.

The angles will be A 42° 22', B 60° 52$\frac{2}{3}$' and C 76° 45$\frac{1}{3}$'.

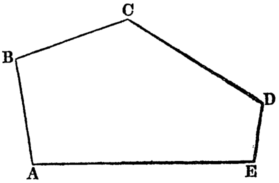
2. Given the three sides 58, 39, and 46; to find the angles.



173. Any right lined figure whatever, whose sides and angles are given, may be constructed, by laying down the sides from a scale of equal parts, and the angles from a line of chords.

Ex. Given the sides AB=20, BC=22, CD=30, DE=12; and the angles B=102°, C=130°, D=108°, to construct the figure.

Draw the side AB=20, make the angle B=102°, draw BC=22, make C=130°, draw CD=30, make D=108°, draw DE=12, and connect E and A.



The last line, EA, may be measured on the scale of equal parts; and the angles E and A, by a line of chords.

SECTION VI.

DESCRIPTION AND USE OF GUNTER'S SCALE.

ART. 174. An expeditious method of solving the problems in trigonometry, and making other logarithmic calculations, in a mechanical way, has been contrived by Mr. Edmund Gunter. The logarithms of numbers, of sines, tangents, &c., are represented by *lines*. By means of these, multiplication, division, the rule of three, involution, evolution, &c., may be performed much more rapidly, than in the usual method by figures.

The logarithmic lines are generally placed on one side only of the scale in common use. They are,

A line of artificial *Sines* divided into *Rhumbs*, and marked,		S. R.
A line of artificial *Tangents*,	do	T. R.
A line of the logarithms of *Numbers*,		Num.
A line of artificial *Sines*, to every *degree*,		SIN.
A line of artificial *Tangents*,	do	TAN.
A line of *Versed Sines*.		V. S.

To these are added a line of *equal parts*, and a line of *Meridional Parts*, which are not logarithmic. The latter is used in Navigation.

The Line of Numbers.

175. Portions of the line of *Numbers*, are intended to represent the *logarithms* of the natural series of numbers 2, 3, 4, 5, &c.

The logarithms of 10, 100, 1000, &c., are 1, 2, 3, &c. (Art. 3.)

If, then, the log. of 10 be represented by a line of 1 foot;
the log. of 100 will be repres'd by one of 2 feet;
the log. of 1000 by one of 3 feet;
the lengths of the several lines being proportional to the corresponding logarithms in the tables. *Portions* of a foot will represent the logarithms of numbers between 1 and 10; and portions of a line 2 feet long, the logarithms of numbers between 1 and 100.

On Gunter's scale, the line of the logarithms of numbers begins at a brass pin on the left, and the divisions are numbered 1, 2, 3, &c., to another pin near the middle. From this the numbers are repeated, 2, 3, 4, &c., which may be read 20, 30, 40, &c. The logarithms of numbers between 1 and 10, are represented by portions of the first half of the line; and the logarithms of numbers between 10 and 100, by portions greater than half the line, and less than the whole.

176. The logarithm of 1, which is 0, is denoted, not by any extent of line, but by a *point* under 1, at the commencement of the scale. The distances from this point to different parts of the line, represent other logarithms, of which the *figures* placed over the several divisions are the *natural numbers*. For the intervening logarithms, the intervals between the figures, are divided into tenths, and sometimes into smaller portions. On the right hand half of the scale, as the divisions which are numbered are *tens*, the subdivisions are units.

Ex. 1. To take from the scale the logarithm of 3.6; set one foot of the dividers under 1 at the beginning of the scale, and extend the other to the 6th division after the first figure 3.

2. For the logarithm of 47; extend from 1 at the beginning, to the 7th subdivision after the second figure 4*.

*** If the dividers will not reach the distance required; first open them so as to take off *half*, or any part of the distance, and then the remaining part.**

177. It will be observed, that the divisions and subdivisions *decrease*, from left to right; as in the tables of *logarithms*, the differences decrease. The difference between the logarithms of 10 and 100, is no greater, than the difference between the logarithms of 1 and 10.

178. The line of numbers, as it has been here explained, furnishes the logarithms of all numbers between 1 and 100.

And if the indices of the logarithms be neglected, the same scale may answer for all numbers whatever. For the *decimal* part of the logarithm of any number is the same, as that of the number multiplied or divided by 10, 100, &c. (Art. 14.) In logarithmic calculations, the use of the indices is to determine the distance of the several figures of the natural numbers from the place of units. (Art. 11.) But in those cases in which the logarithmic line is commonly used, it will not generally be difficult to determine the local value of the figures in the result.

179. We may, therefore, consider the *point* under 1 at the left hand, as representing the logarithm of 1, or 10, or 100; or $\frac{1}{10}$, or $\frac{1}{100}$, &c., for the decimal part of the logarithm of each of these is 0. But if the first 1 is reckoned 10, all the succeeding numbers must also be increased in a tenfold ratio; so as to read, on the first half of the line, 20, 30, 40, &c., and on the other half, 200, 300, &c.

The whole extent of the logarithmic line,

is from 1 to 100, or from 0.1 to 10,
or from 10 to 1000, or from 0.01 to 1,
or from 100 to 10000, &c. or from 0.001 to 0.1, &c.

Different values may, on different occasions, be assigned to the several numbers and subdivisions marked on this line. But for any one calculation, the value must remain the same.

Ex. Take from the scale 365.

As this number is between 10 and 1000, let the 1 at the beginning of the scale, be reckoned 10. Then, from this point to the second 3 is 300; to the 6th dividing stroke is 60; and half way from this to the next stroke is 5.

180. Multiplication, division, &c., are performed by the line of numbers, on the same principle, as by common logarithms. Thus,

To *multiply* by this line, *add* the logarithms of the two factors; (Art. 37.) that is, take off, with the dividers, that length of line which represents the logarithm of *one* of the factors, and apply this so as to extend forward from the end of that which represents the logarithm of the *other* factor. The sum of the two will reach to the end of the line representing the logarithm of the product.

Ex. Multiply 9 into 8. The extent from 1 to 8, added to that from 1 to 9, will be equal to the extent from 1 to 72, the product.

181. To *divide* by the logarithmic line, *subtract* the logarithm of the divisor from that of the dividend; (Art. 41.) that is, take off the logarithm of the divisor, and this extent set back from the end of the logarithm of the dividend, will reach to the logarithm of the quotient.

Ex. Divide 42 by 7. The extent from 1 to 7, set back from 42, will reach to 6, the quotient.

182. *Involution* is performed in logarithms, by multiplying the logarithm of the quantity into the index of the power; (Art. 45.) that is, by *repeating* the logarithms as many times as there are units in the index. To involve a quantity on the scale, then, take in the dividers the linear logarithm, and *double it, treble it, &c.,* according to the index of the proposed power.

Ex. 1. Required the square of 9. Extend the dividers from 1 to 9. *Twice* this extent will reach to 81, the square.

2. Required the cube of 4. The extent from 1 to 4 repeated *three times,* will reach to 64 the cube of 4.

183. On the other hand, to perform *evolution* on the scale;

take *half, one-third, &c.*, of the logarithm of the quantity, according to the index of the proposed root.

Ex. 1. Required the square root of 49. *Half* the extent from 1 to 49, will reach from 1 to 7, the root.

2. Required the cube root of 27. *One third* the distance from 1 to 27, will extend from 1 to 3, the root.

184. The *Rule of Three* may be performed on the scale, in the same manner as in logarithms, by adding the two middle terms, and from the sum, subtracting the first term (Art. 52.) But it is more convenient in practice to *begin* by subtracting the first term from one of the others. If four quantities are proportional, the quotient of the first divided by the second, is equal to the quotient of the third divided by the fourth. (Alg. 315.)

Thus, if $a : b :: c : d$, then $\frac{a}{b}=\frac{c}{d}$, and $\frac{a}{c}=\frac{b}{d}$. (Alg. 344.)

But in logarithms, *subtraction* takes the place of division; so that,

$\log. a - \log. b = \log. c - \log. d$. Or, $\log. a - \log. c = \log. b - \log. d$.

Hence,

185. On the scale, *the difference between the first and second terms of a proportion, is equal to the difference between the third and fourth.* Or, the difference between the first and third terms, is equal to the difference between the second and fourth.

The difference between the two terms is taken, by extending the dividers from one to the other. If the second term be greater than the first; the fourth must be greater than the third; if less, less.* Therefore, if the dividers extend *forward* from *left to right*, that is, from a less number to a greater, from the first term to the second;

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* Euclid, 14, 5.

they must also extend forward from the third to the fourth. But if they extend *backward*, from the first term to the second ; they must extend the same way, from the third to the fourth.

Ex. 1. In the proportion 3 : 8 : : 12 : 32, the extent from 3 to 8, will reach from 12 to 32 ; Or, the extent from 3 to 12, will reach from 8 to 32.

2. If 54 yards of cloth cost 48 dollars, what will 18 yds. cost ?

54 : 48 : : 18 : 16

The extent from 54 to 48, will reach *backwards* from 18 to 16.

3. If 63 gallons of wine cost 81 dollars, what will 35 gallons cost ?

63 : 81 : : 35 : 45

The extent from 63 to 81, will reach from 35 to 45.

The Line of Sines.

186. The line on Gunter's scale marked SIN. is a line of logarithmic sines, made to correspond with the line of numbers. The whole extent of the line of numbers, (Art, 179.)

is from 1 to 100, whose logs. are 0.00000 and 2.00000,
or from 10 to 1000, whose logs. are 1.00000 and 3.00000,
or from 100 to 10000, whose logs. are 2.00000 4.00000,

the *difference of the indices* of the two extreme logarithms being in each case 2.

Now the logarithmic sine of $0^\circ\ 34'\ 22''\ 41'''$ is 8.00000
And the sine of 90° (Art. 95.) is 10.00000

Here also the difference of the indices is 2. If then the point directly beneath one extremity of the line of numbers, be marked for the sine of $0^\circ\ 34'\ 22''\ 41'''$; and the point beneath the other extremity, for the sine of 90°; the interval may furnish the intermediate sine; the divisions on it being made to correspond with the decimal part of the logarithmic sines in the tables.*

The first dividing stroke in the line of Sines is generally at 0° 40′, a little farther to the right than the beginning of the line of numbers. The next division is at 0° 50′; then begins the numbering of the degrees, 1, 2, 3, 4, &c., from left to right.

The Line of Tangents.

187. The first 45 degrees on this line are numbered from left to right, nearly in the same manner as on the line of Sines.

The logarithmic tangent of 0° 34′ 22″ 35‴ is 8.00000
And the tangent of 45°, (Art. 95.) is 10.00000

The difference of the indices being 2, 45 degrees will reach to the end of the line. For those above 45° the scale ought to be continued much farther to the right. But as this would be inconvenient, the numbering of the degrees, after reaching 45, is *carried back* from right to left. The same dividing stroke answers for an arc and its *complement*, one above and the other below 45°. For, (Art. 93. Propor. 9.)

$$\tan : R :: R : \cot.$$

In logarithms, therefore, (Art. 184.)

$$\tan - R = R - \cot.$$

That is, the *difference* between the tangent and radius, is equal to the difference between radius and the cotangent: in other words, one is as much *greater* than the tangent of 45°, as the other is *less*. In taking, then, the tangent of an arc greater than 45°, we are to suppose the distance between 45 and the division marked with a given number of degrees, to be added to the whole line, in the same manner as if the line were continued out. In working proportions, extending the dividers *back*, from a less number to a greater, must be considered the same as carrying them *forward* in other cases. See Art. 185.

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* To represent the sines *less* than 34′ 22″ 41‴, the scale must be extended on the left indefinitely. For, as the sine of an arc approaches to 0, its logarithm, which is negative, increases without limit. (Art. 15.)

Trigonometrical Proportions on the Scale.

188. In working proportions in trigonometry by the scale; *the extent from the first term to the middle term of the same name, will reach from the other middle term to the fourth term.* (Art. 185.)

In a trigonometrical proportion, two of the terms are the lengths of sides of the given triangle; and the other two are tabular sines, tangents, &c. The former are to be taken from the line of numbers; the latter, from the lines of logarithmic sines and tangents. If one of the terms is a *secant*, the calculation cannot be made on the scale, which has commonly no line of secants. It must be kept in mind that *radius* is equal to the sine of 90°, or to the tangent of 45°. (Art. 95.) Therefore, whenever radius is a term in the proportion, one foot of the dividers must be set on the end of the line of sines or of tangents.

189. The following examples are taken from the proportions which have already been solved by numerical calculation.

Ex. 1. In Case I, of right angled triangles, (Art. 134. ex. 1.)

$$R : 45 :: \sin 32^\circ\ 20' : 24$$

Here the third term is a *sine;* the first term radius is, therefore, to be considered as the sine of 90°. Then the extent from 90° to 32° 20′ on the line of sines, will reach from 45 to 24 on the line of numbers. As the dividers are set *back* from 90° to 32° 20′; they must also be set back from 45. (Art. 185.)

2. In the same case, if the base be made radius, (page 60.)

$$R : 38 :: \tan 32^\circ 20' : 24$$

Here, as the third term is a *tangent*, the first term radius is to be considered the tangent of 45°. Then the extent from 45° to 32° 20′ on the line of tangents, will reach from 38 to 24 on the line of numbers.

3. If the perpendicular be made radius, (page 62.)

$$R : 24 :: \tan 57^\circ 40' : 38$$

The extent from 45° to 57° 40′ on the line of tangents, will reach from 24 to 38 on the line of numbers. For the tangent of 57° 40′ on the scale, look for its *complement* 32° 20′. (Art. 187.) In this example, although the dividers extend *back* from 45° to 57° 40′; yet, as this is from a *less* number to a *greater*, they must extend *forward* on the line of numbers. (Arts. 185, 187.)

4. In Art. 135,

$$35 : R :: 26 : \sin 48^\circ$$

The extent from 35 to 26 will reach from 90° to 48°.

5. In Art. 136,

$$R : 48 :: \tan 27\tfrac{1}{4}^\circ : 24\tfrac{3}{4}$$

The extent from 45° to $27\frac{1}{4}$°, will reach from 48 to $24\frac{3}{4}$.

6. In Art. 150, ex. 1.

$$\text{Sin } 74^\circ 30' : 32 :: \sin 56^\circ 20' : 27\tfrac{1}{2}.$$

For other examples, see the several cases in Sections III. and IV.

190. Though the solutions in trigonometry may be ef-

fected by the logarithmic scale, or by geometrical construction, as well as by arithmetical computation; yet the latter method is by far the most accurate. The first is valuable principally for the *expedition* with which the calculations are made by it. The second is of use, in presenting the *form* of the triangle to the eye. But the accuracy which attends arithmetical operations, is not to be expected, in taking lines from a scale with a pair of dividers.*

SECTION VII.

THE FIRST PRINCIPLES OF TRIGONOMETRICAL ANALYSIS.

ART. 191. In the preceding sections, sines, tangents, and secants have been employed in calculating the sides and angles of triangles. But the use of these lines is not confined to this object. Important assistance is derived from them, in conducting many of the investigations in the higher branches of analysis, particularly in physical astronomy. It does not belong to an elementary treatise of trigonometry, to prosecute these inquiries to any considerable extent. But this is the proper place for *preparing the formulæ*, the applications of which are to be made elsewhere.

Positive and Negative SIGNS *in Trigonometry.*

192. Before entering on a particular consideration of the algebraic expressions which are produced by combinations of the several trigonometrical lines, it will be necessary to attend to the positive and negative *signs* in the different

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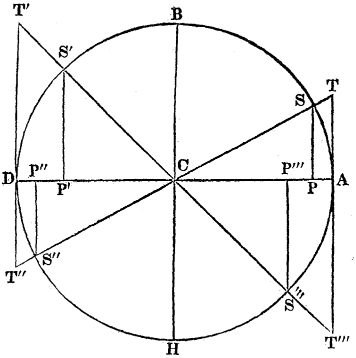
* See note C.

quarters of the circle. The sines, tangents, &c., in the tables, are calculated for a single quadrant only. But these are made to answer for the whole circle. For they are of the same length in each of the four quadrants. (Art. 90.) Some of them however, are *positive;* while others are *negative.* In algebraic processes, this distinction must not be neglected.

193. For the purpose of tracing the changes of the signs, in different parts of the circle, let it be supposed that a straight line CT is fixed at one end C, while the other end is carried round, like a rod moving on a pivot; so that the point S shall describe the circle ABDH. If the two diameters AD and BH, be perpendicular to each other, they will divide the circle into quadrants.



194. In the *first quadrant* AB, the sine, cosine, tangent, &c., are considered *all positive.* In the *second quadrant* BD, the sine P′S′ continues *positive;* because it is still on the *upper* side of the diameter AD, from which it is measured. But the *cosine,* which is measured from BH, becomes *negative,* as soon as it changes from the *right* to the *left* of this line. (Alg. 382.) In the *third quadrant* the *sine* becomes negative, by changing from the upper side to the under side of DA. The *cosine* continues negative, being still on the left of BH. In the *fourth quadrant,* the *sine* continues negative. But the *cosine* becomes positive, by passing to the right of BH.

195. The signs of the *tangents* and *secants* may be derived from those of the sines and cosines. The relations of these several lines to each other must be such, that a uniform method of calculation may extend through the different quadrants.

In the first quadrant, (Art. 93. Propor. 1.)

$$\mathrm{R} : \cos :: \tan : \sin, \text{ that is, } \mathrm{Tan} = \frac{\mathrm{R} \times \sin}{\cos}$$

The sign of the quotient is determined from the signs of the divisor and dividend. (Alg. 100.) The radius is considered as always positive. If then the sine and cosine be both positive or both negative, the tangent will be positive. But if one of these be positive, while the other is negative, the tangent will be negative.

Now by the preceding article,

In the 2d quadrant, the sine is positive, and the cosine negative.

The tangent must therefore be *negative.*

In the 3d quadrant, the sine and cosine are both negative.

The tangent must therefore be *positive.*

In the 4th quadrant, the sine is negative, and the cosine positive.

The tangent must therefore be *negative.*

196. By the 9th, 3d, and 6th proportions in Art. 93.

1. $\mathrm{Tan} : \mathrm{R} :: \mathrm{R} : \cot$, that is $\mathrm{Cot} = \frac{\mathrm{R}^2}{\tan}$.

Therefore, as radius is uniformly positive, the *cotangent* must have the same sign as the tangent.

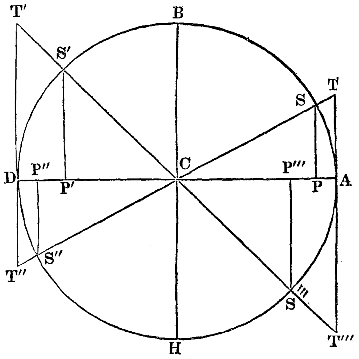
2. $\mathrm{Cos} : \mathrm{R} :: \mathrm{R} : \sec$, that is, $\mathrm{Sec} = \frac{\mathrm{R}^2}{\cos}$.

The *secant,* therefore, must have the same sign as the cosine.

3. Sin : R : : R : cosec, that is, Cosec$=\frac{R^2}{\sin}$.

The *cosecant,* therefore, must have the same sign as the sine.

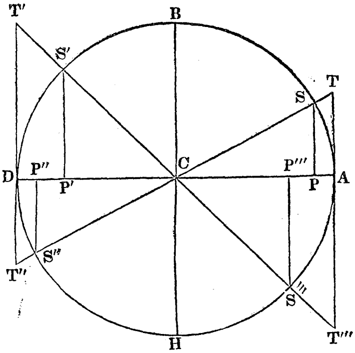
The *versed sine,* as it is measured from A, in one direction only, is invariably positive.



197. The *tangent* AT increases, as the arc extends from A towards B. (See also Fig 11. p. 69.) Near B the increase is very rapid; and when the difference between the arc and 90°, is *less* than any assignable quantity, the tangent is *greater* than any assignable quantity, and is said to be *infinite.* (Alg. 447.) If the arc is *exactly* 90 degrees, it has, strictly speaking, *no* tangent. For a tangent is a line drawn perpendicular to the diameter which passes through one end of the arc, and extended till it *meets* a line proceeding from the centre through the other end. (Art. 84.) But if the arc is 90 degrees, as AB, the angle ACB is a right angle, and therefore AT is *parallel* to CB; so that, if these lines be extended ever so far, they never can meet. Still, as an arc infinitely near to 90° has a tangent infinitely great, it is frequently said, in concise terms, that the tangent of 90° is infinite.

In the second quadrant, the tangent is, at first, infinitely great, and gradually diminishes, till at D it is reduced to nothing. In the third quadrant, it increases again, becomes infinite near H, and is reduced to nothing at A.

The *cotangent* is inversely as the tangent. It is therefore nothing at B and H, and infinite near A and D.



198. The *secant* increases with the tangent, through the first quadrant, and becomes infinite near B; it then diminishes, in the second quadrant, till at D it is equal to the radius CD. In the third quadrant it increases again, becomes infinite near H, after which it diminishes, till it becomes equal to radius.

The *cosecant* decreases, as the secant increases, and *v. v.* It is therefore equal to radius at B and H, and infinite near A and D.

199. The *sine* increases through the first quadrant, till at B it is equal to radius. (See also Fig. 13. page 70.) It then diminishes, and is reduced to nothing at D. In the third quadrant, it increases again, becomes equal to radius at H, and is reduced to nothing at A.

The *cosine* decreases through the first quadrant, and is reduced to nothing at B. In the second quadrant, it increases, till it becomes equal to radius at D. It then diminishes again, is reduced to nothing at H, and afterwards increases till it becomes equal to radius at A.

In all these cases, the arc is supposed to *begin* at A, and to extend round in the direction of BDH.

200. The *sine* and *cosine* vary from nothing to radius, which they never exceed. The *secant* and *cosecant* are never less than radius, but may be greater than any given length.

The *tangent* and *cotangent* have every value from nothing to infinity. Each of these lines, after reaching its *greatest* limit, begins to *decrease;* and as soon as it arrives at its *least* limit, begins to *increase.* Thus, the sine begins to decrease, after becoming equal to radius, which is its greatest limit. But the secant begins to increase after becoming equal to radius, which is its least limit.

201. The substance of several of the preceding articles is comprised in the following tables. The first shows the *signs* of the trigonometrical lines, in each of the quadrants of the circle. The other gives the *values* of these lines, at the extremity of each quadrant.

Quadrant	1st	2d	3d	4th
Sine and cosecant	+	+	—	—
Cosine and secant	+	—	—	+
Tangent and cotangent	+	—	+	—

	0°	90°	180°	270°	360°
Sine	0	r	0	r	0
Cosine	r	0	r	0	r
Tangent	0	∝	0	∝	0
Cotangent	∝	0	∝	0	∝
Secant	r	∝	r	∝	r
Cosecant	∝	r	∝	r	∝

Here r is put for radius, and ∝ for infinite.

202. By comparing these two tables, it will be seen, that each of the trigonometrical lines changes from positive to negative, or from negative to positive, in that part of the circle in which the line is either *nothing* or *infinite.* Thus,

the tangent changes from positive to negative, in passing from the first quadrant to the second, through the place where it is infinite. It becomes positive again, in passing from the second quadrant to the third, through the point in which it is nothing.

203. There can be no more than 360 degrees in any circle. But a body may have a number of successive revolutions in the same circle; as the earth moves round the sun, nearly in the same orbit, year after year. In astronomical calculations, it is frequently necessary to add together parts of different revolutions. The sum may be more than 360°. But a body which has made more than a complete revolution in a circle, is only brought back to a point which it had passed over before. So the sine, tangent, &c., of an arc greater than 360°, is the same as the sine, tangent, &c., of some arc less than 360°. If an entire circumference, or a number of circumferences, be added to any arc, it will terminate in the same point as before. So that, if C be put for a whole circumference, or 360°, and x be any arc whatever;

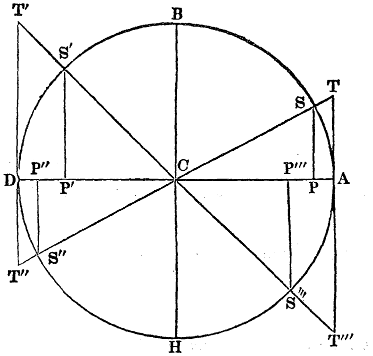
$$\sin x = \sin(C+x) = \sin(2C+x) = \sin(3C+x), \text{ \&c.}$$
$$\tan x = \tan(C+x) = \tan(2C+x) = \tan(3C+x), \text{ \&c.}$$

204. It is evident also, that, in a number of successive revolutions, in the same circle;

The first quadrant must coincide with the	5th, 9th, 13th, 17th,
The second, with the	6th, 10th, 14th, 18th, &c.
The third, with the	7th, 11th, 15th, 19th, &c.
The fourth, with the	8th, 12th, 16th, 20th, &c.

205. If an arc, extending in a certain direction from a given point, be considered *positive;* an arc extending from the same point, in an *opposite* direction, is to be considered

negative. (Alg. 382.) Thus, if the arc extending from A to S, be positive; an arc extending from A to S''' will be negative. The latter will not terminate in the *same quadrant* as the other—and the signs of the tabular lines must be accommodated to this circumstance. Thus, the sine of AS will be positive, while that of AS''' will be negative. (Art. 194.) When a greater arc is subtracted from a less, if the latter be positive, the *remainder* must be negative. (Alg. 40.)



TRIGONOMETRICAL FORMULÆ.

206. From the view which has been here taken of the changes in the trigonometrical lines, it will be easy to see, in what parts of the circle each of them increases or decreases. But this does not determine their exact values, except at the extremities of the several quadrants. In the analytical investigations which are carried on by means of these lines, it is necessary to calculate the changes produced in them, by a given increase or diminution of the arcs to which they belong. In this there would be no difficulty, if the sines, tangents, &c., were *proportioned* to their arcs. But this is far from being the case. If an arc is doubled, its sine is *not* exactly doubled. Neither is its tangent or secant. We have to inquire, then, in what manner the sine,

tangent, &c., of one arc may be obtained, from those of other arcs already known.

The problem on which almost the whole of this branch of analysis depends, consists in deriving, from the sines and cosines of two given arcs, expressions for the sine and cosine of their *sum* and *difference*. For, by addition and subtraction, a few arcs may be so combined and varied, as to produce others of almost every dimension. And the expressions for the tangents and secants may be deduced from those of the sines and cosines.

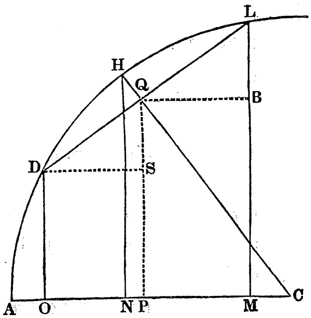
Expressions for the SINE *and* COSINE *of the* SUM *and* DIFFERENCE *of arcs.*

207. Let a=AH, the greater of the given arcs,

And b=HL=HD, the less.

Then $a+b$=AH+HL=AL, the sum of the two arcs,

And $a-b$=AH—HD=AD, their *difference*.



Draw the chord DL, and the radius CH, which may be represented by R. As DH is, by construction, equal to HL; DQ is equal to QL, and therefore DL is perpendicular to CH. (Euc. 3. 3.) Draw DO, HN, QP, and LM, each perpendicular to AC; and DS and QB parallel to AC.

From the definitions of the sine and cosine, (Arts. 82, 9.) it is evident, that

$$\text{The sine}\begin{cases}\text{of AH, that is, } \sin a = HN,\\ \text{of HL,} \quad \sin b = QL,\\ \text{of AL,} \quad \sin (a+b) = LM,\\ \text{of AD,} \quad \sin (a-b) = DO,\end{cases}$$

$$\text{The cosine}\begin{cases}\text{of AH, that is, } \cos a = CN,\\ \text{of HL,} \quad \cos b = CQ,\\ \text{of AL,} \quad \cos (a+b) = CM,\\ \text{of AD,} \quad \cos (a-b) = CO.\end{cases}$$

The triangle CHN is obviously similar to CQP; and it is also similar to BLQ, because the sides of the one are perpendicular to those of the other, each to each. We have, then,

1. $CH : CQ :: HN : QP$, that is, $R : \cos b :: \sin a : QP$,
2. $CH : QL :: CN : BL$, $\quad R : \sin b :: \cos a : BL$,
3. $CH : CQ :: CN : CP$ $\quad R : \cos b :: \cos a : CP$,
4. $CH : QL :: HN : QB$, $\quad R : \sin b :: \sin a : QB$,

Converting each of these proportions into an equation;

1. $QP = \dfrac{\sin a \cos b\text{*}}{R}$ $\qquad$ 3. $CP = \dfrac{\cos a \cos b}{R}$

2. $BL = \dfrac{\sin b \cos a}{R}$ $\qquad$ 4. $QB = \dfrac{\sin a \sin b}{R}$

Then adding the first and second,

$$QP + BL = \frac{\sin a \cos b + \sin b \cos a}{R}$$

Subtracting the second from the first,

$$QP - BL = \frac{\sin a \cos b - \sin b \cos a}{R}$$

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* In these formulæ, the sign of multiplication is omitted; sin a cos b ing put for sin $a \times$ cos b, that is, the product of the sine of a into the cosine of b.

Subtracting the fourth from the third,

$$CP-QB=\frac{\cos a\cos b-\sin a\sin b}{R}$$

Adding the third and fourth,

$$CP+QB=\frac{\cos a\cos b+\sin a\sin b}{R}$$

But it will be seen, from the figure, that

$$QP+BL=BM+BL=LM=\sin(a+b)$$
$$QP-BL=QP-QS=DO=\sin(a-b)$$
$$CP-QB=CP-PM=CM=\cos(a+b)$$
$$CP+QB=CP+SD=CO=\cos(a-b)$$

208. If then, for the first member of each of the four equations above, we substitute its value, we shall have,

$$\text{I. } \sin(a+b=\frac{\sin a\cos b+\sin b\cos a}{R}$$

$$\text{II. } \sin(a-b)=\frac{\sin a\cos b-\sin b\cos a}{R}$$

$$\text{III. } \cos(a+b)=\frac{\cos a\cos b-\sin a\sin b}{R}$$

$$\text{IV. } \cos(a-b)=\frac{\cos a\cos b+\sin a\sin b}{R}$$

Or multiplying both sides by R,

$$R\sin(a+b)=\sin a\cos b+\sin b\cos a$$
$$R\sin(a-b)=\sin a\cos b-\sin b\cos a$$
$$R\cos(a+b)=\cos a\cos b-\sin a\sin b$$
$$R\cos(a-b)=\cos a\cos b+\sin a\sin b$$

That is, the product of radius and the *sine* of the *sum* of two arcs, is equal to the product of the sine of the first arc into the cosine of the second + the product of the sine of the second into the cosine of the first.

The product of radius and the *sine* of the *difference* of two arcs, is equal to the product of the sine of the first arc into the cosine of the second — the product of the sine of the second into the cosine of the first.

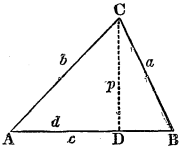
The product of radius and the *cosine* of the *sum* of two arcs, is equal to the product of the cosines of the arcs — the product of their sines.

The product of radius and the *cosine* of the *difference* of two arcs, is equal to the product of the cosines of the arcs + the product of their sines.

These four equations may be considered as fundamental propositions, in what is called the *Arithmetic* of *Sines and Cosines*, or *Trigonometrical Analysis*.

Expression for the *area* of a triangle, in terms of the sides.

209. Let the sides of the triangle ABC be expressed by a, b, and c, the perpendicular CD by p, the segment AD by d, and the area by S.



Then $a^2=b^2+c^2-2cd$, (Euc. 13. 2.)

Transposing and dividing by $2c$;

$d=\frac{b^2+c^2-a^2}{2c}$. Therefore $d^2=\frac{(b^2+c^2-a^2)^2}{4c^2}$. (Alg. 171.)

By Euc. 47, 1, $p^2=b^2-d^2=b^2-\frac{(b^2+c^2-a^2)^2}{4c^2}$

Reducing the fraction, (Alg. 120.) and extracting the root of both sides,

$$p=\frac{\sqrt{4b^2c^2-(b^2+c^2-a^2)^2}}{2c}\ *$$

This gives the length of the *perpendicular* in terms of the sides of the triangle. But the *area* is equal to the product of the base into half the perpendicular height. (Alg. 393.) That is,

$$S=\tfrac{1}{2}cp=\tfrac{1}{4}\sqrt{4b^2c^2-(b^2+c^2-a^2)^2}$$

Here we have an expression for the area, in terms of the sides. But this may be reduced to a form much better adapted to arithmetical computation. It will be seen, that the quantities $4b^2c^2$, and $(b^2+c^2-a^2)^2$ are both *squares*; and that the whole expression under the radical sign is the *difference* of these squares. But the difference of two squares is equal to the product of the sum and difference of their roots. (Alg. 191.) Therefore, $4b^2c^2-(b^2+c^2-a^2)^2$ may be resolved into the two factors,

$$\begin{cases} 2bc+(b^2+c^2-a^2) \text{ which is equal to } (b+c)^2-a^2 \\ 2bc-(b^2+c^2-a^2) \text{ which is equal to } a^2-(b-c)^2 \end{cases}$$

Each of these also, as will be seen in the expressions on the right, is the difference of two squares; and may, on the same principle, be resolved into factors, so that,

$$\begin{cases} (b+c)^2-a^2=(b+c+a)\times(b+c-a) \\ a^2-(b-c)^2=(a+b-c)\times(a-b+c) \end{cases}$$

---

* The expression for the perpendicular is the same, when one of the angles is *obtuse*, as in Fig. 24. page 86. Let AD$=d$.

Then $a^2=b^2+c^2+2cd$. (Euc. 12, 2.) And $d=\frac{-b^2-c^2+a^2}{2c}$

Therefore, $d^2=\frac{(-b^2-c^2+a^2)^2}{4c^2}=\frac{(b^2+c^2-a^2)^2}{4c^2}$ (Alg. 169.)

And $p=\frac{\sqrt{4b^2c^2-(b^2+c^2-a^2)^2}}{2c}$. as above

Substituting, then, these four factors, in the place of the quantity which has been resolved into them, we have,

$$S=\tfrac{1}{4}\sqrt{(b+c+a)\times(b+c-a)\times(a+b-c)\times(a-b+c},$$

Here it will be observed, that all the three sides, a, b, and c, are in each of these factors.

Let $h=\frac{1}{2}(a+b+c)$ *half the sum* of the sides. Then

$$S=\sqrt{h\times(h-a)\times(h-b)\times(h-c)}$$

210. For finding the area of a triangle, then, when the three sides are given, we have this general rule;

From half the sum of the sides, subtract each side severally; multiply together the half sum and the three remainders; and extract the square root of the product.

APPLICATION OF TRIGONOMETRY

TO THE

MENSURATION

OF

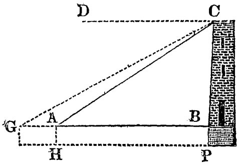
HEIGHTS AND DISTANCES.

ART. 1. The most direct and obvious method of determining the distance or height of any object, is to apply to it some known measure of length, as a foot, a yard, or a rod. In this manner, the height of a room is found, by a joiner's rule; or the side of a field by a surveyor's chain. But in many instances, the object, or a part, at least, of the line which is to be measured, is *inaccessible.* We may wish to determine the breadth of a river, the height of a cloud, or the distances of the heavenly bodies. In such cases it is necessary to measure some *other* line; from which the required line may be obtained, by geometrical construction, or more exactly, by trigonometrical calculation. The line first measured is frequently called a *base* line.

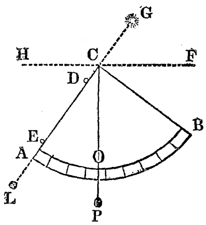
2. In measuring *angles,* some instrument is used which contains a portion of a graduated circle divided into degrees and minutes. For the proper measure of an angle is an arc of a circle, whose centre is the angular point. (Trig. 74.) The instruments used for this purpose are made in different forms, and with various appendages. The essential parts are a graduated circle, and an index with sight-holes, for taking the directions of the lines which include the angles.

3. Angles of *elevation,* and of *depression* are in a plane

perpendicular to the horizon, which is called a *vertical plane.* An angle of *elevation* is contained between a parallel to the horizon, and an ascending line, as BAC. An angle of *depression* is contained between a parallel to the horizon, and a descending line, as DCA. The *complement* of this is the angle ACB.



4. The instrument by which angles of elevation, and of depression, are commonly measured, is called a *Quadrant.* In its most simple form, it is a portion of a circular board ABC, on which is a graduated arc of 90 degrees, AB, a plumb line CP, suspended from the central point C, and two sight-holes D and E, for taking the direction of the object.



To measure an angle of *elevation* with this, hold the plane of the instrument perpendicular to the horizon, bring the centre C to the angular point, and direct the edge AC in such a manner, that the object G may be seen through the two sight-holes. Then the arc BO measures the angle BCO, which is equal to the angle of elevation FCG. For as the plumb line is perpendicular to the horizon, the angle FCO is a right angle, and therefore equal to BCG. Taking from these the common angle BCF, there will remain the angle BCO=FCG.

In taking an angle of *depression,* as HCL, the eye is placed at C, so as to view the object at L, through the sight-holes D and E.

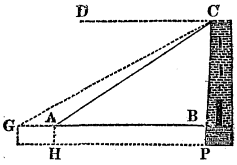
5. In treating of the mensuration of heights and dis-

tances, no *new principles* are to be brought into view. We have only to make an application of the rules for the solution of triangles, to the particular circumstances in which the observer may be placed, with respect to the line to be measured. These are so numerous, that the subject may be divided into a great number of distinct cases. But as they are all solved upon the same general principles, it will not be necessary to give examples under each. The following problems may serve as a specimen of those which most frequently occur in practice.

PROBLEM I.

TO FIND THE PERPENDICULAR HEIGHT OF AN ACCESSIBLE OBJECT STANDING ON A HORIZONTAL PLANE.

6. MEASURE FROM THE OBJECT TO A CONVENIENT STATION, AND THERE TAKE THE ANGLE OF ELEVATION SUBTENDED BY THE OBJECT.



If the distance AB be measured, and the angle of elevation BAC; there will be given in the right angled triangle ABC, the base and the angles, to find the perpendicular. (Trig. 137.)

As the instrument by which the angle at A is measured, is commonly raised a few feet above the ground; a point B must be taken in the object, so that AB shall be parallel to the horizon. The part BP, may afterwards be added to the height BC, found by trigonometrical calculation.

Ex. 1. What is the height of a tower BC, if the distance AB, on a horizontal plane, be 93 feet; and the angle BAC $35\frac{1}{2}$ degrees?

Making the hypothenuse radius (Trig 121.)

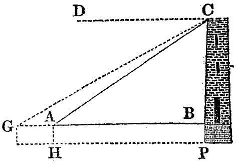
Cos. BAC : AB : : Sin. BAC : BC=69.9 feet.

For the *geometrical construction* of the problem, see Trig. 169.

2. What is the height of the perpendicular sheet of water at the falls of Niagara, if it subtends an angle of 40 degrees, at the distance of 163 feet from the bottom, measured on a horizontal plane ? Ans. 136$\frac{3}{4}$ feet.

7. If the height of the object be *known*, its *distance* may be found by the angle of elevation. In this case the angles, and the perpendicular of the triangle are given, to find the base.

Ex. A person on shore, taking an observation of a ship's mast, which is known to be 99 feet high, finds the angle of elevation 3$\frac{1}{2}$ degrees. What is the distance of the ship from the observer ? Ans. 98 rods.



8. If the observer be stationed at the *top* of the perpendicular BC, whose height is known ; he may find the length of the base line AB, by measuring the angle of *depression* ACD, which is equal to BAC.

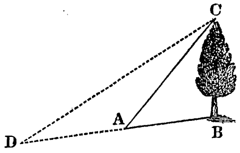
Ex. A seaman at the top of a mast 66 feet high, looking at another ship, finds the angle of depression 10 degrees. What is the distance of the two vessels from each other ?
Ans. 22$\frac{2}{3}$ rods.

We may find the distance between *two objects* which are in the same vertical plane with the perpendicular, by calculating the distance of each from the perpendicular. Thus AG is equal to the difference between AB and GB.

Problem II.

TO FIND THE HEIGHT OF AN ACCESSIBLE OBJECT STANDING ON AN INCLINED PLANE.

9. Measure the distance from the object to a convenient station, and take the angles which this base makes with lines drawn from its two ends to the top of the object.



If the base AB be measured and the angles BAC and ABC; there will be given, in the oblique angled triangle ABC, the side AB, and the angles, to find BC. (Trig. 150.)

Or the height BC may be found by measuring the distances BA, AD, and taking the angles, BAC and BDC. There will then be given in the triangle ADC, the angles and the side AD, to find AC; and consequently, in the triangle ABC, the sides AB and AC with the angle BAC, to find BC.

Ex. If AB be 76 feet, the angle B 101° 25′, and the angle A 44° 42′; what is the height of the tree BC?

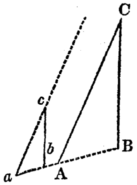
Sin. C : AB : : Sin. A : : BC=95.9 feet.

For the *geometrical construction* of the problem, see Trig. 169.

10. The following are some of the methods by which the height of an object may be found, without measuring the angle of elevation.

1. *By shadows.* Let the staff *bc* be parallel to an ob-

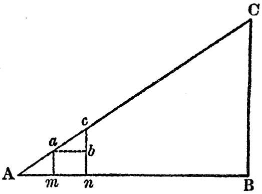
ject BC, whose height is required. If the shadow of BC extend to A, and that of *bc* to *a;* the rays of light CA and *ca* coming from the sun may be considered parallel; and therefore the triangles ABC and *abc* are similar; so that



$$ab : bc :: AB : BC.$$

Ex. If *ab* be 3 feet, *bc* 5 feet, and AB 69 feet, what is the height of BC? Ans. 115 feet.

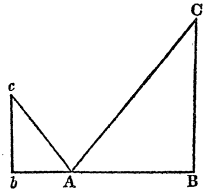
2. *By parallel rods.* If two poles *am* and *cn* be placed parallel to the object BC, and at such distances as to bring the points C, *c*, *a*, in a line, and if *ab* be made parallel to AB; the triangles ABC, and *abc* will be similar; and we shall have



$$ab : bc :: AB : BC.$$

One pole will be sufficient, if the observer can place his eye at the point A, so as to bring A, *a*, and C in a line.

3. *By a mirror.* Let the smooth surface of a body of water at A, or any plane mirror parallel to the horizon, be so situated, that the eye of the observer at *c* may view the top of the object C reflected from the mirror. By a law of Optics, the angle BAC is equal to *b*A*c*; and if *bc* be made parallel to BC, the triangle *b*A*c* will be similar to BAC; so that

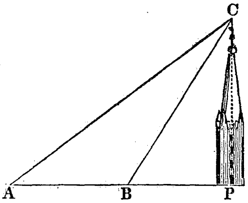


$$Ab : bc :: AB : BC.$$

PROBLEM III.

TO FIND THE HEIGHT OF AN INACCESSIBLE OBJECT ABOVE A HORIZONTAL PLANE.

11. TAKE TWO STATIONS IN A VERTICAL PLANE PASSING THROUGH THE TOP OF THE OBJECT, MEASURE THE DISTANCE FROM ONE STATION TO THE OTHER, AND THE ANGLE OF ELEVATION AT EACH.



If the base AB be measured, with the angle CBP and CAB; as ABC is the supplement of CBP, there will be given, in the oblique angled triangle ABC, the side AB and the angles, to find BC; and then in the right angled triangle BCP, the hypothenuse and the angles, to find the perpendicular CP.

Ex. 1. If C be the top of a spire, the horizontal base line AB 100 feet, the angle of elevation BAC 40°, and the angle PBC 60°; what is the perpendicular height of the spire?

The difference between the angles PBC and BAC is equal to ACB. (Euc. 32. 1.)

Then Sin ACB : AB : : Sin BAC : BC=187.9
And R : BC : : Sin PBC : CP=$162\frac{3}{4}$ feet.

2. If two persons 120 rods from each other, are standing on a horizontal plane, and also in a vertical plane passing through a *cloud*, both being on the same side of the cloud: and if they find the angles of elevation at the two stations to be 68° and 76°; what is the height of the cloud?

Ans. 2 miles 135.7 rods.

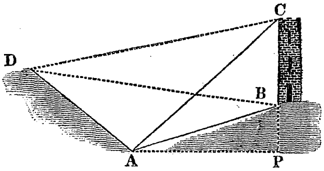
12. The preceding problems are useful in particular cases.

But the following is a *general* rule, which may be used for finding the height of any object whatever, within moderate distances.

PROBLEM IV.

TO FIND THE HEIGHT OF ANY OBJECT, BY OBSERVATIONS AT TWO STATIONS.

13. MEASURE THE BASE LINE BETWEEN THE TWO STATIONS, THE ANGLES BETWEEN THIS BASE AND LINES DRAWN FROM EACH OF THE STATIONS TO EACH END OF THE OBJECT, AND THE ANGLE SUBTENDED BY THE OBJECT, AT ONE OF THE STATIONS.



If BC be the object whose height is required, and if the distance between the stations A and D be measured, with the angles ADC, DAC, ADB, DAB, and BAC; there will be given in the triangle ADC, the side AD and the angles, to find AC; in the triangle ADB, the side AD and the angles, to find AB; and then, in the triangle BAC, the sides AB and AC with the included angle, to find the required height BC.

If the two stations A and D be in the *same plane* with BC, the angle BAC will be equal to the difference between BAD and CAD. In this case it will not be necessary to measure BAC.

Ex. If AD=83 feet,

$\begin{cases} ADC=51^\circ \\ DAC=95^\circ \end{cases}$ $\begin{cases} ADB=33^\circ \\ DAB=121^\circ \end{cases}$ $BAC=26^\circ$,

What is the height of the object BC?

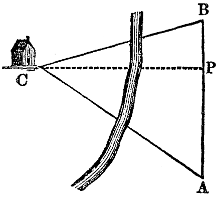
$$\text{Sin ACD} : \text{AD} :: \text{ADC} : \text{AC}=115.3 \text{ (Fig. 8.)}$$
$$\text{Sin ABD} : \text{AD} :: \text{ADB} : \text{AB}=103.1.$$
$$(\text{AC}+\text{AB}) : (\text{AC}-\text{AB}) :: \text{Tan } \tfrac{1}{2} (\text{ABC}+\text{ACB}) : \text{Tan } \tfrac{1}{2} (\text{ABC}-\text{ACB})=13^\circ\ 38'$$
$$\text{Sin ACB} : \text{AB} :: \text{Sin BAC} : \text{BC}=50.57 \text{ feet.}$$

If the object BC be perpendicular to the horizon, its height, after obtaining AB and AC as before, may be found by taking the *angles* of *elevation* BAP and CAP. The difference of the perpendiculars in the right angled triangles ABP and ACP, will be the height required.

Problem V.

TO FIND THE DISTANCE OF AN INACCESSIBLE OBJECT.

14. MEASURE A BASE LINE BETWEEN TWO STATIONS, AND THE ANGLES BETWEEN THIS AND LINES DRAWN FROM EACH OF THE STATIONS TO THE OBJECT.



If C be the object, and if the distance between the stations A and B be measured, with the angles at B and A; there will be given, in the oblique angled triangle ABC, the side AB and the angles, to find AC and BC, the distances of the object from the two stations.

For the geometrical construction, see Trig. 169.

Ex. 1. What are the distances of the two stations A and B from the house C, on the opposite side of a river; if AB be 26.6 rods, B 92° 46′, and A 38° 40′?

The angle $\text{C}=180-(\text{A}+\text{B})=48^\circ\ 34'$. Then

$$\text{Sin C} : \text{AB} :: \begin{cases} \text{Sin A} : \text{BC}=22.17 \\ \text{Sin B} : \text{AC}=35.44. \end{cases}$$

2. Two ships in a harbor, wishing to ascertain how far they are from a fort on shore, find that their mutual distance is 90 rods, and that the angles formed between a line from one to the other, and lines drawn from each to the fort are 45° and 56° 15′. What are their respective distances from the fort? Ans. 76.3 and 64.9 rods.

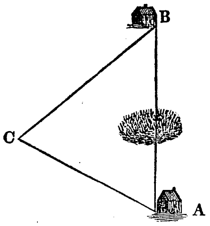
15. The *perpendicular* distance of the object from the line joining the two stations may be easily found, after the distance from one of the stations is obtained. The perpendicular distance PC is one of the sides of the right angled triangle BCP. Therefore

$$R : BC :: \text{Sin } B : PC.$$

PROBLEM VI.

TO FIND THE DISTANCE BETWEEN TWO OBJECTS, WHEN THE PASSAGE FROM ONE TO THE OTHER, IN A STRAIGHT LINE IS OBSTRUCTED.

16. MEASURE THE RIGHT LINES FROM ONE STATION TO EACH OF THE OBJECTS, AND THE ANGLE INCLUDED BETWEEN THESE LINES.



If A and B be the two objects, and if the distances BC and AC be measured, with the angle at C; there will be given, in the oblique angled triangle ABC, two sides and the included angle, to find the other two angles, and the remaining side. (Trig. 153.)

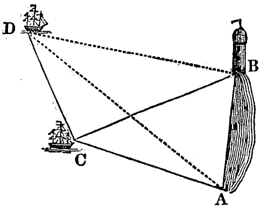
Ex. The passage between the two objects A and B being obstructed by a morass, the line BC was measured and found to be 109 rods, the line AC 76 rods, and the angle at C 101° 30′. What is the distance AB?

Ans. 144.7 rods.

PROBLEM VII.

TO FIND THE DISTANCE BETWEEN TWO INACCESSIBLE OBJECTS.

17. MEASURE A BASE LINE BETWEEN TWO STATIONS AND THE ANGLES BETWEEN THIS BASE AND LINES DRAWN FROM EACH OF THE STATIONS TO EACH OF THE OBJECTS.



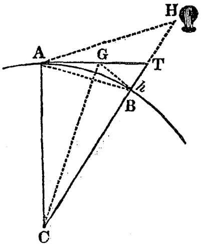
If A and B be the two objects, and if the distance between the stations C and D be measured, with the angles BDC, BCD, ADC, and ACD; the lines AC and BC may be found as in Problem V, and then the distance AB as in Problem VI.

This rule is substantially the same as that in Art. 13. The two stations are supposed to be in the *same plane* with the objects. If they are not, it will be necessary to measure the angle ACB.

18. The same process by which we obtain the distance of *two* objects from each other, will enable us to find the distance between one of these and a third, between that and a fourth, and so on, till a connection is formed between a great number of remote points. This is the plan of the great *Trigonometrical Surveys,* which have been lately carried on, with surprising exactness, particularly in England and France.

19. In the preceding problems for determining altitudes, the objects are supposed to be at such moderate distances, that the observations are not sensibly affected by the *spherical figure of the earth.* The height of an object is measured from an *horizontal plane,* passing through the station at which the angle of elevation is taken. But in an extent

of several miles, the figure of the earth ought to be taken into account.



Let AB be a portion of the earth's surface, H an object above it, and AT a tangent at the point A, or a horizontal line passing through A. Then HT, the oblique height of the object above the horizon of A, is only a *part* of the height above the surface of the earth, or the level of the ocean. To obtain the true altitude, it is necessary to add BT to the height HT found by observation. The height BT may be calculated, if the diameter of the earth and the distance AT be previously known. Or if the height BT be first determined from observation, with the distance AT; the diameter of the earth may be thence deduced.

Problem VIII.

TO FIND THE DIAMETER OF THE EARTH, FROM THE KNOWN HEIGHT OF A DISTANT MOUNTAIN, WHOSE SUMMIT IS JUST VISIBLE IN THE HORIZON.

20. FROM THE SQUARE OF THE DISTANCE DIVIDED BY THE HEIGHT, SUBTRACT THE HEIGHT.

If BT (above figure) be a mountain whose height is known, with the distance AT; and if the summit T be just visible in the horizon at A; then AT is a *tangent* at the point A.

Let $2BC = D$, the diameter of the earth,

$AT = d$, the distance of the mountain,

$BT = h$, its height.

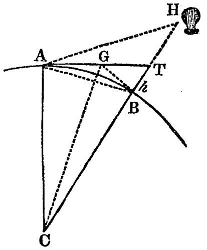
Then considering AT as a straight line, and the earth as a sphere, we have (Euc. 36. 3.)*

$$(2BC+BT)\times BT=AT^2;\ \text{that is,}\ (D+h)\times h=d^2,$$

and reducing the equation,

$$D=\frac{d^2}{h}-h.$$

Ex. The highest point of the Andes is about 4 miles above the level of the ocean. If a straight line from this touch the surface of the water at the distance of $178\frac{1}{4}$ miles; what is the diameter of the earth? Ans. 7940 miles.



21. If the distance AT be *unknown,* it may be found by measuring with a quadrant the angle ATC. Draw BG perpendicular to BC; and join CG. The triangles ACG and BCG are equal, because each has a right angle, the sides AC and BC are equal, and the hypothenuse CG is common. Therefore BG and AG are equal. In the right angled triangle BGT, the angle BTG is given, and the perpendicular BT. From these may be found BG and TG, whose sum is equal to AT, the distance required.†

22. In the common measurement of angles, the light is supposed to come from the object to the eye in a *straight line.* But this is not strictly true. The direction of the light is affected by the *refraction of the atmosphere.* If the object be near, the deviation is very inconsiderable. But in an extent of several miles, and particularly in such nice ob-

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* Thomson's Legendre, 30. 4.

† This method of determining the diameter of the earth is not as accurate as that by measuring a degree of Latitude.

servations as determining the height of distant mountains, and the diameter of the earth, it is necessary to make allowance for the refraction.

PROBLEM IX.

TO FIND THE GREATEST DISTANCE AT WHICH A GIVEN OBJECT CAN BE SEEN ON THE SURFACE OF THE EARTH.

23. TO THE PRODUCT OF THE HEIGHT OF THE OBJECT INTO THE DIAMETER OF THE EARTH, ADD THE SQUARE OF THE HEIGHT; AND EXTRACT THE SQUARE ROOT OF THE SUM.

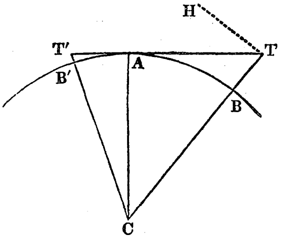
Let $2BC=D$, the diameter of the earth, (Fig. 12.)
$BT=h$, the height of the object,
$AT=d$, the distance required.

Then $(D+h)\times h=d^2$. And $d=\sqrt{Dh+h^2}$.

Ex. If the diameter of the earth be 7940 miles, and Mount Ætna 2 miles high; how far can its summit be seen at sea? Ans. 126 miles.

The actual distance at which an object can be seen, is increased by the refraction of the air.

24. In this problem the eye is supposed to be placed at the level of the ocean. But if the observer be elevated above the surface, as on the deck of a ship, he can see to a greater distance. If BT be the height of the object, and B′T′ the height of the eye above the level of the ocean; the distance at which the object can be seen, is evidently equal to the *sum* of the tangents AT and AT′.



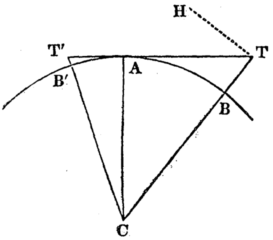
Ex. The top of a ship's mast 132 feet high is just visible in the horizon, to an observer whose eye is 33 feet above the surface of the water. What is the distance of the ship?
Ans. $21\frac{1}{8}$ miles.

25. The distance to which a person can see the smooth surface of the ocean, if no allowance be made for refraction, is equal to a tangent to the earth drawn from his eye, as T′A. (Fig. 13.)

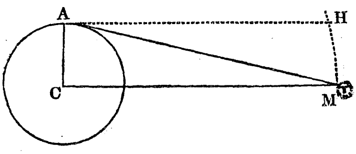
Ex. If a man standing on the level of the ocean, has his eye raised $5\frac{1}{2}$ feet above the water: to what distance can he see the surface?
Ans. $2\frac{7}{8}$ miles.

26. If the distance AT, with the diameter of the earth be given, and the *height* BT be required; the equation in Art. 23 gives

$$h=\sqrt{\tfrac{1}{4}D^2+d^2}-\tfrac{1}{2}D$$



27. When the diameter of the earth is ascertained, this may be made a *base line* for determining the distance of the *heavenly bodies*. A right angled triangle may be formed, the perpendicular sides of which shall be the distance required, and the semi-diameter of the earth. If then one of the angles be found by observation, the required side may be easily calculated.



Let AC be the semi-diameter of the earth, AH the sensible horizon at A, and CM the rational horizon, parallel to AH, passing through the moon M. The angle HAM may be found by astronomical observation. This angle, which is called the *Horizontal Parallax,* is equal to AMC, the angle at the moon subtended by the semi-diameter of the earth. (Euc. 29. 1.)

PROBLEM X.

TO FIND THE DISTANCE OF ANY HEAVENLY BODY WHOSE HORIZONTAL PARALLAX IS KNOWN.

28. AS RADIUS, TO THE SEMI-DIAMETER OF THE EARTH; SO IS THE CO-TANGENT OF THE HORIZONTAL PARALLAX, TO THE DISTANCE.

In the right angled triangle ACM, (Fig. 14.) if AC be made radius;

$$R : AC :: \text{Cot. } AMC : CM.$$

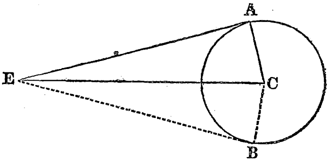
Ex. If the horizontal parallax of the moon be 0° 57′, and the diameter of the earth 7940 miles; what is the distance of the moon from the centre of the earth?

Ans. 239,414 miles.

29. The *fixed stars* are too far distant to have any sensible horizontal parallax. But from late observations it would seem, that some of them are near enough, to suffer a small apparent change of place, from the revolution of the earth round the sun. The distance of the sun, then, which is the semi-diameter of the earth's orbit, may be taken as a *base line,* for finding the distance of the stars.

We thus proceed by degrees from measuring a line on the surface of the earth, to calculate the distances of the heavenly bodies. From a base line on a plane, is determined the height of a mountain; from the height of a mountain, the diameter of the earth; from the diameter of the earth, the distance of the sun, and from the distance of the sun the distance of the stars.

30. After finding the distance of a heavenly body, its *magnitude* is easily ascertained; if it have an apparent diameter, sufficiently large to be measured by the instruments which are used for taking angles.



Let AEB be the angle which a heavenly body subtends at the eye. Half this angle, if C be the centre of the body, is AEC; the line EA is a tangent to the surface, and therefore EAC is a right angle. Then making the distance EC radius,

$$R : EC :: \text{Sin. } AEC : AC.$$

That is, radius is to the distance, as the sine of half the angle which the body subtends, to its semi-diameter.

Ex. If the sun subtends an angle of 32′ 2″, and if his distance from the earth be 95 million miles; what is his diameter? Ans. 885 thousand miles.

Promiscuous Examples.

1. On the bank of a river, the angle of elevation of a tree on the opposite side is found to be 46°; and at another station 100 feet directly back on the same level, 31°. What is the height of the tree? Ans. 143 feet.

2. On a horizontal plane, observations were taken of a tower standing on the top of a hill. At one station the angle of elevation of the top of the tower was found to be 50°; that at the bottom 39°; and at another station 150 feet directly back, the angle of elevation of the top of the tower was 32°. What are the heights of the hill and the tower?

Ans. The hill is 134 feet high; the tower 63.

3. What is the altitude of the sun, when the shadow of a tree, cast on a horizontal plane, is to the height of the tree as 4 to 3? Ans. 36° 52′ 12″.

4. If a straight line from the top of the White Mountains in New Hampshire touch the ocean at the distance of $103\frac{1}{3}$ miles? what is the height of the mountains?

Ans. 7100 feet.

5. From the top of a perpendicular rock 55 yards high, the angle of depression of the nearest bank of a river is found to be 55° 54′, that of the opposite bank 33° 20′. Required the breadth of the river, and the distance of its nearest bank from the bottom of the rock.

The breadth of the river is 46.4 yards;
Its distance from the rock 37.2.

6. If the moon subtend an angle of 31′ 14″, when her distance is 240,000 miles; what is her diameter?

Ans. 2180 miles.

7. Observations are made on the altitude of a balloon, by two persons standing on the same side of the balloon, and in a vertical plane passing through it. The distance of the stations is half a mile. At one, the angle of elevation is 30° 58′, at the other 36° 52′. What is the height of the balloon above the ground? Ans. $1\frac{1}{2}$ miles.

8. The shadow of the top of a mountain, when the altitude of the sun on the meridian is 32°, strikes a certain point on a level plain below; but when the meridian altitude of the sun is 67°, the shadow strikes half a mile farther south, on the same plain. What is the height of the mountain above the plain? Ans. 2245 feet.

NOTES.

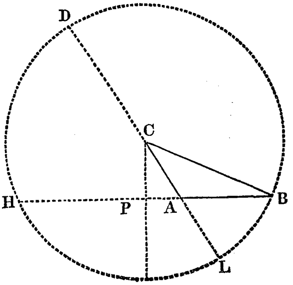
Note A. p. 13.

It is common to *define* logarithms to be a series of numbers in arithmetical progression, corresponding with another series in geometrical progression. This is calculated to perplex the learner, when, upon opening the tables, he finds that the natural numbers, as they stand there, instead of being in *geometrical,* are in *arithmetical* progression; and that the logarithms are *not* in arithmetical progression.

It is true, that a geometrical series may be obtained, by taking out, here and there, a few of the natural numbers; and that the logarithms of these will form an arithmetical series. But the definition is not applicable to the whole of the numbers and logarithms, as they stand in the tables.

Note B. p. 89.

If the perpendicular be drawn from the angle opposite the longest side, it will always fall *within* the triangle; because the other two angles must, of course, be acute. But if one of the angles at the base be *obtuse,* the perpendicular will fall *without* the triangle, as CP.



In this case, the side on which the perpendicular falls, is to the sum of the other two; as the difference of the latter, to the *sum* of the segments made by the perpendicular.

The demonstration is the same, as in the other case, except that AH=BP+PA, instead of BP—PA.

Thus, in the circle BDHL, of which C is the centre,

$$AB \times AH = AL \times AD; \text{ therefore } AB : AD :: AL :: AH.$$

$$\begin{aligned} &\text{But } AD = CD + CA = CB + CA \\ &\text{And } AL = CL - CA = CB - CA \\ &\text{And } AH = HP + PA = BP + PA \end{aligned}$$

Therefore,

$$AB : CB + CA :: CB - CA : BP + PA$$

When the three sides are given, it may be known whether one of the angles is obtuse. For any angle of a triangle is obtuse or acute, according as the square of the side subtending the angle is *greater*, or *less*, than the sum of the squares of the sides containing the angle. (Euc. 12, 13. 2.)*

NOTE C. p. 000.

Gunter's *Sliding Rule*, is constructed upon the same principle as his scale, with the addition of a slider, which is so contrived as to answer the purpose of a pair of dividers, in working proportions, multiplying, dividing, &c. The lines on the *fixed part* are the same as on the scale. The *slider* contains two lines of numbers, a line of logarithmic sines, and a line of logarithmic tangents.

To *multiply* by this, bring 1 on the slider, against one of the factors on the fixed part; and against the other factor on the slider, will be the product on the fixed part. To divide, bring the divisor on the slider, against the dividend on the fixed part; and against 1 on the slider, will be the quotient on the fixed part. To work a *proportion*, bring the first term on the slider, against one of the middle terms on the fixed part; and against the other middle term on the slider, will be the fourth term on the fixed part. Or the first term may be taken on the fixed part; and then the fourth term will be found on the slider.

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* Thomson's Legendre, 12, 13. 4.

Another instrument frequently used in trigonometrical constructions, is

THE SECTOR.

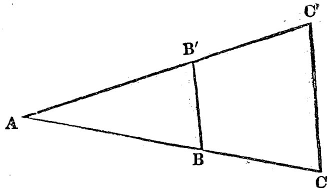
This consists of two equal scales movable about a point as a centre. The lines which are drawn on it are of two kinds, some being parallel to the sides of the instrument, and others diverging from the central point, like the radii of a circle. The latter are called the *double* lines, as each is repeated upon the two scales. The *single* lines are of the same nature, and have the same use, as those which are put upon the common scale; as the lines of equal parts, of chords, of latitude, &c., on one face; and the logarithmic lines of numbers, of sines, and of tangents, on the other.

The *double* lines are

A line of *Lines*, or equal parts, marked	Lin. or L.
A line of *Chords*,	Cho. or C.
A line of natural *Sines*,	Sin. or S.
A line of natural *Tangents* to 45°,	Tan. or T.
A line of tangents, *above* 45°,	Tan. or T.
A line of natural *Secants*,	Sec. or S.
A line of *Polygons*,	Pol. or P.

The double lines of *chords*, of *sines*, and of *tangents* to 45°, are all of the same radius; beginning at the central point, and terminating near the other extremity of each scale; the chords at 60°, the sines at 90°, and the tangents at 45°. (See Art. 95.) The line of *lines* is also of the same length, containing ten equal parts which are numbered, and which are again subdivided. The radius of the lines of secants and of tangents above 45°, is about one-fourth of the length of the other lines. From the end of the radius, which for the secants is at 0, and for the tangents at 45°, these lines extend to between 70° and 80°. The line of polygons is numbered 4, 5, 6, &c., from the extremity of each scale, towards the centre.

The simple principle on which the utility of these several pairs of lines depends is this, that *the sides of similar triangles are proportional.* (Euc. 4. 6.) So that sines, tangents, &c., are furnished *to any radius,* within the extent of the opening of the two scales. Let AC and AC′ be any pair of lines on the sector, and AB and AB′ equal portions of these lines. As AC and AC′ are equal, the triangle ACC′ is isosceles, and similar to ABB′. Therefore,



$$AB : AC :: BB' : CC'.$$

Distances measured from the centre on either scale, as AB and AC, are called *lateral distances.* And the distances between corresponding points of the two scales, as BB′ and CC′, are called *transverse distances.*

Let AC and CC′ be radii of two circles. Then if AB be the chord, sine, tangent, or secant, of any number of degrees in one; BB′ will be the chord, sine, tangent, or secant of the same number of degrees in the other. (Art. 119.) Thus, to find the *chord* of 30°, to a radius of four inches, open the sector so as to make the transverse distance from 60 to 60, on the lines of chords, four inches; and the distance from 30 to 30, on the same lines, will be the chord required. To find the *sine* of 28°, make the distance from 90 to 90, on the lines of sines, equal to radius; and the distance from 28 to 28 will be the sine. To find the *tangent* of 37°, make the distance from 45 to 45, on the lines of tangents, equal to radius; and the distance from 37 to 37 will be the tangent. In finding *secants*, the distance from 0 to 0 must be made radius. (Art. 201.)

To lay down an *angle* of 34°, describe a circle, of any convenient radius, open the sector, so that the distance from 60 to 60 on the lines of chords shall be equal to this radius, and to the circle apply a chord equal to the distance from 34 to 34. (Art. 161.) For an angle above 60°, the chord of *half* the number of degrees may be taken, and applied *twice* on the arc, as in Art. 161.

The line of *polygons* contains the chords of arcs of a circle which is divided into equal portions. Thus, the distances from the centre of the sector to 4, 5, 6, and 7, are the chords of $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, and $\frac{1}{7}$ of a circle. The distance 6 is the radius. (Art. 95.) This line is used to make a regular polygon, or to inscribe one in a given circle. Thus, to make a *pentagon* with the transverse distance from 6 to 6 for radius, describe a circle, and the distance from 5 to 5 will be the length of one of the sides of a pentagon inscribed in that circle.

The line of *lines* is used to divide a line into equal or proportional parts, to find fourth proportionals, &c. Thus, to divide a line into 7 equal parts, make the length of the given line the transverse distance from 7 to 7, and the distance from 1 to 1 will be one of the parts. To find $\frac{3}{5}$ of a line, make the transverse distance from 5 to 5 equal to the given line; and the distance from 3 to 3 will be $\frac{3}{5}$ of it.

In working the *proportions in trigonometry* on the sector, the lengths of the sides of triangles are taken from the line of lines, and the degrees and minutes from the lines of sines, tangents, or secants. Thus, in Art. 135, ex. 1,

$$35 : R :: 26 : \sin 48°.$$

To find the fourth term of this proportion by the sector, make the lateral distance 35 on the line of lines, a transverse distance from 90 to 90 on the lines of sines; then the lateral distance 26 on the line of lines, will be the transverse distance from 48 to 48 on the lines of sines.

For a more particular account of the construction and uses of the Sector, see Stone's edition of Bion on Mathematical Instruments, Hutton's Dictionary, and Robertson's Treatise on Mathematical Instruments.

Day and Thomson's Series.

A

PRACTICAL APPLICATION

OF

THE PRINCIPLES OF GEOMETRY

TO THE

MENSURATION

OF

SUPERFICIES AND SOLIDS.

ADAPTED TO

THE METHOD OF INSTRUCTION IN SCHOOLS AND ACADEMIES.

BY JEREMIAH DAY, D.D. LL.D.

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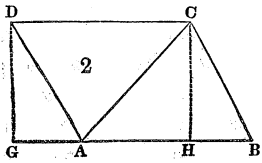
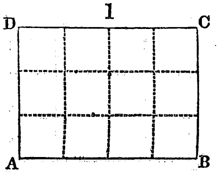
SECTION I.

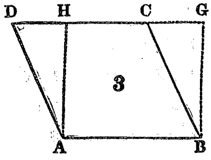
AREAS OF FIGURES BOUNDED BY RIGHT LINES.

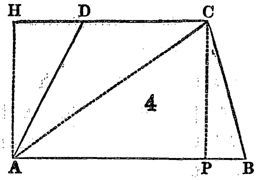
Art. 1. The following definitions, which are nearly the same as in Euclid, are inserted here for the convenience of reference.

I. *Four-sided* figures have different names, according to the relative position and length of the sides. A *parallelogram* has its opposite sides equal and parallel, as ABCD. (Fig. 2.) A *rectangle,* or *right parallelogram,* has its opposite sides equal, and all its angles right angles; as AC. (Fig. 1.) A *square* has all its sides equal, and all its angles right angles; as ABGH. (Fig. 3.) A *rhombus* has all its sides equal, and its angles oblique; as ABCD. (Fig. 3.) A *rhomboid* has its opposite sides equal, and its angles oblique; as ABCD. (Fig. 2.) A *trapezoid* has only two of its sides parallel; as ABCD. (Fig. 4.) Any other four sided figure is called a *trapezium.*



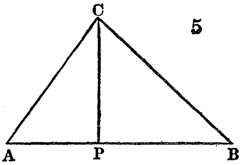






II. A figure which has more than four sides is called a *polygon*. A *regular* polygon has all its sides equal, and all its angles equal.

III. The *height* of a *triangle* is the length of a perpendicular, drawn from one of the angles to the opposite side; as CP. The *height* of a *four sided* figure is the perpendicular distance between two of its parallel sides; as CP. (Fig. 4.)



IV. The *area* or *superficial contents* of a figure is the *space* contained within the line or lines by which the figure is bounded.

2. In calculating areas, some particular portion of surface is fixed upon, as the *measuring unit*, with which the given figure is to be compared. This is commonly a *square;* as a square inch, a square foot, a square rod, &c. For this reason, determining the quantity of surface in a figure is called *squaring it*, or finding its *quadrature;* that is, finding a square or number of squares to which it is equal.

3. The *superficial* unit has generally the same name, as the *linear* unit which forms the side of the square.

The side of a square inch is a linear inch;
of a square foot, a linear foot;
of a square rod, a linear rod, &c.

There are some superficial measures, however, which have no corresponding denominations of length. The *acre*, for instance, is not a square which has a line of the same name for its side.

The following tables contain the linear measures in common use, with their corresponding square measures.

Linear Measures.	*Square Measures.*
12 inches = 1 foot.	144 inches = 1 foot.
3 feet = 1 yard.	9 feet = 1 yard.
6 feet = 1 fathom.	36 feet = 1 fathom.
$16\frac{1}{2}$ feet = 1 rod.	$272\frac{1}{4}$ feet = 1 rod.
$5\frac{1}{2}$ yards = 1 rod.	$30\frac{1}{4}$ yards = 1 rod.
4 rods = 1 chain.	16 rods = 1 chain.
40 rods = 1 furlong.	1600 rods = 1 furlong.
320 rods = 1 mile.	102400 rods = 1 mile.

An *acre* contains 160 square rods, or 10 square chains.

By reducing the denominations of square measure, it will be seen that

1 sq. mile=640 acres=102400 rods=27878400 feet=4014489600 inches.
1 acre=10 chains=160 rods=43560 feet=6272640 inches.

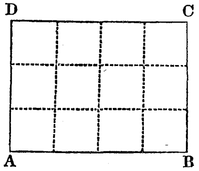
The fundamental problem in the mensuration of superficies is the very simple one of determining the area of a *right parallelogram*. The contents of other figures, particularly those which are rectilinear, may be obtained by finding parallelograms which are equal to them, according to the principles laid down in Euclid.

Problem I.

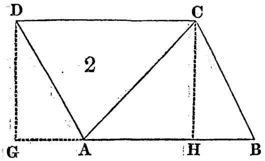
To find the area of a PARALLELOGRAM, *square, rhombus, or rhomboid.*

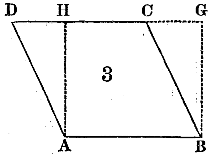
4. MULTIPLY THE LENGTH BY THE PERPENDICULAR HEIGHT OR BREADTH.

It is is evident that the number of *square* inches in the parallelogram AC is equal to the number of *linear* inches in the length AB, repeated as many times as there are inches in the breadth BC. For more particular illustration of this see Alg. 386—389.



The oblique parallelogram or rhomboid ABCD, (Fig. 2.) is equal to the right parallelogram GHCD. (Euc. 36. 1.)* The





area, therefore, is equal to the length AB multiplied into the perpendicular height HC. And the rhombus ABCD, (Fig. 3.) is equal to the *parallelogram* ABGH. As the sides of a *square* are all equal, its area is found, by *multiplying one of the sides into itself.*

Ex. 1. How many square feet are there in a floor $23\frac{1}{2}$ feet long, and 18 feet broad? Ans. $23\frac{1}{2}\times 18=423$.

2. What are the contents of a piece of ground which is 66 feet square? Ans. 4356 sq. feet=16 sq. rods.

3. How many square feet are there in the four sides of a room which is 22 feet long, 17 feet broad, and 11 feet high? Ans. 858.

Art. 5. If the sides and angles of a parallelogram are given, the perpendicular height may be easily found by trigonometry. Thus, CH (Fig. 2.) is the perpendicular of a right angled triangle, of which BC is the hypothenuse. Then, (Trig. 134.)

$$R : BC :: \sin B : CH.$$

The area is obtained by multiplying CH thus found, into the length AB.

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* Thomson's Legendre, 1. 5.

Or, to reduce the two operations to one,

As radius,
To the sine of any angle of a parallelogram ;
So is the product of the sides including that angle,
To the area of the parallelogram.

For *the area*$=AB\times CH$, (Fig. 2.) But $CH=\frac{BC\times\sin B}{R}$

Therefore,

The area$=\frac{AB\times BC\times\sin B}{R}$. Or, $R:\sin B::AB\times BC:$ *the area.*

Ex. If the side AB be 58 rods, BC 42 rods, and the angle B 63°, what is the area of the parallelogram ?

As radius		10.00000
To the sine of B	63°	9.94988
So is the product of AB	58	1.76343
Into BC (Trig. 39.)	42	1.62325
To the area	2170.5 sq. rods	3.33656

2. If the side of a rhombus is 67 feet, and one of the angles 73°, what is the area ? Ans. 4292.7 feet.

6. When the dimensions are given in feet and inches, the multiplication may be conveniently performed by the arithmetical rule of *Duodecimals ;* in which each inferior denomination is one-twelfth of the next higher. Considering a foot as the measuring *unit,* a prime is the twelfth part of a foot; a second, the twelfth part of a prime, &c. It is to be observed, that, in measures of *length, inches* are *primes ;* but in *superficial* measure they are *seconds.* In both, a prime is $\frac{1}{12}$ of a foot. But $\frac{1}{12}$ of a *square* foot is a parallelogram, a foot long and an inch broad. The twelfth part of this is a square inch, which is $\frac{1}{144}$ of a square foot.

Ex. 1. What is the surface of a board 9 feet 5 inches, by 2 feet 7 inches.

$$\begin{array}{rrl}
\text{F} & & \\
9 & 5' & \\
2 & 7 & \\
\hline
18 & 10 & \\
5 & 5 & 11 \\
\hline
24 & 3 & 11'', \text{ or 24 feet 47 inches.}
\end{array}$$

2. How many feet of glass are there in a window 4 feet 11 inches high, and 3 feet 5 inches broad ?

Ans. 16 F. $9'$ $7''$, or 16 feet 115 inches.

7. If the area and one side of a parallelogram be given, the other side may be found by *dividing the area by the given side.* And if the area of a *square* be given, the side may be found by *extracting the square root of the area.* This is merely reversing the rule in Art. 4. See Alg. 520, 521.

Ex. 1. What is the breadth of a piece of cloth which is 36 yds. long, and which contains 63 square yards.

Ans. $1\frac{3}{4}$ yds.

2. What is the side of a square piece of land containing 289 square rods ?

3. How many yards of carpeting $1\frac{1}{4}$ yard wide, will cover a floor 30 feet long and $22\frac{1}{2}$ broad ?

Ans. $30\times22\frac{1}{2}$ feet$=10\times7\frac{1}{2}=75$ yds. And $75\div1\frac{1}{4}=60$.

4. What is the side of a square which is equal to a parallelogram 936 feet long and 104 broad ?

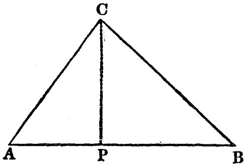
5. How many panes of 8 by 10 glass are there, in a window 5 feet high, and 2 feet 8 inches broad ?

PROBLEM II.

To find the area of a TRIANGLE.

8. RULE I. MULTIPLY ONE SIDE BY HALF THE PERPENDICULAR FROM THE OPPOSITE ANGLE. Or, multiply half the side by the perpendicular, Or, multiply the whole side by the perpendicular and take half the product.

The area of the triangle ABC, is equal to $\frac{1}{2}$ PC×AB, because a parallelogram of the same base and height is equal to PC ×AB, (Art. 4.) and by Euc. 41, 1,* the triangle is half the parallelogram.

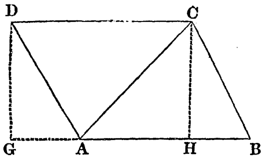


Ex. 1. If AB be 65 feet, and PC 31.2, what is the area of the triangle? Ans. 1014 square feet.

2. What is the surface of a triangular board, whose base is 3 feet 2 inches, and perpendicular height 2 feet 9 inches? Ans. 4 F. 4′ 3″, or 4 feet 51 inches.

9. If two sides of a triangle and the included angle, are given, the perpendicular on one of these sides may be easily found by rectangular trigonometry. And the area may be calculated in the same manner as the area of a parallelogram in Art. 5. In the triangle ABC,

$$R : BC :: \sin B : CH$$



And because the triangle is half the parallelogram of the same base and height,

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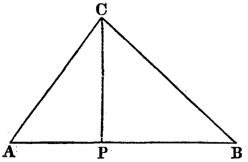
* Thomson's Legendre, 2. 4.

As radius,
To the sine of any angle of a triangle;
So is the product of the sides including that angle,
To twice the area of the triangle. (Art. 5.)

Ex. If AC be 39 feet, AB 65 feet, and the angle at A 53° 7′ 48″, what is the area of the triangle? Ans. 1014 square feet.



9. *b.* If *one side* and the *angles* are given; then

As the product of radius and the sine of the angle opposite the given side,
To the product of the sines of the two other angles;
So is the square of the given side,
To twice the area of the triangle.

If PC be perpendicular to AB.

$$R : \sin B :: BC : CP$$
$$\sin ACB : \sin A :: AB : BC$$

Therefore, (Alg. 351, 345.)

$R \times \sin ACB : \sin A \times \sin B :: AB \times BC : CP \times BC :: \overline{AB}^2 : AB \times CP =$ twice the area of the triangle.

Ex. If one side of a triangle be 57 feet, and the angles at the ends of this side 50° and 60°, what is the area?

Ans. 1147 sq. feet.

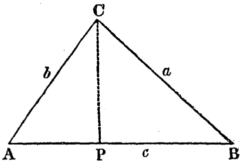
10. If the *sides* only of a triangle are given, an angle may be found, by oblique trigonometry, Case IV, and then the perpendicular and the area may be calculated. But the area may be more directly obtained, by the following method.

Rule II. When the three sides are given, *from half their sum subtract each side severally, multiply together the half sum and the three remainders, and extract the square root of the product.*

If the sides of the triangle are a, b, and c, and if h=half their sum, then

$$\textit{The area}=\sqrt{h\times(h-a)\times(h-b)\times(h-c)}$$

Ex. 1. In the triangle ABC, given the sides a 52 feet, b 39, and c 65; to find the side of a square which has the same area as the triangle.



$\frac{1}{2}(a+b+c)=h=78$ $\qquad h-b=39$

$h-a=26$ $\qquad h-c=13$

Then the area $=\sqrt{78\times26\times39\times13}=1014$ square feet.

By logarithms.

The half sum	=78	1.89209
First remainder	=26	1.41497
Second do.	=39	1.59106
Third do.	=13	1.11394
		2)6.01206
The area required	=1014	2)3.00603
Side of the square	=31.843 (Trig. 47.)	1.50301

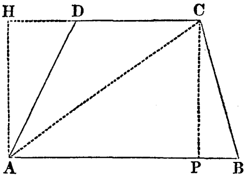
2. If the sides of a triangle are 134, 108, and 80 rods, what is the area? Ans. 4319.

3. What is the area of a triangle whose sides are 371, 264, and 225 feet?

PROBLEM III.

To find the area of a TRAPEZOID.

21. MULTIPLY HALF THE SUM OF THE PARALLEL SIDES INTO THEIR PERPENDICULAR DISTANCE.



The area of the trapezoid ABCD, is equal to half the sum of the sides AB and CD, multiplied into the perpendicular distance PC or AH. For the whole figure is made up of the two triangles ABC and ADC; the area of the first of which is equal to the product of half the base AB into the perpendicular PC, (Art. 8.) and the area of the other is equal to the product of half the base DC into the perpendicular AH or PC.

Ex. If AB be 46 feet, BC 31, DC 38, and the angle B 70°, what is the area of the trapezoid?

$R : BC :: \sin B : PC = 29.13$. And $42 \times 29.13 = 1223\frac{1}{2}$.

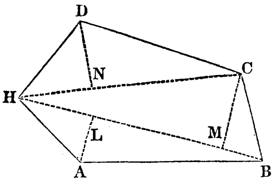
2. What are the contents of a field which has two parallel sides 65 and 38 rods, distant from each other 27 rods?

Problem IV.

To find the area of a TRAPEZIUM, *or of an irregular* POLYGON.

13. Divide the whole figure into triangles, by drawing diagonals, and find the sum of the areas of these triangles. (Alg. 394.)

If the perpendiculars in two triangles fall upon the *same diagonal*, the area of the trapezium formed of the two triangles, is equal to half the product of the diagonal into the sum of the perpendiculars.



Thus the area of the trapezium ABCH, is

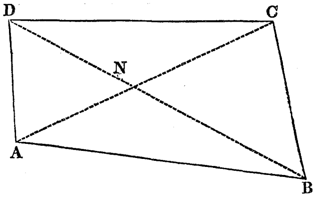
$$\tfrac{1}{2}BH\times AL+\tfrac{1}{2}BH\times CM=\tfrac{1}{2}BH\times(AL+CM.)$$

Ex. In the irregular polygon ABCDH,

$$\text{if the diagonals}\left\{\begin{matrix}BH=36,\\ CH=32,\end{matrix}\right. \text{and the perpendiculars}\left\{\begin{matrix}AL=5.3\\ CM=9.3\\ DN=7.3\end{matrix}\right.$$

The area$=18\times14.6+16\times7.3=379.6$.

14. If the diagonals of a *trapezium* are given, the area may be found, nearly in the same manner as the area of a parallelogram in Art. 5, and the area of a triangle in Art. 9.



In the trapezium ABCD, the sines of the four angles at N, the point of intersection of the diagonals, are all equal. For the two acute angles are *supplements* of the other two, and therefore have the same sine. (Trig. 90.) Putting, then, sin N for the sine of each of these angles, the areas of the four triangles of which the trapezium is composed, are given by the following proportions; (Art. 9.)

$$R : \sin N :: \left\{\begin{array}{l}BN\times AN : 2\ \textit{area}\ ABN\\ BN\times CN : 2\ \textit{area}\ BCN\\ DN\times CN : 2\ \textit{area}\ CDN\\ DN\times AN : 2\ \textit{area}\ ADN\end{array}\right.$$

And by addition, (Alg. 349, Cor. 1.)*

$$R : \sin N :: BN\times AN+BN\times CN+DN\times CN+DN\times AN : 2\ \textit{area}\ ABCD.$$

The 3d term$=(AN+CN)\times(BN+DN)=AC\times BD$, by the figure.

Therefore $R : \sin N :: AC\times BD : 2$ *area* ABCD. That is,

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[*] Euclid, 2, 5. Cor.

As Radius,
To the sine of the angle at the intersection of the diagonals of a trapezium;
So is the product of the diagonals,
To twice the area of the trapezium.

It is evident that this rule is applicable to a parallelogram, as well as to a trapezium.

If the diagonals intersect at *right angles,* the sine of N is equal to radius; (Trig. 95.) and therefore the product of the diagonals is equal to twice the area. (Alg. 356.)*

Ex. 1. If the two diagonals of a trapezium are 37 and 62, and if they intersect at an angle of 54°, what is the area of the trapezium? Ans. 928.

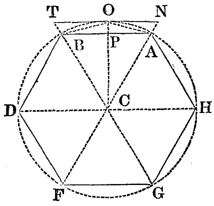
2. If the diagonals are 85 and 93, and the angle of intersection 74°, what is the area of the trapezium?

PROBLEM V.

To find the area of a REGULAR POLYGON.

15. MULTIPLY ONE OF ITS SIDES INTO HALF ITS PERPENDICULAR DISTANCE FROM THE CENTRE, AND THIS PRODUCT INTO THE NUMBER OF SIDES.

A regular polygon contains as many equal triangles as the figure has sides. Thus, the hexagon ABDFGH contains six triangles, each equal to ABC. The area of one of them is equal to the product of the side AB, into half the perpendicular CP. (Art. 8.) The area of the whole, therefore, is equal to this product multiplied into the *number* of sides.



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* Euclid, 14. 5.

Ex. 1. What is the area of a regular octagon, in which the length of a side is 60, and the perpendicular from the centre 72.426? Ans. 17382.

2. What is the area of a regular decagon whose sides are 46 each, and the perpendicular 70.7867?

16. If only the length and number of sides of a regular polygon be given, the *perpendicular* from the centre may be easily found by trigonometry. The periphery of the circle in which the polygon is inscribed, is divided into as many equal parts as the polygon has sides. (Euc. 16. 4. Schol.)* The arc, of which one of the sides is a chord, is therefore known; and of course, the angle at the centre subtended by this arc.

Let AB be one side of a regular polygon inscribed in the circle ABDG. The perpendicular CP bisects the line AB, and the angle ACB. (Euc. 3. 3.)† Therefore, BCP is the same part of 360°, which BP is of the perimeter of the polygon. Then, in the right angled triangle BCP, if BP be radius, (Trig. 122.)

$$R : BP :: \cot BCP : CP. \quad \text{That is,}$$

As Radius,
To half of one of the sides of the polygon;
So is the cotangent of the opposite angle,
To the perpendicular from the centre.

Ex. 1. If the side of a regular hexagon be 38 inches, what is the area?

$$\text{The angle } BCP = \tfrac{1}{12} \text{ of } 360° = 30°. \quad \text{Then,}$$

$$R : 19 :: \cot 30° : 32.909 = CP, \text{ the perpendicular,}$$

$$\text{And the area} = 19 \times 32.909 \times 6 = 3751.6$$

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* Thomson's Legendre, 2. 5. Schol.

† Ibid. 6. 2.

2. What is the area of a regular decagon whose sides are each 62 feet? Ans. 29576.

17. From the proportion in the preceding article, a *table* of perpendiculars and areas may be easily formed, for a series of polygons, of which each side is a unit. Putting R=1, (Trig. 100.) and n=the number of sides, the proportion becomes

$$1 : \tfrac{1}{2} : : \cot\frac{360^\circ}{2n} : \textit{the perpendicular}$$

$$\text{So that, } \textit{the perp.} = \tfrac{1}{2}\cot\frac{360}{2n}$$

And the *area* is equal to half the product of the perpendicular into the number of sides. (Art. 15.)

Thus, in the trigon, or equilateral triangle, the perpendicular$=\frac{1}{2}\cot\frac{360^\circ}{6}=\frac{1}{2}\cot 60^\circ=0.2886752$.

And the area=0.4330127.

In the tetragon, or square, the perpendicular$=\frac{1}{2}\cot\frac{360^\circ}{8}=\frac{1}{2}\cot 45^\circ=0.5$. And the area=1.

In this manner, the following table is formed, in which the side of each polygon is supposed to be a unit.

A TABLE OF REGULAR POLYGONS.

Names.	Sides.	Angles.	Perpendiculars.	Areas.
Trigon,	3	60°	0.2886752	0.4330127
Tetragron,	4	45°	0.5000000	1.0000000
Pentagon,	5	36°	0.6881910	1.7204774
Hexagon,	6	30°	0.8660254	2.5980762
Heptagon,	7	$25\frac{5}{7}$	1.0382601	3.6339124
Octagon,	8	$22\frac{1}{2}$	1.2071069	4.8284271
Nonagon,	9	20°	1.3737385	6.1818242
Decagon,	10	18°	1.5388418	7.6942088
Undecagon,	11	$16\frac{4}{11}$	1.7028439	9.3656399
Dodecagon,	12	15°	1.8660252	11.1961524

By this table may be calculated the area of any other regular polygon, of the same number of sides with one of these. For the areas of similar polygons are as the *squares* of their homologous sides. (Euc. 20. 6.)*

To find, then, the area of a regular polygon, *multiply the square of one of its sides by the area of a similar polygon of which the side is a unit.*

Ex. 1. What is the area of a regular decagon whose sides are each 102 rods? Ans. 80050.5 rods.

2. What is the area of a regular dodecagon whose sides are each 87 feet?

SECTION II.

THE QUADRATURE OF THE CIRCLE AND ITS PARTS.

Art. 18. *Definition* I. A circle is a plane bounded by a line which is equally distant in all its parts from a point within called the centre. The bounding line is called the *circumference* or periphery. An *arc* is any portion of the circumference. A semi-circle is half, and a quadrant one-fourth of a circle.

II. A *Diameter* of a circle is a straight line drawn through the centre, and terminated both ways by the circumference. A *Radius* is a straight line extending from the centre to the circumference. A *Chord* is a straight line which joins the two extremities of an arc.

III. A Circular *Sector* is a space contained between an arc and the two radii drawn from the extremities of the arc.

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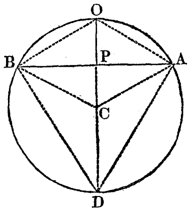
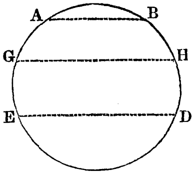
* Thomson's Legendre 1, 5. Cor.

It may be *less* than a semi-circle, as ACBO, or *greater*, as ACBD.



IV. A Circular *Segment* is the space contained between an arc and its chord, as ABO or ABD. The chord is sometimes called the *base* of the segment. The *height* of a segment is the perpendicular from the middle of the base to the arc, as PO.



V. A Circular *Zone* is the space between two parallel chords, as AGHB. It is called the *middle* zone, when the two chords are equal, as GHDE.

VI. A Circular *Ring* is the space between the peripheries of two concentric circles, as AA′, BB′. (Fig. 13.)

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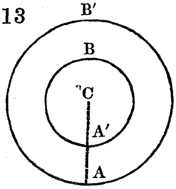

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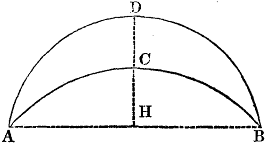
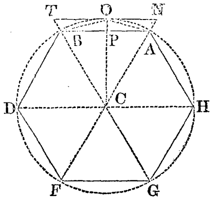


VII. A *Lune* or Crescent is the space between two circular arcs which intersect each other, as ACBD. (Fig. 14.)

19. The *Squaring of the Circle* is a problem which has exercised the ingenuity of distinguished mathematicians for many centuries. The result of their efforts has been only an *approximation* to the value of the area. This can be carried to a degree of exactness far beyond what is necessary for practical purposes.

20. If the *circumference* of a circle of given diameter were known, its area could be easily found. For the area is equal to the product of half the circumference into half the diameter. (Sup. Euc. 5, 1.[*])[†] But the circumference of a circle has never been exactly determined. The method of approximating to it is by inscribing and circumscribing *polygons,* or by some process of calculation which is, in principle, the same. The perimeters of the polygons can be easily and exactly determined. That which is circumscribed is *greater,* and that which is inscribed is *less,* than the periphery of the circle; and by increasing the number of sides, the difference of the two polygons may be made less than any given quantity. (Sup. Euc. 4, 1.)



21. The side of a *hexagon* inscribed in a circle, as AB, is the chord of an arc of 60°, and therefore equal to the radius. (Trig. 95.) The chord of *half* this arc, as BO, is the side of a polygon of 12 equal sides. By repeatedly bisecting the arc, and finding the chord, we may obtain the side of a polygon of an immense number of sides. Or we may calculate the *sine,* which will be half the chord of double the arc, (Trig. 82, cor.,) and the *tangent,* which will be half the side of a similar *circumscribed* polygon. Thus the sine AP, is half of AB, a side of the inscribed hexagon; and the tangent NO is half of NT, a side of the circumscribed hexagon. The difference between the sine and the arc AO is less than the difference between the sine and the tangent. In the section on the computation of the canon, (Trig. 223.) by 12 successive bisections, beginning with 60 degrees, an arc is obtained which is the $\frac{1}{24576}$ of the whole circumference.

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[*] In this manner, the *Supplement* to *Playfair's Euclid* is referred to in this work.

[†] Thomson's Legendre, 11. 5.

The *cosine* of this, if radius be 1, is found to be .99999996732
The *sine* is .00025566346
And the tangent$=\frac{\text{sine}}{\text{cosine}}$(Trig. 93.) =.00025566347
The diff. between the sine and tangent is only .00000000001
And the difference between the sine and the *arc* is still less.

Taking then, .000255663465 for the length of the arc, multiplying by 24576, and retaining 8 places of decimals, we have 6.28318531 for the whole circumference, the radius being 1. Half of this,

3.14159265

is the circumference of a circle whose radius is $\frac{1}{2}$, and *diameter* 1.

22. If this be multiplied by 7, the product is 21.99+or 22 nearly. So that,

Diam : Circum : : 7 : 22, nearly.

If 3.14159265 be multiplied by 113, the product is 354.9999+, or 355, very nearly. So that,

Diam : Circum : : 113 : 355, very nearly.

The first of these ratios was demonstrated by Archimedes.

There are various methods, principally by infinite series and fluxions, by which the labor of carrying on the approximation to the periphery of a circle may be very much abridged. The calculation has been extended to nearly 150 places of decimals. But four or five places are sufficient for most practical purposes.

After determining the ratio between the diameter and the circumference of a circle, the following problems are easily solved.

PROBLEM I.

To find the CIRCUMFERENCE *of a circle from its diameter.*

23. MULTIPLY THE DIAMETER BY **3.14159.***

Or,

Multiply the diameter by 22 *and divide the product by* 7. Or, multiply the diameter by 355, and divide the product by 113. (Art. 22.)

Ex. 1. If the diameter of the earth be 7930 miles, what is the circumference? Ans. 249128 miles.

2. How many miles does the earth move, in revolving round the sun; supposing the orbit to be a circle whose diameter is 190 million miles? Ans. 596,902,100.

3. What is the circumference of a circle whose diameter is 769843 rods?

PROBLEM II.

To find the DIAMETER *of a circle from its circumference.*

24. DIVIDE THE CIRCUMFERENCE BY **3.14159.**

Or,

Multiply the circumference by 7, *and divide the product by* 22. Or, multiply the circumference by 113, and divide the product by 355. (Art. 22.)

Ex. 1. If the circumference of the sun be 2,800,000 miles, what is his diameter? Ans. 891,267.

2. What is the diameter of a tree which is $5\frac{1}{2}$ feet round?

25. As multiplication is more easily performed than division, there will be an advantage in exchanging the *divisor*

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* In many cases, 3.1416 will be sufficiently accurate.

3.14159 for a *multiplier* which will give the same result. In the proportion

3.14159 : 1 : : Circum : Diam.

to find the fourth term, we may divide the second by the first, and multiply the quotient into the third. Now, 1÷3.14159=0.31831. If, then, the circumference of a circle be multiplied by .31831, the product will be the diameter.

Ex. 1. If the circumference of the moon be 6850 miles, what is her diameter? Ans. 2180.

2. If the whole extent of the orbit of Saturn be 5650 million miles, how far is he from the sun?

3. If the periphery of a wheel be 4 feet 7 inches, what is its diameter?

PROBLEM III.

To find the length of an ARC *of a circle.*

26. *As* 360°, *to the number of degrees in the arc;*
So is the circumference of the circle, to the length of the arc.

The circumference of a circle being divided into 360°, (Trig. 73.) it is evident that the length of an arc of any less number of degrees must be a proportional part of the whole.

Ex. What is the length of an arc of 16°, in a circle whose radius is 50 feet?

The circumference of the circle is 314.159 feet. (Art. 23.)

Then 360 : 16 : : 314.159 : 13.96 feet.

2. If we are 95 millions of miles from the sun, and if the earth revolves round it in $365\frac{1}{4}$ days, how far are we carried in 24 hours? Ans. 1 million 634 thousand miles.

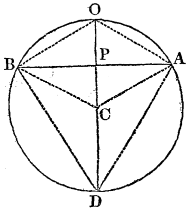
27. The length of an arc may also be found, by multiplying the diameter into the number of degrees in the arc, and

this product into .0087266, which is the length of *one* degree, in a circle whose diameter is 1. For $3.14159 \div 360 = 0.0087266$. And in different circles, the circumferences, and of course the degrees, are as the diameters. (Sup. Euc. 8, 1.)*

Ex. 1. What is the length of an arc of $10° 15'$ in a circle whose radius is 68 rods? Ans. 12.165 rods.

2. If the circumference of the earth be 24913 miles, what is the length of a degree at the equator?

28. The length of an arc is frequently required, when the *number of degrees* is not given. But if the radius of the circle, and either the *chord* or the *height* of the arc, be known; the number of degrees may be easily found.



Let AB be the chord, and PO the height, of the arc AOB. As the angles at P are right angles, and AP is equal to BP; (Art. 18. Def. 4.) AO is equal to BO. (Euc. 4, 1.)† Then,

$$\left.\begin{array}{l}\text{BP is the } \textit{sine}\text{, and CP the } \textit{cosine}\text{,} \\ \text{OP the } \textit{versed sine}\text{, and BO the } \textit{chord}\text{,}\end{array}\right\} \text{ of } \textit{half} \text{ the arc AOB.}$$

And in the right angled triangle CBP,

$$CB : R :: \left\{\begin{array}{l} BP : \sin BCP \text{ or } BO. \\ CP : \cos BCP \text{ or } BO. \end{array}\right.$$

Ex. 1. If the radius $CO = 25$, and the chord $AB = 43.3$; what is the length of the arc AOB?

$$CB : R :: BP : \sin BCP \text{ or } BO = 60° \text{ very nearly.}$$

The circumference of the circle $= 3.14159 \times 50 = 157.08$.

And $360° : 60° :: 157.08 : 26.18 = OB$. Therefore, $AOB = 52.36$.

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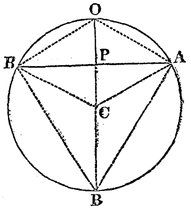
* Thomson's Legendre, 10. 5.

† Ibid., 5. 1.

2. What is the length of an arc whose chord is $216\frac{1}{2}$, in a circle whose radius is 125? Ans. 261.8.

29. If only the *chord* and the *height* of an arc be given, the radius of the circle may be found, and then the length of the arc.



If BA be the chord, and PO the the height of the arc AOB, then (Euc. 35. 3.)*

$$DP=\frac{\overline{BP}^2}{OP}. \quad \text{And } DO=OP+DP=OP+\frac{\overline{BP}^2}{OP}.$$

That is, the *diameter* is equal to the height of the arc, + the square of half the chord divided by the height.

The diameter being found, the length of the arc may be calculated by the two preceding articles.

Ex. 1. If the chord of an arc be 173.2, and the height 50, what is the length of the arc?

The diameter $=50+\frac{\overline{86.6}^2}{50}=200$. The arc contains 120°, (Art. 28.) and its length is 209.44. (Art. 26.)

2. What is the length of an arc whose chord is 120, and height 45? Ans. 160.8.

PROBLEM IV.

To find the AREA *of a* CIRCLE.

30. **MULTIPLY THE SQUARE OF THE DIAMETER BY THE DECIMALS .7854.**

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* Thomson's Legendre, 10. 5.

Or,

MULTIPLY HALF THE DIAMETER INTO HALF THE CIRCUMFERENCE. Or, multiply the whole diameter into the whole circumference, and take $\frac{1}{4}$ of the product.

The area of a circle is equal to the product of half the diameter into half the circumference; (Sup. Euc. 5, 1.) or, which is the same thing, $\frac{1}{4}$ the product of the diameter and circumference. If the diameter be 1, the circumference is 3.14159; (Art. 23.) one-fourth of which is 0.7854 nearly. But the areas of different circles are to each other, *as the squares of their diameters.* (Sup. Euc. 7, 1.)* The area of any circle, therefore, is equal to the product of the square of its diameter into 0.7854, which is the area of a circle whose diameter is 1.

Ex. 1. What is the area of a circle whose diameter is 623 feet? Ans. 304836 square feet.

2. How many acres are there in a circular island whose diameter is 124 rods. Ans. 75 acres, and 76 rods.

3. If the diameter of a circle be 113, and the circumference 355, what is the area? Ans. 10029.

4. How many square yards are there in a circle whose diameter is 7 feet?

31. If the *circumference* of a circle be given, the area may be obtained, by first finding the diameter; or, without finding the diameter, by multiplying the square of the circumference by .07958.

For, if the circumference of a circle be 1, the diameter $=1 \div 3.14159 = 0.31831$; and $\frac{1}{4}$ the product of this into the circumference is .07958 the area. But the areas of different circles, being as the squares of their diameters, are also as the squares of their *circumferences.* (Sup. Euc. 8, 1.)

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* Thomson's Legendre, 28. 4. Cor.

Ex. 1. If the circumference of a circle be 136 feet, what is the area? Ans. 1472 feet.

2. What is the surface of a circular fish-pond, which is 10 rods in circumference?

32. If the area of a circle be *given*, the diameter may be found, by dividing the area by .7854, and extracting the square root of the quotient.

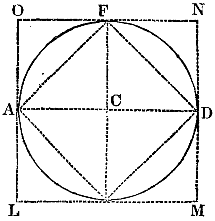
This is reversing the rule in Art. 30.

Ex. 1. What is the diameter of a circle whose area is 380.1336 feet?

Ans. $380.1336 \div .7854 = 484$. And $\sqrt{484} = 22$.

2. What is the diameter of a circle whose area is 19.635?

33. The area of a circle, is to the area of the *circumscribed square*; as .7854 to 1; and to that of the *inscribed* square as .7854 to $\frac{1}{2}$.



Let ABDF be the inscribed square, and LMNO the circumscribed square, of the circle ABDF. The area of the circle is equal to $\overline{AD}^2 \times .7854$. (Art. 30.) But the area of the circumscribed square (Art. 4.) is equal to $\overline{ON}^2 = \overline{AD}^2$. And the smaller square is half of the larger one. For the latter contains 8 equal triangles, of which the former contains only 4.

Ex. What is the area of a square inscribed in a circle whose area is 159? Ans. $.7854 : \frac{1}{2} :: 159 : 101.22$.

PROBLEM V.

To find the area of a SECTOR *of a circle.*

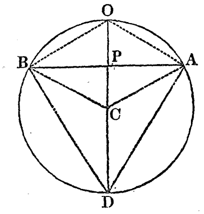
34. MULTIPLY THE RADIUS INTO HALF THE LENGTH OF THE ARC.

Or,

As 360, TO THE NUMBER OF DEGREES IN THE ARC;
So IS THE AREA OF THE CIRCLE, TO THE AREA OF THE SECTOR.

It is evident, that the area of the sector has the same ratio to the area of the circle, which the length of the arc has to the length of the whole circumference; or which the number of *degrees* in the arc has to the number of degrees in the circumference.

Ex. 1. If the arc AOB be 120°, and the diameter of the circle 226; what is the area of the sector AOBC?



The area of the whole circle is 40115. (Art. 30.)

And 360° : 120° : : 40115 : $13371\frac{2}{3}$, the area of the sector.

2. What is the area of a quadrant whose radius is 621?

3. What is the area of a semi-circle, whose diameter is 328?

4. What is the area of a sector which is less than a semicircle, if the radius be 15, and the chord of its arc 12?

Half the chord is the sine of $23° \ 34\frac{3}{4}'$ nearly. (Art. 28.)
The whole arc, then, is $47° \ 9\frac{1}{2}'$
The area of the circle is 706.86
And 360° : $47° \ 9\frac{1}{2}'$: : 706.86 : 92.6 the area of the sector.

5. If the arc ADB be 240 degrees, and the radius of the circle 113, what is the area of the sector ADBC?

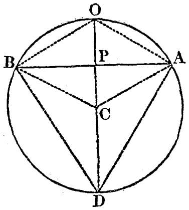
PROBLEM VI.

To find the area of a SEGMENT *of a circle.*

35. FIND THE AREA OF THE SECTOR WHICH HAS THE

SAME ARC, AND ALSO THE AREA OF A TRIANGLE FORMED BY THE CHORD OF THE SEGMENT AND THE RADII OF THE SECTOR.

THEN, IF THE SEGMENT BE LESS THAN A SEMI-CIRCLE, SUBTRACT THE AREA OF THE TRIANGLE FROM THE AREA OF THE SECTOR. BUT, IF THE SEGMENT BE GREATER THAN A SEMI-CIRCLE, ADD THE AREA OF THE TRIANGLE TO THE AREA OF THE SECTOR.



If the triangle ABC, be taken from the sector AOBC, it is evident the difference will be the segment AOBP, less than a semi-circle. And if the same triangle be added to the sector ADBC, the sum will be the segment ADBP, greater than a semi-circle.

The area of the triangle (Art. 8.) is equal to the product of half the chord AB into CP, which is the difference between the radius and PO the height of the segment. Or CP is the *cosine* of half the arc BOA. If this cosine and the chord of the segment are not given, they may be found from the arc and the radius.

Ex. 1. If the arc AOB be 120°, and the radius of the circle be 113 feet, what is the area of the segment AOBP ?

In the right angled triangle BCP,

R : BC : : sin BCO : BP=97.86, half the chord. (Art. 28.)

The cosine $PC=\frac{1}{2}$ CO (Trig. 96, Cor.)	=56.5
The area of the sector AOBC (Art. 34.)	=13371.67
The area of the triangle ABC=BP×PC	= 5528.97
The area of the segment, therefore,	= 7842.7

2. If the base of a segment, less than a semi-circle, be 10

feet, and the radius of the circle 12 feet, what is the area of the segment ?

The arc of the segment contains $49\frac{1}{4}$ degrees. (Art. 28.)
The area of the sector =61.89 (Art. 34.)
The area of the triangle =54.54
And the area of the segment = 7.35 square feet.

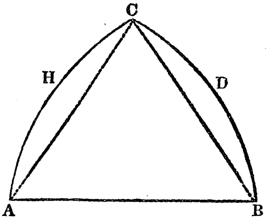
3. What is the area of a circular segment, whose height is 19.2 and base 70 ? Ans. 947.86.

4. What is the area of the segment ADBP, (Fig. 9.) if the base AB be 195.7, and the height PD 169.5 ?
Ans. 32272.

36. The area of any figure which is bounded *partly* by arcs of circles, and partly by right lines, may be calculated, by finding the areas of the segments under the arcs, and then the area of the rectilinear space between the chords of the arcs and the other right lines.

Thus, the Gothic arch ACB, contains the two segments ACH, BCD, and the plane triangle ABC.

Ex. If AB be 110, each of the lines AC and BC 100, and the height of each of the segments ACH, BCD 10.435; what is the area of the whole figure ?



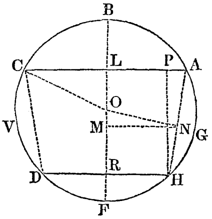
The areas of the two segments are	1404
The area of the triangle ABC is	4593.4
And the whole figure is	5997.4

PROBLEM VII.

To find the area of a circular ZONE.

37. FROM THE AREA OF THE WHOLE CIRCLE, SUBTRACT THE TWO SEGMENTS ON THE SIDES OF THE ZONE.

If from the whole circle there be taken the two segments ABC and DFH, there will remain the zone ACDH.



Or, the area of the zone may be found by subtracting the segment ABC from the segment HBD: Or, by adding the two small segments GAH and VDC, to the trapezoid ACDH. (See Art. 36.)

The latter method is rather the most expeditious in practice, as the two segments at the end of the zone are *equal*.

Ex. 1. What is the area of the zone ACDH, if AC is 7.75, DH 6.93, and the diameter of the circle 8?

The area of the whole circle is		50.26
of the segment ABC	17.32	
of the segment DFH	9.82	
of the zone ACDH		23.12

2. What is the area of a zone, one side of which is 23.25, and the other side 20.8, in a circle whose diameter is 24?
Ans. 208.

38. If the *diameter* of the circle is not given, it may be found from the sides and the breadth of the zone.

Let the centre of the circle be at O. Draw ON perpendicular to AH, NM perpendicular to LR, and HP perpendicular to AL. Then,

$AN=\frac{1}{2}AH$, (Euc. 3. 3.)[*] $MN=\frac{1}{2}(LA+RH)$
$LM=\frac{1}{2}LR$, (Euc. 2. 6.)[†] $PA=LA-RH$.

The triangles APH and OMN are similar, because the sides of one are perpendicular to those of the other, each to each. Therefore,

$$PH : PA :: MN : MO$$

MO being found, we have $ML-MO=OL$.

And the *radius* $CO=\sqrt{OL^2+CL^2}$. (Euc. 47. 1.)[‡]

Ex. If the breadth of the zone ACDH (Fig. 12.) be 6.4, and the sides 6.8 and 6; what is the radius of the circle ?

$PA=3.4-3=0.4$. And, $MN=\frac{1}{2}(3.4+3)=3.2$.

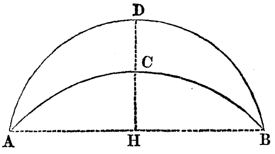
Then, $6.4 : 0.4 :: 3.2 : 0.2=MO$. And, $3.2-0.2=3=OL$

And the radius $CO=\sqrt{3^2+(3.4)^2}=4.534$.

Problem VIII.

To find the area of a LUNE *or crescent.*

39. FIND THE DIFFERENCE OF THE TWO SEGMENTS WHICH ARE BETWEEN THE ARCS OF THE CRESCENT AND ITS CHORD.



If the segment ABC, be taken from the segment ABD; there will remain the lune or crescent ACBD.

Ex. If the chord AB be 88, the height CH 20, and the height DH 40; what is the area of the crescent ACBD ?

The area of the segment ABD is	2698
of the segment ABC	1220
of the crescent ACBD	1478

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[*] Thomson's Legendre, 6. 2. [†] Ibid. 15. 4. [‡] Ibid. 11. 4.

PROBLEM IX.

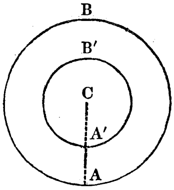
To find the area of a RING, included between the peripheries of two concentric circles.

40. FIND THE DIFFERENCE OF THE AREAS OF THE TWO CIRCLES.

Or,

Multiply the product of the sum and difference of the two diameters by .7854.

The area of the ring is evidently equal to the difference between the areas of the two circles AB and A′B′.



But the area of each circle is equal to the square of its diameter multiplied into .7854. (Art. 30.) And the *difference* of these squares is equal to the product of the sum and difference of the diameters. (Alg. 191.) Therefore the area of the ring is equal to the product of the sum and difference of the two diameters multiplied by .7854.

Ex. 1. If AB be 221, and A′B′ 106, what is the area of the ring? Ans. $(\overline{221}^2\times.7854)-(\overline{106}^2\times.7854)=29535$.

2. If the diameters of Saturn's larger ring be 205,000 and 190,000 miles, how many square miles are there on one side of the ring?

Ans. $395000\times15000\times.7854=4{,}653{,}495{,}000$.

PROMISCUOUS EXAMPLES OF AREAS.

Ex. 1. What is the expense of paving a street 20 rods long and 2 rods wide, at 5 cents for a square foot?

Ans. 544½ dollars.

2. If an equilateral triangle contains as many square feet as there are inches in one of its sides; what is the area of the triangle?

Let x=the number of square feet in the area.

Then $\frac{x}{12}$=the number of linear feet in one of the sides.

And, (Art. 11.) $x=\frac{1}{4}\left(\frac{x}{12}\right)^2\times\sqrt{3}=\frac{x^2}{576}\times\sqrt{3}$.

Reducing the equation, $x=\frac{576}{\sqrt{3}}=332.55$ the area.

3. What is the side of a square whose area is equal to that of a circle 452 feet in diameter?

Ans. $\sqrt{(452)^2\times.7854}=400.574$. (Arts. 30 and 7.)

4. What is the diameter of a circle which is equal to a square whose side is 36 feet?

Ans. $\sqrt{(36)^2\div0.7854}=40.6217$. (Arts. 4 and 32.)

5. What is the area of a square inscribed in a circle whose diameter is 132 feet?

Ans. 8712 square feet. (Art. 33.)

6. How much carpeting, a yard wide, will be necessary to cover the floor of a room which is a regular octagon, the sides being eight feet each? Ans. $34\frac{1}{3}$ yards.

7. If the diagonal of a square be 16 feet, what is the area? Ans. 128 feet. (Art. 14.)

8. If a carriage-wheel four feet in diameter revolve 300 times, in going round a circular green; what is the area of the green?

Ans. $4154\frac{1}{3}$ sq. rods, or 25 acres, 3 qrs. and $34\frac{1}{3}$ rods.

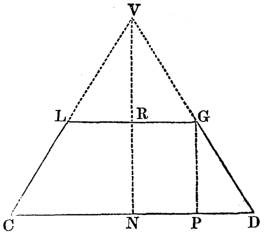
9. What will be the expense of papering the sides of a room, at 10 cents a square yard; if the room be 21 feet long,

18 feet broad, and 12 feet high; and if there be deducted 3 windows, each 5 feet by 3, two doors 8 feet by $4\frac{1}{2}$, and one fire-place 6 feet by $4\frac{1}{2}$? Ans. 8 dollars 80 cents.

10. If a circular pond of water 10 rods in diameter be surrounded by a gravelled walk $8\frac{1}{4}$ feet wide; what is the area of the walk? Ans. $16\frac{1}{2}$ sq. rods. (Art. 40.)

11. If CD, the base of the isosceles triangle VCD, be 60 feet, and the area 1200 feet; and if there be cut off, by the line LG parallel to CD, the triangle VLG, whose area is 432 feet; what are the sides of the latter triangle?

Ans. 30, 30, and 36 feet.



12. What is the area of an equilateral triangle inscribed in a circle whose diameter is 52 feet?

Ans. 878.15 sq. ft.

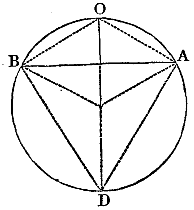
13. If a circular piece of land is inclosed by a fence, in which 10 rails make a rod in length; and if the field contains as many square rods, as there are rails in the fence; what is the value of the land at 120 dollars an acre?

Ans. 942.48 dollars.

14. If the area of the equilateral triangle ABD be 219.5375 feet; what is the area of the circle OBDA, in which the triangle is inscribed?

The sides of the triangle are each 22.5167. (Art. 11.)

And the area of the circle is 530.93.



15. If 6 concentric circles are so drawn, that the space
between the least or 1st, and the 2d is 21.2058,
between the 2d and the 3d is 35.343,
between the 3d and the 4th is 49.4802,
between the 4th and the 5th is 63.6174,
between the 5th and the 6th is 77.7546;
what are the several diameters, supposing the longest to be equal to 6 times the shortest?

Ans. 3, 6, 9, 12, 15, and 18.

16. If the area between two concentric circles be 1202.64 square inches, and the diameter of the lesser circle be 19 inches, what is the diameter of the other?

17. What is the area of a circular segment, whose height is 9, and base 24?

SECTION III.

SOLIDS BOUNDED BY PLANE SURFACES.

Art. 41. Definition I. A *prism* is a solid bounded by plane figures or faces, two of which are parallel, similar, and equal; and the others are parallelograms.

II. The parallel planes are sometimes called the *bases* or *ends;* and the other figures the *sides* of the prism. The latter taken together constitute the *lateral surface.*

III. A prism is *right* or *oblique,* according as the sides are perpendicular or oblique to the bases.

IV. The *height* of a prism is the perpendicular distance between the planes of the bases. In a right prism, therefore, the height is equal to the length of one of the sides.

V. A *Parallelopiped* is a prism whose bases are parallelograms.

VI. A *Cube* is a solid bounded by six equal squares. It is a right prism whose sides and bases are all equal.

VII. A *Pyramid* is a solid bounded by a plane figure called the base, and several triangular planes, proceeding from the sides of the base, and all terminating in a single point. These triangles taken together constitute the *lateral* surface.

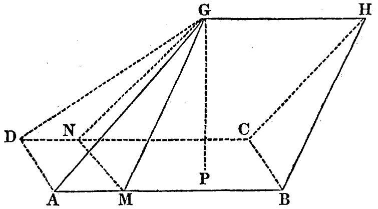
VIII. A pyramid is *regular*, if its base is a regular polygon, and if a line from the centre of the base to the vertex of the pyramid is *perpendicular* to the base. This line is called the *axis* of the pyramid.

IX. The *height* of a pyramid is the perpendicular distance from the summit to the plane of the base. In a *regular* pyramid, it is the length of the *axis*.

X. The *slant-height* of a regular pyramid, is the distance from the summit to the middle of one of the sides of the base.

XI. A *frustum* or *trunk* of a pyramid is a portion of the solid next the base, cut off by a plane parallel to the base. The *height* of the frustum is the perpendicular distance of the two parallel planes. The *slant-height* of a frustum of a *regular* pyramid, is the distance from the middle of one of the sides of the base, to the middle of the corresponding side in the plane above. It is a line passing on the surface of the frustum, through the middle of one of its sides.

XII. A *Wedge* is a solid of five sides, viz. a rectangular base, two rhomboidal sides meeting in an edge, and two triangular ends; as ABHG. The base is ABCD, the sides are ABHG and DCHG, meeting in the edge GH, and the ends are BCH and ADG. The *height* of the wedge is a



perpendicular drawn from any point in the edge, to the plane of the base, as GP.

XIII. A *Prismoid* is a solid whose ends or bases are parallel, but not similar, and whose sides are quadrilateral. It differs from a prism or a frustum of a pyramid, in having its ends dissimilar. It is a *rectangular* prismoid, when its ends are right parallelograms.

XIV. A *linear side* or *edge* of a solid is the line of intersection of two of the planes which form the surface.

42. The common *measuring unit* of solids is a *cube,* whose sides are squares of the same name. The sides of a cubic inch are square inches; of a cubic foot, square feet, &c. Finding the *capacity, solidity,** or *solid contents* of a body, is finding the number of cubic measures, of some given denomination contained in the body.

In solid measure.

1728 cubic inches = 1 cubic foot,
27 cubic feet = 1 cubic yard,
$4492\frac{1}{8}$ cubic feet = 1 cubic rod,
32768000 cubic rods = 1 cubic mile,
282 cubic inches = 1 ale gallon,
231 cubic inches = 1 wine gallon,
2150.42 cubic inches = 1 bushel,
1 cubic foot of pure water weighs 1000 avoirdupois ounces, or $62\frac{1}{2}$ pounds.

PROBLEM I.

To find the SOLIDITY *of a* PRISM.

43. MULTIPLY THE AREA OF THE BASE BY THE HEIGHT.

This is a general rule, applicable to parallelopipeds whether right or oblique, cubes, triangular prisms, &c.

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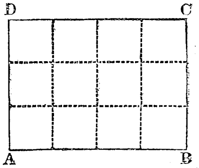
* See note A.

As *surfaces* are measured, by comparing them with a right *parallelogram* (Art. 3.); so *solids* are measured, by comparing them with a right *parallelopiped.*



If ABCD be the base of a right parallelopiped, as a stick of timber standing erect, it is evident that the number of *cubic* feet contained in *one* foot of the height, is equal to the number of *square* feet in the area of the base. And if the solid be of any other height, instead of one foot, the contents must have the same ratio. For parallelopipeds of the same base are to each other as their heights. (Sup. Euc. 9. 3.)* The solidity of a right parallelopiped, therefore, is equal to the *product of its length, breadth, and thickness.* See Alg. 397.

And an *oblique* parallelopiped being equal to a right one of the same base and altitude, (Sup. Euc. 7. 3)† is equal to the area of the base multiplied into the perpendicular height. This is true also of *prisms,* whatever be the form of their bases. (Sup. Euc. 2. Cor. to 8, 3. Thomson's Legendre, 12. 7.)

44. As the sides of a *cube* are all *equal,* the solidity is found by *cubing one of its edges.* On the other hand, if the solid contents be given, the length of the edges may be found, by *extracting the cube root.*

45. When solid measure is cast by *Duodecimals,* it is to be observed that *inches* are not *primes* of feet, but *thirds.* If the unit is a cubic foot, a solid which is an inch thick and a foot square is a prime; a parallelopiped a foot long, an inch broad, and an inch thick is a second, or the twelfth part of a prime; and a cubic inch is a third, or the twelfth part of a second. A linear inch is $\frac{1}{12}$ of a foot, a square inch $\frac{1}{144}$ of a foot, and a cubic inch $\frac{1}{1728}$ of a foot.

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* Thomson's Legendre, 9. 7.

† Ibid., 7. 7.

Ex. 1. What are the solid contents of a stick of timber which is 31 feet long, 1 foot 3 inches broad, and 9 inches thick? Ans. 29 feet 9'', or 29 feet 108 inches.

2. What is the solidity of a wall which is 22 feet long, 12 feet high, and 2 feet 6 inches thick?

Ans. 660 cubic feet.

3. What is the capacity of a cubical vessel which is 2 feet 3 inches deep?

Ans. 11 F. 4' 8'' 3''', or 11 feet 675 inches.

4. If the base of a prism be 108 square inches, and the height 36 feet, what are the solid contents?

Ans. 27 cubic feet.

5. If the height of a square prism be $2\frac{1}{4}$ feet, and each side of the base $10\frac{1}{3}$ feet, what is the solidity?

The area of the base $= 10\frac{1}{3}\times10\frac{1}{3}=106\frac{7}{9}$ sq. feet.

And the solid contents$=106\frac{7}{9}\times\ 2\frac{1}{4}=240\frac{1}{4}$ cubic feet.

6. If the height of a prism be 23 feet, and its base a regular pentagon, whose perimeter is 18 feet, what is the solidity? Ans. 512.84 cubic feet.

46. The number of *gallons* or *bushels* which a vessel will contain may be found, by calculating the capacity in *inches*, and then dividing by the number of inches in 1 gallon or bushel.

The *weight of water* in a vessel of given dimensions is easily calculated; as it is found by experiment, that a cubic foot of pure water weighs 1000 ounces avoirdupois. For the weight in ounces, then, multiply the cubic feet by 1000; or for the weight in pounds, multiply by $62\frac{1}{2}$.

Ex. 1. How many ale gallons are there in a cistern which is 11 feet 9 inches deep, and whose base is 4 feet 2 inches square?

The cistern contains 352500 cubic inches;
And $352500 \div 282 = 1250$.

2. How many wine gallons will fill a ditch 3 feet 11 inches wide, 3 feet deep, and 462 feet long? Ans. 40608.

3. What weight of water can be put into a cubical vessel 4 feet deep? Ans. 4000 lbs.

Problem II.

To find the LATERAL SURFACE *of a* RIGHT PRISM.

47. MULTIPLY THE LENGTH INTO THE PERIMETER OF THE BASE.

Each of the sides of the prism is a right parallelogram, whose area is the product of its length and breadth. But the breadth is one side of the base; and therefore, the sum of the breadths is equal to the perimeter of the base.

Ex. 1. If the base of a right prism be a regular hexagon whose sides are each 2 feet 3 inches, and if the height be 16 feet, what is the lateral surface?

Ans. 216 square feet.

If the areas of the two ends be added to the lateral surface, the sum will be the whole surface of the prism. And the superficies of any solid bounded by planes, is evidently equal to the areas of all its sides.

2. If the base of a prism be an equilateral triangle whose perimeter is 6 feet, and if the height be 17 feet, what is the surface?

The area of the triangle is 1.732. (Art. 11.)
And the whole surface is 105.464.

PROBLEM III.

To find the SOLIDITY *of a* PYRAMID.

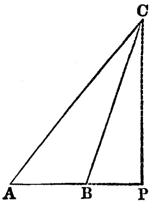
48. MULTIPLY THE AREA OF THE BASE INTO $\frac{1}{3}$ OF THE HEIGHT.

The solidity of a *prism* is equal to the product of the area of the base into the height. (Art. 43.) And a pyramid is $\frac{1}{3}$ of a prism of the same base and altitude. (Sup. Euc. 15, 3. Cor. 1.)* Therefore the solidity of a pyramid whether right or oblique, is equal to the product of the base into $\frac{1}{3}$ of the perpendicular height.

Ex. 1. What is the solidity of a triangular pyramid, whose height is 60, and each side of whose base is 4?

The area of the base is 6.928
And the solidity is 138.56.

2. Let ABC be one side of an oblique pyramid whose base is 6 feet square; let BC be 20 feet, and make an angle of 70 degrees with the plane of the base; and let CP be perpendicular to this plane. What is the solidity of the pyramid?



In the right angled triangle BCP, (Trig. 134.)

$$R : BC :: \sin B : PC = 18.79.$$

And the solidity of the pyramid is 225.48 feet.

3. What is the solidity of a pyramid whose perpendicular height is 72, and the sides of whose base are 67, 54, and 40? Ans. 25920.

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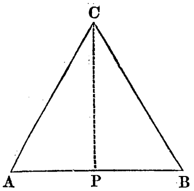
* Thomson's Legendre, 15 and 18. 7.

PROBLEM IV.

To find the LATERAL SURFACE *of a* REGULAR PYRAMID.

49. MULTIPLY HALF THE SLANT-HEIGHT INTO THE PERIMETER OF THE BASE.



Let the triangle ABC be one of the sides of a regular pyramid. As the sides AC and BC are equal, the angles A and B are equal. Therefore a line drawn from the vertex C to the middle of AB is *perpendicular* to AB. The area of the triangle is equal to the product of half this perpendicular into AB. (Art. 8.) The perimeter of the base is the sum of its sides, each of which is equal to AB. And the areas of all the equal triangles which constitute the lateral surface of the pyramid, are together equal to the product of the perimeter into half the slant-height CP.

The *slant-height* is the hypothenuse of a right angled triangle, whose legs are the axis of the pyramid, and the distance from the centre of the base to the middle of one of the sides. See Def. 10.

Ex. 1. What is the lateral surface of a regular hexagonal pyramid, whose axis is 20 feet, and the sides of whose base are each 8 feet ?

The square of the distance from the centre of the base to one of the sides. (Art. 16.)$=48$.

The slant-height (Euc. 47. 1.)*$=\sqrt{48+(20)^2}=21.16$

And the lateral surface$=21.16\times4\times6=507.84$ sq. feet.

2. What is the whole surface of a regular triangular pyr-

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* Thomson's Legendre, 11. 4.

amid whose axis is 8, and the sides of whose base are each 20.78 ?

The lateral surface is	312
The area of the base is	187
And the whole surface is	499

3. What is the lateral surface of a regular pyramid whose axis is 12 feet, and whose base is 18 feet square ?

Ans. 540 square feet.

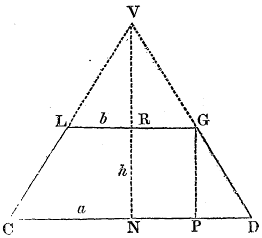
The lateral surface of an *oblique* pyramid may be found, by taking the sum of the areas of the unequal triangles which form its sides.

PROBLEM V.

To find the SOLIDITY *of a* FRUSTUM *of a pyramid.*

50. ADD TOGETHER THE AREAS OF THE TWO ENDS, AND THE SQUARE ROOT OF THE PRODUCT OF THESE AREAS; AND MULTIPLY THE SUM BY $\frac{1}{3}$ OF THE PERPENDICULAR HEIGHT OF THE SOLID.

Let CDGL be a vertical section, through the middle of a frustum of a right pyramid CDV, whose base is a *square*.



Let $CD=a$, $LG=b$, $RN=h$.

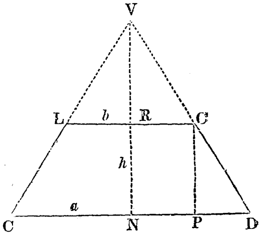
By similar triangles,

$$LG : CD :: RV : NV.$$

Subtracting the antecedents, (Alg. 349.)

$$LG : CD-LG :: RV : NV-RV=RN.$$

$$\text{Therefore } RV=\frac{RN\times LG}{CD-LG}=\frac{hb}{a-b}$$



The square of CD is the base of the pyramid CDV;

And the square of LG is the base of the small pyramid LGV.

Therefore, the solidity of the larger pyramid (Art. 48) is

$$\overline{CD}^2\times\tfrac{1}{3}(RN+RV)=a^2\times\tfrac{1}{3}\left(h+\frac{hb}{a-b}\right)=\frac{ha^3}{3a-3b}$$

And the solidity of the smaller pyramid is equal to

$$\overline{LG}^2\times\tfrac{1}{3}RV=b^2\times\frac{hb}{3a-3b}=\frac{hb^3}{3a-3b}.$$

If the smaller pyramid be taken from the larger, there will remain the frustum CDLG, whose solidity is equal to

$$\frac{ha^3-hb^3}{3a-3b}=\tfrac{1}{3}h\times\frac{a^3-b^3}{a-b}=\tfrac{1}{3}h\times(a^2+ab+b^2.)\ \text{(Alg. 194. }a.\text{)}$$

Or, because $\sqrt{a^2b^2}=ab$, (Alg. 210. *a.*)

$$\tfrac{1}{3}h\times(a^2+b^2+\sqrt{a^2b^2})$$

Here h, the height of the frustum, is multiplied into a and b^2, the areas of the two ends, and into $\sqrt{a^2b^2}$ the square root of the products of these areas.

In this demonstration the pyramid is supposed to be *square*. But the rule is equally applicable to a pyramid of any other form. For the solid contents of pyramids are equal, when they have equal heights and bases, whatever be the *figure* of their bases. (Sup. Euc. 14. 3.)* And the sections parallel to the bases, and at equal distances, are equal to one another. (Sup. Euc. 12. 3. Cor. 2.)*

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* Thomson's Legendre, 14. 7.

Ex. 1. If one end of the frustum of a pyramid be 9 feet square, the other end 6 feet square, and the height 36 feet, what is the solidity ?

The areas of the two ends are 81 and 36.
The square root of their product is 54.
And the solidity of the frustum$=(81+36+54)\times12=2052$.

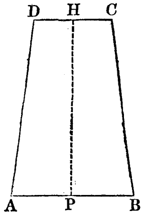
2. If the height of a frustum of a pyramid be 24, and the areas of the two ends 441 and 121 ; what is the solidity ? Ans. 6344.

3. If the height of a frustum of a hexagonal pyramid be 48, each side of one end 26, and each side of the other end 16 ; what is the solidity ? Ans. 56034.

PROBLEM VI.

To find the LATERAL SURFACE *of a* FRUSTUM *of a regular pyramid.*

51. MULTIPLY HALF THE SLANT-HEIGHT BY THE SUM OF THE PERIMETERS OF THE TWO ENDS.



Each side of a frustum of a regular pyramid is a *trapezoid,* as ABCD. The slant-height HP, (Def. 11.) though it is oblique to the base of the solid, is perpendicular to the line AB. The area of the trapezoid is equal to the product of half this perpendicular into the sum of the parallel sides AB and DC. (Art. 12.) Therefore the area of all the equal trapezoids which form the lateral surface of the frustum, is equal to the product of half the slant-height into the sum of the perimeters of the ends.

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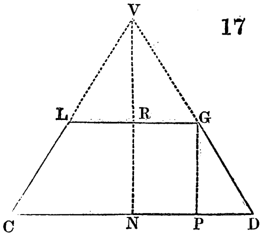
* Thomson's Legendre, 13. 7. Cor.

Ex. If the slant-height of a frustum of a regular octagonal pyramid be 42 feet, the sides of one end 5 feet each, and the sides of the other end 3 feet each; what is the lateral surface? Ans. 1344 square feet.

52. If the slant-height be not given, it may be obtained from the perpendicular height and the dimensions of the two ends. Let GD be the slant-height of the frustum CDGL, RN or GP the perpendicular height, ND and RG the radii of the circles inscribed in the perimeters of the two ends. Then, PD is the difference of the two radii:



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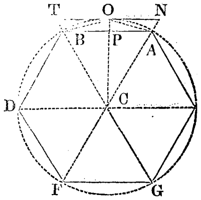
And the slant-height $GD=\sqrt{(GP^2+PD^2)}$.

Ex. If the perpendicular height of a frustum of a regular hexagonal pyramid be 24, the sides of one end 13 each, and the sides of the other end 8 each; what is the whole surface?



$\sqrt{(BC^2-BP^2)}=CP$, that is, $\sqrt{(13^2-6.5^2)}=11.258$

And $\sqrt{8^2-4^2} = 6.928$

The difference of the two radii is, therefore 4.33

The slant-height $=\sqrt{(24^2+4.33^2)}=24.3875$.

The lateral surface is 1536.4

And the whole surface, 2141.75.

The height of the *whole pyramid* may be calculated from the dimensions of the frustum. Let VN (Fig. 17.) be the height of the pyramid, RN or GP the height of the frustum, ND and RG the radii of the circles inscribed in the perimeters of the ends of the frustum.

Then, in the similar triangles GPD and VND,

DP : GP : : DN : VN.

The height of the frustum subtracted from VN, gives VR the height of the small pyramid VLG. The *solidity* and *lateral surface* of the frustum may then be found, by subtracting from the whole pyramid, the part which is above the cutting plane. This method may serve to verify the calculations which are made by the rules in Arts. 50 and 51.

Ex. If one end of the frustum CDGL (Fig. 17.) be 90 feet square, the other end 60 feet square, and the height RN 36 feet; what is the height of the whole pyramid VCD: and what are the solidity and lateral surface of the frustum?

DP=DN—GR=45—30=15. And, GP=RN=36.

Then, 15 : 36 : : 45 : 108=VN, the height of the whole pyramid.

And, 108—36=72=VR, the height of the part VLG.

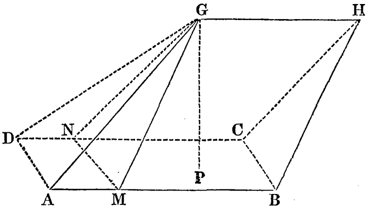
The solidity of the large pyramid is	291600	(Art. 48.)
of the small pyramid	86400	
of the frustum CDGL	205200	

The lateral surface of the large pyramid is	21060	(Art. 49.)
of the small pyramid	9360	
of the frustum	11700	

PROBLEM VII.

To find the SOLIDITY *of a* WEDGE.

54. ADD THE LENGTH OF THE EDGE TO TWICE THE LENGTH OF THE BASE, AND MULTIPLY THE SUM BY $\frac{1}{6}$ OF THE PRODUCT OF THE HEIGHT OF THE WEDGE AND THE BREADTH OF THE BASE.



Let $L = AB$ the length of the base.
Let $l = GH$ the length of the edge.
Let $b = BC$ the breadth of the base.
Let $h = PG$ the height of the wedge.

Then, $L-l = AB-GH = AM$.

If the length of the base and the edge be *equal*, as BM and GH, the wedge MBHG is half a parallelopiped of the same base and height. And the solidity (Art. 43.) is equal to half the product of the height, into the length and breadth of the base; that is $\frac{1}{2}\,bhl$.

If the length of the base be *greater* than that of the edge, as ABGH; let a section be made by the plane GMN, parallel to HBC. This will divide the whole wedge into two parts MBHG and AMG. The latter is a pyramid, whose solidity (Art. 48.) is $\frac{1}{3}\,bh\times(L-l)$

The solidity of the parts together, is, therefore,

$$\tfrac{1}{2}bhl+\tfrac{1}{3}bh\times(L-l)=\tfrac{1}{6}bh3l+\tfrac{1}{6}bh2L-\tfrac{1}{6}bh2l=\tfrac{1}{6}bh\times(2L+l)$$

If the length of the base be *less* than that of the edge, it is evident that the pyramid is to be *subtracted* from half the parallelopiped, which is equal in height and breadth to the wedge, and equal in length to the edge.

The solidity of the wedge is, therefore,

$$\tfrac{1}{2}bhl - \tfrac{1}{3}bh\times(l-\mathrm{L}) = \tfrac{1}{6}bh3l - \tfrac{1}{6}bh2l + \tfrac{1}{6}bh2\mathrm{L} = \tfrac{1}{6}bh\times(2\mathrm{L}+l).$$

Ex. 1. If the base of a wedge be 35 by 15, the edge 55, and the perpendicular height 12.4; what is the solidity?

Ans. $(70+55)\times\frac{15\times12.4}{6}=3875.$

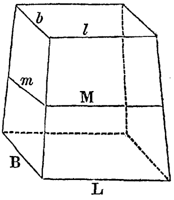
2. If the base of a wedge be 27 by 8, the edge 36, and the perpendicular height 42; what is the solidity?

Ans. 5040.

PROBLEM VIII.

To find the SOLIDITY *of a rectangular* PRISMOID.

55. TO THE AREAS OF THE TWO ENDS, ADD FOUR TIMES THE AREA OF A PARALLEL SECTION EQUALLY DISTANT FROM THE ENDS, AND MULTIPLY THE SUM BY $\frac{1}{6}$ OF THE HEIGHT.



Let L and B be the length and breadth of one end,

Let l and b be the length and breadth of the other end,

Let M and m be the length and breadth of the section in the middle.

And h be the height of the prismoid.

The solid may be divided into two wedges whose bases are the ends of the prismoid, and whose edges are L and l. The solidity of the whole, by the preceding article is,

$$\tfrac{1}{6}Bh\times(2\mathrm{L}+l)+\tfrac{1}{6}bh\times(2l+\mathrm{L})=\tfrac{1}{6}h(2\mathrm{BL}+\mathrm{B}l+2bl+b\mathrm{L})$$

As M is equally distant from L and l,

$$2\mathrm{M}=\mathrm{L}+l,\ 2m=\mathrm{B}+b, \text{ and } 4\,\mathrm{M}m=(\mathrm{L}+l)(\mathrm{B}+b)=\mathrm{BL}+\mathrm{B}l+b\mathrm{L}+lb.$$

Substituting 4 Mm for its value, in the preceding expression for the solidity, we have

$$\tfrac{1}{6}h\,(\mathrm{BL}+bl+4\mathrm{M}m)$$

That is, the solidity of the prismoid is equal to $\frac{1}{6}$ of the height, multiplied into the areas of the two ends, and 4 times the area of the section in the middle.

This rule may be applied to prismoids of other forms. For, whatever be the figure of the two ends, there may be drawn in each, such a number of small rectangles, that the sum of them shall differ less, than by any given quantity, from the figure in which they are contained. And the solids between these rectangles will be rectangular prismoids.

Ex. 1. If one end of a rectangular prismoid be 44 feet by 23, the other end 36 by 21, and the perpendicular height 72; what is the solidity?

The area of the larger end $=44\times23=1012$
of the smaller end $=36\times21=\ 756$
of the middle section $=40\times22=\ 880$
And the solidity$=(1012+756+4\times880)\times12=63456$ feet.

2. What is the solidity of a stick of hewn timber, whose ends are 30 inches by 27, and 24 by 18, and whose length is 48 feet? Ans. 204 feet.

Other solids not treated of in this section, if they be bounded by plane surfaces, may be measured by supposing them to be divided into prisms, pyramids, and wedges. And, indeed, every such solid may be considered as made up of triangular pyramids.

THE FIVE REGULAR SOLIDS.

56. A SOLID IS SAID TO BE REGULAR, WHEN ALL ITS SOLID ANGLES ARE EQUAL, AND ALL ITS SIDES ARE EQUAL AND REGULAR POLYGONS.

The following figures are of this description;

1. The *Tetraedron*,	whose sides are	four triangles;
2. The *Hexaedron or cube*,		six squares;
3. The *Octaedron*,		eight triangles;
4. The *Dodecaedron*,		twelve pentagons;
5. The *Icosaedron*,		twenty triangles.*

Besides these five there can be no other regular solids. The only plane figures which can form such solids, are triangles, squares, and pentagons. For the plane angles which contain any solid angle, are together less than four right angles or 360°. (Sup. Euc. 21, 2.) And the least number which can form a solid angle is three. (Sup. Euc. Def. 8, 2.) If they are angles of equilateral *triangles*, each is 60°. The sum of *three* of them is 180°, of *four* 240°, of *five* 300°, and of *six* 360°. The latter number is too great for a solid angle.

The angles of *squares* are 90° each. The sum of *three* of these is 270°, of four 360°, and of any other greater number, still more.

The angles of regular *pentagons* are 108° each. The sum of *three* of them is 324°; of four, or any other greater number, more than 360°. The angles of all other regular polygons are still greater.

In a regular solid, then, each solid angle must be contained by three, four, or five equilateral triangles, by three squares, or by three regular pentagons.

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* For the geometrical construction of these solids, see Legendre's Geometry; Appendix to Books VI. and VII., or Thomson's Legendre, p. 214.

57. As the sides of a regular solid are similar and equal, and the angles are also alike; it is evident that the sides are all equally distant from a central point in the solid. If then, planes be supposed to proceed from the several edges to the centre, they will divide the solid into as many equal *pyramids*, as it has sides. The base of each pyramid will be one of the sides; their common vertex will be the central point; and their height will be a perpendicular from the centre to one of the sides.

PROBLEM IX.

To find the SURFACE *of a* REGULAR SOLID.

58. MULTIPLY THE AREA OF ONE OF THE SIDES BY THE NUMBER OF SIDES.

Or,

MULTIPLY THE SQUARE OF ONE OF THE EDGES, BY THE SURFACE OF A SIMILAR SOLID WHOSE EDGES ARE 1.

As all the sides are *equal*, it is evident that the area of one of them, multiplied by the number of sides, will give the area of the whole.

Or, if a *table* is prepared, containing the surfaces of the several regular solids whose linear edges are *unity*; this may be used for other regular solids, upon the principle, that the areas of similar polygons are as the squares of their homologous sides. (Euc. 20. 6.)* Such a table is easily formed, by multiplying the area of one of the sides, as given in Art. 17, by the number of sides. Thus, the area of an equilateral triangle whose side is 1, is 0.4330127. Therefore, the surface

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* Thomson's Legendre, 27. 4.

Of a regular tetraedron $=.4330127\times4 \;=1.7320508$.
Of a regular octaedron $=.4330127\times8 \;=3.4641016$.
Of a regular icosaedron $=.4330127\times20=8.6602540$.

See the table in the following article.

Ex. 1. What is the surface of a regular dodecaedron whose edges are each 25 inches?

The area of one of the sides is 1075.3
And the surface of the whole solid $=1075.3\times12=12903.6$.

2. What is the surface of a regular icosaedron whose edges are each 102? Ans. 90101.3.

PROBLEM X.

To find the SOLIDITY *of a* REGULAR SOLID.

59. MULTIPLY THE SURFACE BY $\frac{1}{3}$ OF THE PERPENDICULAR DISTANCE FROM THE CENTRE TO ONE OF THE SIDES.

Or,

MULTIPLY THE CUBE OF ONE OF THE EDGES, BY THE SOLIDITY OF A SIMILAR SOLID WHOSE EDGES ARE 1.

As the solid is made up of a number of equal pyramids, whose bases are the sides, and whose height is the perpendicular distance of the sides from the centre (Art. 57.); the solidity of the whole must be equal to the areas of all the sides multiplied into $\frac{1}{3}$ of this perpendicular. (Art. 48.)

If the contents of the several regular solids whose edges are 1, be inserted in a *table*, this may be used to measure other similar solids. For two similar regular solids contain the same number of similar pyramids; and these are to each other as the *cubes* of their linear sides or edges. (Sup. Euc. 15. 3. Cor. 3.)*

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* Thomson's Legendre, 20. 7.

A TABLE OF REGULAR SOLIDS WHOSE EDGES ARE 1.

Names.	No. of sides.	Surfaces.	Solidities.
Tetraedron	4	1.7320508	0.1178513
Hexaedron	6	6.0000000	1.0000000
Octaedron	8	3.4641016	0.4714045
Dodecaedron	12	20.6457288	7.6631189
Icosaedron	20	8.6602540	2.1816950

For the method of calculating the last column of this table, see Hutton's Mensuration, Part. III. Sec. 2.

Ex. What is the solidity of a regular octaedron whose edges are each 32 inches ? Ans. 15447 inches.

SECTION IV.

THE CYLINDER, CONE, AND SPHERE.

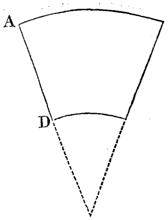
Art. 61. Definition I. A *right cylinder* is a solid described by the revolution of a rectangle about one of its sides. The *ends or bases* are evidently equal and parallel circles. And the *axis,* which is a line passing through the middle of the cylinder, is perpendicular to the bases.

The ends of an *oblique* cylinder are also equal and parallel circles ; but they are not perpendicular to the axis. The *height* of a cylinder is the perpendicular distance from one base to the plane of the other. In a right cylinder, it is the length of the axis.

II. A *right cone* is a solid described by the revolution of a right angled triangle about one of the sides which contain the right angle. The *base* is a circle, and is perpendicular to the *axis*, which proceeds from the middle of the base to the vertex.

The base of an *oblique* cone is also a circle, but is not perpendicular to the axis. The *height* of a cone is the perpendicular distance from the vertex to the plane of the base. In a right cone, it is the length of the axis. The *slant-height* of a right cone is the distance from the vertex to the circumference of the base.

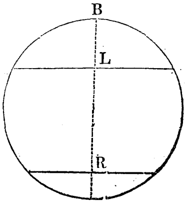
III. A *frustum* of a cone is a portion cut off by a plane parallel to the base. The *height* of the frustum is the perpendicular distance of the two ends. The *slant-height* of a frustum of a right cone, is the distance between the peripheries of the two ends, measured on the outside of the solid; as AD.



IV. A *sphere* or *globe* is a solid which has a centre equally distant from every part of the surface. It may be described by the revolution of a semicircle about a diameter. A *radius* of the sphere is a line drawn from the centre to any part of the surface. A *diameter* is a line passing through the centre, and terminated at both ends by the surface. The *circumference* is the same as the circumference of a circle whose plane passes through the centre of the sphere. Such a circle is called a *great circle.*

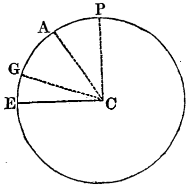
V. A *segment* of a sphere is a part cut off by any plane. The *height* of the segment is a perpendicular from the middle of the base to the convex surface, as LB.

VI. A *spherical zone* or frustum is a part of the sphere included between two parallel planes. It is called the *middle zone*, if the planes are equally distant from the centre.



The *height* of a zone is the distance of the two planes, as LR.*

VII. A *spherical sector* is a solid produced by a circular sector, revolving in the same manner as the semicircle which describes the whole sphere. Thus a spherical sector is described by the circular sector ACP or GCE revolving on the axis CP.



VIII. A solid described by the revolution of any figure about a fixed axis, is called a *solid of revolution.*

PROBLEM I.

To find the CONVEX SURFACE *of a* RIGHT CYLINDER.

62. MULTIPLY THE LENGTH INTO THE CIRCUMFERENCE OF THE BASE.

If a right cylinder be covered with a thin substance like paper, which can be spread out into a plane ; it is evident that the plane will be a *parallelogram,* whose length and breadth will be equal to the length and circumference of the cylinder. The area must, therefore, be equal to the length multiplied into the circumference. (Art. 4.)

Ex. 1. What is the convex surface of a right cylinder which is 42 feet long, and 15 inches in diameter?

Ans. $42 \times 1.25 \times 3.14159 = 164.933$ sq. feet.

2. What is the whole surface of a right cylinder, which is 2 feet in diameter and 36 feet long?

---

* According to some writers, a spherical *segment* is either a solid which is cut off from the sphere by a single plane, or one which is included between two planes: and a *zone* is the *surface* of either of these. In this sense, the term zone is commonly used in geography.

The convex surface is	226.1945
The area of the two ends (Art. 30.) is	6.2832
The whole surface is	232.4777

3. What is the whole surface of a right cylinder whose axis is 82, and circumference 71 ? Ans. 6624.32.

63. It will be observed that the rules for the *prism* and *pyramid* in the preceding section, are substantially the same, as the rules for the *cylinder* and *cone* in this. There may be some advantage, however, in considering the latter by themselves.

In the base of a *cylinder,* there may be inscribed a polygon, which shall differ from it less than by any given space. (Sup. Euc. 6. 1. Cor.)[*] If the polygon be the base of a *prism,* of the same height as the cylinder, the two solids may differ less than by any given quantity. In the same manner, the base of a *pyramid* may be a polygon of so many sides, as to differ less than by any given quantity, from the base of a *cone* in which it is inscribed. A cylinder is therefore considered, by many writers, as a prism of an infinite number of sides; and a cone, as a pyramid of an infinite number of sides. (For the meaning of the term "infinite," when used in the mathematical sense, see Alg. Sec. XV.)

PROBLEM II.

To find the SOLIDITY *of a* CYLINDER.

64. MULTIPLY THE AREA OF THE BASE BY THE HEIGHT.

The solidity of a *parallelopiped* is equal to the product of the base into the perpendicular altitude. (Art. 43.) And a parallelopiped and a cylinder which have equal bases and altitudes are equal to each other. (Sup. Euc. 17. 3.)[†]

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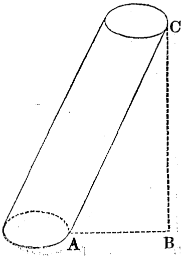
[*] **Thomson's Legendre, 9. 5.**

[†] **Ibid., 2. 8.**

Ex. 1. What is the solidity of a cylinder, whose height is 121, and diameter 45.2?

Ans. $\overline{45.2^2}\times.7854\times121=194156.6$.

2. What is the solidity of a cylinder, whose height is 424, and circumference 213? Ans. 1530837.



3. If the side AC of an oblique cylinder be 27, and the area of the base 32.61, and if the side make an angle of 62° 44′ with the base, what is the solidity?

R : AC : : sin A : BC=24 the perpendicular height.

And the solidity is 782.64.

4. The Winchester bushel is a hollow cylinder, $18\frac{1}{2}$ inches in diameter, and 8 inches deep. What is its capacity?

The area of the base $=(18.5)^2\times.7853982=268.8025$.

And the capacity is 2150.42 cubic inches. See the table in Art. 42.

Problem III.

To find the CONVEX SURFACE *of a* RIGHT CONE.

65. MULTIPLY HALF THE SLANT-HEIGHT INTO THE CIRCUMFERENCE OF THE BASE.

If the convex surface of a right cone be spread out into a plane, it will evidently form a *sector* of a circle whose radius is equal to the slant-height of the cone. But the area of the sector is equal to the product of half the radius into the length of the arc. (Art. 34.) Or if the cone be considered as a pyramid of an infinite number of sides, its lateral surface is equal to the product of half the slant-height into the perimeter of the base. (Art. 49.)

Ex. 1. If the slant-height of a right cone be 82, and the diameter of the base 24, what is the convex surface?

Ans. $41\times24\times3.14159=3091.3$ square feet.

2. If the axis of a right cone be 48, and the diameter of the base 72, what is the whole surface?

The slant-height $=\sqrt{(36^2+48^2)}=60$. (Euc. 47. 1.)

The convex surface is	6786
The area of the base	4071.6
And the whole surface	10857.6

3. If the axis of a right cone be 16, and the circumference of the base 75.4; what is the whole surface?

Ans. 1206.4.

PROBLEM IV.

To find the SOLIDITY *of a* CONE.

66. MULTIPLY THE AREA OF THE BASE INTO $\frac{1}{3}$ OF THE HEIGHT.

The solidity of a *cylinder* is equal to the product of the base into the perpendicular height. (Art. 64.) And if a cone and a cylinder have the same base and altitude, the cone is $\frac{1}{3}$ of the cylinder. (Sup. Euc. 18. 3.)* Or if a cone be considered as a pyramid of an infinite number of sides, the solidity is equal to the product of the base into $\frac{1}{3}$ of the height, by Art. 48.

Ex. 1. What is the solidity of a right cone whose height is 663, and the diameter of whose base is 101?

Ans. $\overline{101^2}\times.7854\times221=1770622$.

---

* Thomson's Legendre, 4. 8. Cor.

2. If the axis of an oblique cone be 738, and make an angle of 30° with the plane of the base; and if the circumference of the base be 355, what is the solidity?

Ans. 1233536.

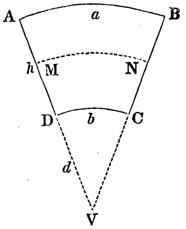
PROBLEM V.

To find the CONVEX SURFACE *of a* FRUSTUM *of a right cone.*

67. MULTIPLY HALF THE SLANT-HEIGHT BY THE SUM OF THE PERIPHERIES OF THE TWO ENDS.

This is the rule for a frustum of a *pyramid*; (Art. 51.) and is equally applicable to a frustum of a *cone*, if a cone be considered as a pyramid of an infinite number of sides. (Art. 63.)

Or thus,



Let the sector ABV represent the convex surface of a right cone, (Art. 65.) and DCV the surface of a portion of the cone, cut off by a plane parallel to the base. Then will ABCD be the surface of the frustum.

Let $AB=a$, $DC=b$, $VD=d$, $AD=h$.

Then the area $ABV=\frac{1}{2}a\times(h+d)=\frac{1}{2}ah+\frac{1}{2}ad$. (Art. 34.)

And the area $DCV=\frac{1}{2}bd$.

Subtracting the one from the other,

The area $ABDC=\frac{1}{2}ah+\frac{1}{2}ad-\frac{1}{2}bd$.

But $d : d+h :: b : a$. (Sup. Euc. 8. 1.)* Therefore $\frac{1}{2}ad-\frac{1}{2}bd=\frac{1}{2}bh$.

The surface of the frustum then, is equal to

$\frac{1}{2}ah+\frac{1}{2}bh$. or $\frac{1}{2}h\times(a+b)$

---

* Thomson's Legendre, 10. 5. Cor.

Cor. The surface of the frustum is equal to the product of the slant-height into the circumference of a circle which is *equally distant* from the two ends. Thus, the surface ABCD is equal to the product of AD into MN. For MN is equal to half the sum of AB and DC.

Ex. 1. What is the convex surface of a frustum of a right cone, if the diameters of the two ends be 44 and 33, and the slant-height 84? Ans. 10159.8.

2. If the perpendicular height of a frustum of a right cone be 24, and the diameters of the two ends 80 and 44, what is the whole surface?

Half the difference of the diameters is 18.

And $\sqrt{18^2+24^2}=30$, the slant-height, (Art. 52.)

The convex surface of the frustum is	5843
The sum of the areas of the two ends is	6547
And the whole surface is	12390

PROBLEM VI.

To find the SOLIDITY *of a* FRUSTUM *of a cone.*

68. ADD TOGETHER THE AREAS OF THE TWO ENDS, AND THE SQUARE ROOT OF THE PRODUCT OF THESE AREAS; AND MULTIPLY THE SUM BY $\frac{1}{3}$ OF THE PERPENDICULAR HEIGHT.

This rule, which was given for the frustum of a *pyramid*, (Art. 50.) is equally applicable to the frustum of a cone; because a cone and a pyramid which have equal bases and altitudes are equal to each other.

Ex. 1. What is the solidity of a mast which is 72 feet long, 2 feet in diameter at one end, and 18 inches at the other? Ans. 174.36 cubic feet.

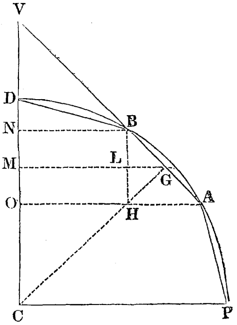
2. What is the capacity of a conical cistern which is 9 feet deep, 4 feet in diameter at the bottom, and 3 feet at the top? Ans. 87.18 cubic feet=652.15 wine gallons.

3. How many gallons of ale can be put into a vat in the form of a conic frustum, if the larger diameter be 7 feet, the smaller diameter 6 feet, and the depth 8 feet?

PROBLEM VII.

To find the SURFACE *of a* SPHERE.

69. MULTIPLY THE DIAMETER BY THE CIRCUMFERENCE.



Let a hemisphere be described by the quadrant CPD, revolving on the line CD. Let AB be the side of a regular polygon inscribed in the circle of which DBP is an arc. Draw AO and BN perpendicular to CD, and BH perpendicular to AO. Extend AB till it meets CD continued. The triangle AOV, revolving on OV as an axis, will describe a right *cone*. (Defin. 2.) AB will be the slant-height of a *frustum* of this cone extending from AO to BN. From G the middle of AB, draw GM parallel to AO. The surface of the frustum described by AB. (Art. 67. Cor.) is equal to

$$AB \times circ\ GM.*$$

From the centre C draw CG, which will be perpendicular to AB, (Euc. 3. 3.) and the radius of a circle inscribed in the polygon. The triangles ABH and CGM are similar, because the sides are perpendicular, each to each. Therefore,

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* By *circ* GM is meant the circumference of a circle the radius of which is GM.

$$\text{HB or ON} : \text{AB} :: \text{GM} : \text{GC} :: circ\ \text{GM} : circ\ \text{GC}.$$

So that $\text{ON} \times circ\ \text{GC} = \text{AB} \times circ\ \text{GM}$, that is, the surface of the frustum is equal to the product of ON the perpendicular height, into *circ* GC, the perpendicular distance from the centre of the polygon to one of the sides.

In the same manner it may be proved, that the surfaces produced by the revolution of the lines BD and AP about the axis DC, are equal to

$$\text{ND} \times circ\ \text{GC}, \qquad \text{and } \text{CO} \times circ\ \text{GC}.$$

The surface of the whole solid, therefore, (Euc. 1. 2.) is equal to

$$\text{CD} \times circ\ \text{GC}.$$

The demonstration is applicable to a solid produced by the revolution of a polygon of *any* number of sides. But a polygon may be supposed which shall differ less than by any given quantity from the circle in which it is inscribed; (Sup. Euc. 4. 1.)* and in which the perpendicular GC shall differ less than by any given quantity from the radius of the circle. Therefore, the surface of a *hemisphere* is equal to the product of its radius into the circumference of its base; and *the surface of a sphere is equal to the product of its diameter into its circumference.*

Cor. 1. From this demonstration it follows, that the surface of any *segment* or *zone* of a sphere is equal to the product of the height of the segment or zone into the circumference of the sphere. The surface of the zone produced by the revolution of the arc AB about ON, is equal to $\text{ON} \times circ$ CP. And the surface of the segment produced by the revolution of BD about DN is equal to DN× *circ* CP.

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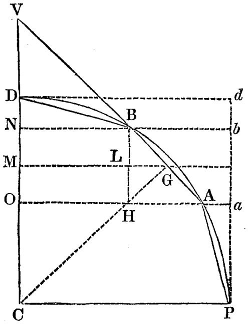
* Thomson's Legendre, 9. 5.

Cor. 2. The surface of a sphere is equal to four times the area of a circle of the same diameter; and therefore, the convex surface of a hemisphere is equal to twice the area of its base. For the area of a circle is equal to the product of half the diameter into half the circumference; (Art. 30.) that is, to $\frac{1}{4}$ the product of the diameter and circumference.

Cor. 3. The surface of a sphere, or the convex surface of any spherical segment or zone, is equal to that of the circumscribing cylinder. A hemisphere described by the revolution of the arc DBP, is circumscribed by a cylinder produced by the revolution of the parallelogram D*d*CP. The convex surface of the cylinder is equal to its height multiplied by its circumference. (Art. 62.) And this is also the surface of the hemisphere.



So the surface produced by the revolution of AB is equal to that produced by the revolution of *ab*. And the surface produced by BD is equal to that produced by *bd*.

Ex. 1. Considering the earth as a sphere 7930 miles in diameter, how many square miles are there on its surface?

Ans. 197,558,500.

2. If the circumference of the sun be 2,800,000, what is his surface? Ans. 2,495,547,600,000 sq. miles.

3. How many square feet of lead will it require, to cover a hemispherical dome whose base is 13 feet across?

Ans. $265\frac{1}{2}$.

PROBLEM VIII.

To find the SOLIDITY *of a* SPHERE.

70. 1. MULTIPLY THE CUBE OF THE DIAMETER BY **.5236.**

Or,

2. MULTIPLY THE SQUARE OF THE DIAMETER BY $\frac{1}{6}$ OF THE CIRCUMFERENCE.

Or,

3. MULTIPLY THE SURFACE BY $\frac{1}{6}$ OF THE DIAMETER.

1. A sphere is *two-thirds* of its circumscribing cylinder. (Sup. Euc. 21. 3.)* The height and diameter of the cylinder are each equal to the diameter of the sphere. The solidity of the cylinder is equal to its height multiplied into the area of its base, (Art. 64.) that is putting D for the diameter,

$$D \times D^2 \times .7854 \quad \text{or} \quad D^3 \times .7854.$$

And the solidity of the *sphere*, being $\frac{2}{3}$ of this, is

$$D^3 \times .5236.$$

2. The base of the circumscribing cylinder is equal to half the circumference multiplied into half the diameter; (Art. 30.) that is, if C be put for the circumference,

$\frac{1}{4}C \times D$; and the solidity is $\frac{1}{4}C \times D^2$.

Therefore, the solidity of the sphere is

$$\tfrac{2}{3} \text{ of } \tfrac{1}{4}C \times D^2 = D^2 \times \tfrac{1}{6}C.$$

3. In the last expression, which is the same as $C \times D \times \frac{1}{6}D$,

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* Thomson's Legendre, 12. 8.

we may substitute S, the surface, for $C\times D$. (Art. 69.) We then have the solidity of the sphere equal to

$$S\times\tfrac{1}{6}D.$$

Or, the sphere may be supposed to be filled with small *pyramids*, standing on the surface of the sphere, and having their common vertex in the centre. The number of these may be such, that the difference between their sum and the sphere shall be less than any given quantity. The solidity of each pyramid is equal to the product of its base into $\frac{1}{3}$ of its height. (Art. 48.) The solidity of the whole, therefore, is equal to the product of the surface of the sphere into $\frac{1}{3}$ of its radius, or $\frac{1}{6}$ of its diameter.

71. The numbers 3.14159, .7854, .5236, should be made perfectly familiar. The first expresses the ratio of the *circumference* of a circle to the *diameter;* (Art. 23.) the second, the ratio of the *area* of a circle to the square of the diameter (Art. 30.); and the third, the ratio of the *solidity* of a sphere to the *cube* of the diameter. The second is $\frac{1}{4}$ of the first, and the third is $\frac{1}{6}$ of the first.

As these numbers are frequently occurring in mathematical investigations, it is common to represent the first of them by the Greek letter π. According to this notation,

$$\pi=\mathbf{3.14159},\qquad \tfrac{1}{4}\pi=\mathbf{.7854},\qquad \tfrac{1}{6}\pi=\mathbf{.5236}.$$

If D=the *diameter*, and R=the *radius* of any circle or sphere;

$$\text{Then,}\quad D=2R\quad D^2=4R^2\quad D^3=8R^3.$$

$$\left.\begin{matrix}\text{And } \pi D\\ \text{Or, } 2\pi R\end{matrix}\right\}=\text{the } \textit{periph.}\ \left.\begin{matrix}\frac{1}{4}\pi D^2\\ \text{or } \pi R^2\end{matrix}\right\}\begin{matrix}=\text{the } \textit{area} \text{ of}\\ \text{the circ. or}\end{matrix}\ \left.\begin{matrix}\frac{1}{6}\pi D^3\\ \frac{4}{3}\pi R^3\end{matrix}\right\}=\text{the}$$

solidity of the sphere.

Ex 1. What is the solidity of the earth, if it be a sphere 7930 miles in diameter?

Ans. 261,107,000,000 cubic miles.

2. How many wine gallons will fill a hollow sphere 4 feet in diameter?

Ans. The capacity is 33.5104 feet$=250\frac{2}{3}$ gallons.

3. If the diameter of the moon be 2180 miles, what is its solidity? Ans. 5,424,600,000 miles.

72. If the solidity of a sphere be *given,* the diameter may be found by reversing the first rule in the preceding article; that is, *dividing by* .5236 and *extracting the cube root of the quotient.*

Ex. 1. What is the diameter of a sphere whose solidity is 65.45 cubic feet? Ans. 5 feet.

2. What must be the diameter of a globe to contain 16755 pounds of water? Ans. 8 feet.

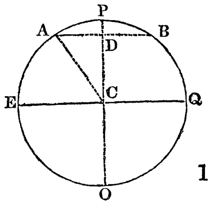
PROBLEM IX.

To find the CONVEX SURFACE *of a* SEGMENT *or* ZONE *of a sphere.*

73. MULTIPLY THE HEIGHT OF THE SEGMENT OR ZONE INTO THE CIRCUMFERENCE OF THE SPHERE.

For the demonstration of this rule, see Art. 69.

Ex. 1. If the earth be considered a perfect sphere 7930 miles in diameter, and if the polar circle be 23° 28′ from the pole, how many square miles are there in one of the frigid zones?



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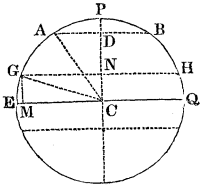
If PQOE be a meridian on the earth, ADB one of the polar circles, and P the pole; then the frigid zone is a spherical segment described by the revolution of the arc APB about PD. The angle ACD subtended by the arc AP is 23° 28′. And in the right angled triangle ACD, R : AC :: cos ACD : CD=3637.

Then, CP—CD=3965—3637=328=PD the height of the segment.

And 328×7930×3.14159=8171400 the surface.

2. If the diameter of the earth be 7930 miles, what is the surface of the torrid zone, extending 23° 28′ on each side of the equator?



If EQ be the equator, and GH one of the tropics, then the angle ECG is 23° 28′. And in the right angled triangle GCM,

R : CG : : sin ECG : GM=CN=1578.9 the height of half the zone.

The surface of the whole zone is 78669700.

3. What is the surface of each of the temperate zones?

The height DN=CP—CN—PD=2058.1

And the surface of the zone is 51273000.

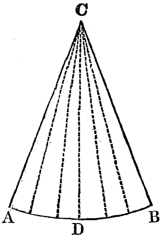
The surface of the two temperate zones is	102,546,000
of the two frigid zones	16,342,800
of the torrid zone	78,669,700
of the whole globe	197,558,500

Problem X.

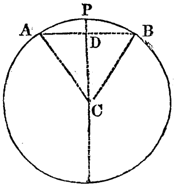
To find the SOLIDITY *of a spherical* SECTOR.

74. MULTIPLY THE SPHERICAL SURFACE BY $\frac{1}{3}$ OF THE RADIUS OF THE SPHERE.

The spherical sector produced by the revolution of ACBD



about CD, may be supposed to be filled with *small pyramids*, standing on the spherical surface ADB, and terminating in the point C. Their number may be so great, that the height of each shall differ less than by any given length from the radius CD, and the sum of their bases shall differ less than by any given quantity from the surface ABD. The solidity of each is equal to the product of its base into $\frac{1}{3}$ of the radius CD. (Art. 48.) Therefore, the solidity of all of them, that is, of the sector ADBC, is equal to the product of the spherical surface into $\frac{1}{3}$ of the radius.



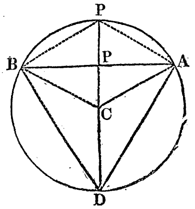
Ex. Supposing the earth to be a sphere. 7930 miles in diameter, and the polar circle ADB to be 23° 28′ from the pole; what is the solidity of the spherical sector ACBP ?

Ans. 10,799,867,000 miles.

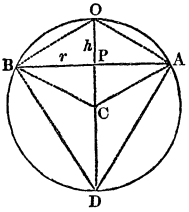
PROBLEM XI.

To find the SOLIDITY *of a spherical* SEGMENT.

75. MULTIPLY HALF THE HEIGHT OF THE SEGMENT INTO THE AREA OF THE BASE, AND THE CUBE OF THE HEIGHT INTO .5236 ; AND ADD THE TWO PRODUCTS.



As the *circular* sector AOBC consists of two parts, the segment AOBP and the triangle ABC; (Art. 35.) so the *spherical* sector produced by the revolution of AOC about OC consists of two parts, the *segment* produced by the revolution of AOP, and the *cone* produced by the revolution of ACP. If then the cone be subtracted from the sector, the remainder will be the segment.



Let CO$=$R, the radius of the sphere,
PB$=r$, the radius of the base of the segment.
PO$=h$, the height of the segment,
Then PC$=$R$-h$, the axis of the cone.

The sector$=2\pi\mathrm{R}\times h\times\frac{1}{3}\mathrm{R}$ (Arts. 71, 73, 74.)$=\frac{2}{3}\pi h\mathrm{R}^2$.
The cone$=\pi r^2\times\frac{1}{3}(\mathrm{R}-h)$ (Arts. 71, 66.)$=\frac{1}{2}\pi r^2\mathrm{R}-\frac{1}{3}\pi hr^2$.

Subtracting the one from the other,

$$\text{The segment }=\frac{2}{3}\pi h\mathrm{R}^2-\frac{1}{3}\pi r^2\mathrm{R}+\frac{1}{3}\pi hr^2.$$

But DO$\times$PO$=\overline{\mathrm{BO}}^2$ (Trig. 97.*)$=\overline{\mathrm{PO}}^2+\overline{\mathrm{PB}}^2$ (Euc. 47. 1.)

That is, $2\mathrm{R}h=h^2+r^2$. So that, $\mathrm{R}=\dfrac{h^2+r^2}{2h}$

$$\text{And }\mathrm{R}^2=\left(\frac{h^2+r^2}{2h}\right)^2=\frac{h^4+2h^2r^2+r^4}{4h^2}$$

Substituting then, for R and R^2, their values, and multiplying the factors,

$$\text{The segment}=\frac{1}{6}\pi h^3+\frac{1}{3}\pi hr^2+\frac{1}{6}\frac{\pi r^4}{h}-\frac{1}{6}\pi hr^2-\frac{1}{6}\frac{\pi r^4}{h}+\frac{1}{3}\pi hr^2$$

Which, by uniting the terms, becomes

$$\frac{1}{2}\pi hr^2+\frac{1}{6}\pi h^3.$$

The first term here is $\frac{1}{2}h\times\pi r^2$, half the height of the segment multiplied into the area of the base; (Art. 71.) and the other $h^3\times\frac{1}{6}\pi$, the cube of the height multiplied into .5236.

---

* Euclid 31, 3, and 8, 6. Cor.

If the segment be *greater* than a hemisphere, as ABD; the cone ABC must be *added* to the sector ACBD.

Let $PD = h$ the height of the segment,
Then $PC = h - R$ the axis of the cone.

$$\text{The sector } ACBD = \tfrac{2}{3}\pi h R^2$$

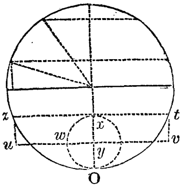
$$\text{The cone} = \pi r^2 \times \tfrac{1}{3}(h - R) = \tfrac{1}{3}\pi h r^2 - \tfrac{1}{3}\pi r^2 R$$

Adding them together, we have as before,

$$\text{The segment} = \tfrac{2}{3}\pi h R^2 - \tfrac{1}{3}\pi r^2 R + \tfrac{1}{3}\pi h r^2.$$

Cor. The solidity of a spherical segment is equal to half a cylinder of the same base and height + a sphere whose diameter is the height of the segment. For a cylinder is equal to its height multiplied into the area of its base; and a sphere is equal to the cube of its diameter multiplied by .5236.

Thus, if Oy be half Ox, the spherical segment produced by the revolution, of Oxt is equal to the cylinder produced by $tvyx$ + the sphere produced by $Oyxz$; supposing each to revolve on the line Ox.



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Ex. 1. If the height of a spherical segment be 8 feet, and the diameter of its base 25 feet; what is the solidity?
Ans. $(25)^2 \times .7854 \times 4 + 8^3 \times .5236 = 2231.58$ feet.

2. If the earth be a sphere 7930 miles in diameter, and the polar circle 23° 28′ from the pole, what is the solidity of one of the frigid zones? Ans. 1,303,000,000 miles.

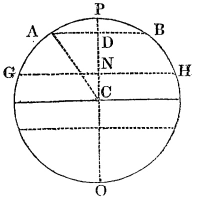
PROBLEM XII.

To find the SOLIDITY *of a spherical* ZONE *or frustum.*

76. FROM THE SOLIDITY OF THE WHOLE SPHERE, SUBTRACT THE TWO SEGMENTS ON THE SIDES OF THE ZONE.

Or,

ADD TOGETHER THE SQUARES OF THE RADII OF THE TWO ENDS, AND $\frac{1}{3}$ THE SQUARE OF THEIR DISTANCE; AND MULTIPLY THE SUM BY THREE TIMES THIS DISTANCE, AND THE PRODUCT BY .5236.



If from the whole sphere, there be taken the two segments ABP and GHO, there will remain the zone or frustum ABGH.

Or, the zone ABGH is equal to the difference between the segments GHP and ABP.

Let $NP=H$, $DP=h$, the *heights* of the two segments.
$GN=R$, $AD=r$, the *radii* of their bases.
$DN=d=H-h$ the *distance* of the two bases, or the height of the zone.

Then the larger segment $=\frac{1}{2}\pi HR^2+\frac{1}{6}\pi H^3$
And the smaller segment $=\frac{1}{2}\pi hr^2+\frac{1}{6}\pi h^3$ (Art. 75.)

Therefore the zone $ABGH=\frac{1}{6}\pi\,(3HR^2+H^3-3hr^2-h^3)$

By the properties of the circle, (Euc. 35, 3.)

$ON\times H=R^2$. Therefore, $(ON+H)\times H=R^2+H^2$

$$\text{Or, } OP=\frac{R^2+H^2}{H}$$

In the same manner, $\mathrm{OP}=\dfrac{r^2+h^2}{h}$

Therefore, $3\mathrm{H}\times(r^2+h^2)=3h\times(\mathrm{R}^2+\mathrm{H}^2.)$

Or, $3\mathrm{H}r^2+3\mathrm{H}h^2-3h\mathrm{R}^2-3h\mathrm{H}^2=0.$ (Alg. 178.)

To reduce the expression for the solidity of the zone to the required form, without altering its value, let these terms be added to it: and it will become

$$\tfrac{1}{6}\pi(3\mathrm{HR}^2+3\mathrm{H}r^2-3h\mathrm{R}^2-3hr^2+\mathrm{H}^3-3\mathrm{H}^2h+3\mathrm{H}h^2-h^3)$$

Which is equal to

$$\tfrac{1}{6}\pi\times 3(\mathrm{H}-h)\times(\mathrm{R}^2+r^2+\tfrac{1}{3}(\mathrm{H}-h)^2)$$

Or, as $\frac{1}{6}\pi$ equals .5236 (Art. 71.) and $\mathrm{H}-h$ equals d,

The zone$=.5236\times 3d\times(\mathrm{R}^2+r^2+\frac{1}{3}d^2.)$

Ex. 1. If the diameter of one end of a spherical zone is 24 feet, the diameter of the other end 20 feet, and the distance of the two ends, or the height of the zone 4 feet; what is the solidity? Ans. 1566.6 feet.

2. If the earth be a sphere 7930 miles in diameter, and the obliquity of the ecliptic 23° 28′; what is the solidity of one of the temperate zones?

Ans. 55,390,500,000 miles.

3. What is the solidity of the torrid zone?

Ans. 147,720,000,000 miles.

The solidity of the two temperate zones is	110,781,000,000
of the two frigid zones	2,606,000,000
of the torrid zone	147,720,000,000
of the whole globe	261,107,000,000

4. What is the convex surface of a spherical zone, whose breadth is 4 feet, on a sphere of 25 feet diameter?

5. What is the solidity of a spherical segment, whose height is 18 feet, and the diameter of its base 40 feet?

PROMISCUOUS EXAMPLES OF SOLIDS.

Ex. 1. How much water can be put into a cubical vessel three feet deep, which has been previously filled with cannon balls of the same size, 2, 4, 6, or 9 inches in diameter, regularly arranged in tiers, one directly above another?

Ans. $96\frac{1}{5}$ wine gallons.

2. If a cone or pyramid, whose height is three feet, be divided into three equal portions, by sections parallel to the base; what will be the heights of the several parts?

Ans. 24.961, 6.488, and 4.551 inches.

3. What is the solidity of the greatest square prism which can be cut from a cylindrical stick of timber, 2 feet 6 inches in diameter and 56 feet long?*

Ans. 175 cubic feet.

4. How many such globes as the earth are equal in bulk to the sun; if the former is 7930 miles in diameter, and the latter 890,000? Ans. 1,413,678.

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* The common rule for measuring *round timber* is to multiply the square of the *quarter-girt* by the length. The quarter-girt is one-fourth of the circumference. This method does not give the whole solidity. It makes an allowance of about one-fifth, for waste in hewing, bark, &c. The solidity of a cylinder is equal to the product of the height into the area of the base.

If C=the circumference, and π=3.14159, then (Art. 31.)

$$\text{The area of the base}=\frac{C^2}{4\pi}=\left(\frac{C}{\sqrt{4\pi}}\right)^2=\left(\frac{C}{3.545}\right)^2$$

If then the circumference were divided by 3.545, instead of 4, and the quotient squared, the area of the base would be correctly found. See note B.

5. How many cubic feet of wall are there in a conical tower 66 feet high, if the diameter of the base be 20 feet from outside to outside, and the diameter of the top 8 feet; the thickness of the wall being 4 feet at the bottom, and decreasing regularly, so as to be only two feet at the top?

Ans. 7188.

6. If a metallic globe filled with wine, which cost as much at 5 dollars a gallon, as the globe itself at 20 cents for every square inch of its surface; what is the diameter of the globe?

Ans. 55.44 inches.

7. If the circumference of the earth be 25,000 miles, what must be the diameter of a metallic globe, which, when drawn into a wire $\frac{1}{20}$ of an inch in diameter, would reach round the earth? Ans. 15 feet and 1 inch.

8. If a conical cistern be 3 feet deep, $7\frac{1}{2}$ feet in diameter at the bottom, and 5 feet at the top; what will be the depth of a fluid occupying half its capacity?

Ans. 14.535 inches.

9. If a globe 20 inches in diameter, be perforated by a cylinder 16 inches in diameter, the axis of the latter passing through the centre of the former; what part of the solidity, and the surface of the globe, will be cut away by the cylinder?

Ans. 3284 inches of the solidity, and 502,655 of the surface.

10. What is the solidity of the greatest cube which can be cut from a sphere three feet in diameter?

Ans. $5\frac{1}{5}$ feet.

11. What is the solidity of a conic frustum, the altitude of which is 36 feet, the greater diameter 16, and the lesser diameter 8?

12. What is the solidity of a spherical segment 4 feet high, cut from a sphere 16 feet in diameter?

SECTION V.

ISOPERIMETRY.

Art. 77. It is often necessary to compare a number of different figures or solids, for the purpose of ascertaining which has the *greatest area*, within a given perimeter, or the *greatest capacity* under a given surface. We may have occasion to determine, for instance, what must be the form of a fort, to contain a given number of troops, with the least extent of wall; or what the shape of a metallic pipe to convey a given portion of water, or of a cistern to hold a given quantity of liquor, with the least expense of materials.

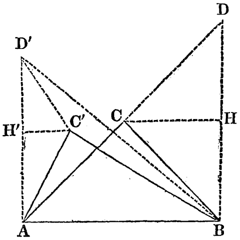
78. Figures which have equal perimeters are called *Isoperimeters.* When a quantity is *greater* than any other of the same class, it is called a *maximum.* A multitude of straight lines, of different lengths, may be drawn within a circle. But among them all, the *diameter* is a *maximum.* Of all *sines* of angles, which can be drawn in a circle, the sine of 90° is a *maximum.*

When a quantity is *less* than any other of the same class, it is called a *minimum.* Thus, of all straight lines drawn from a given point to a given straight line, that which is *perpendicular* to the given line is a *minimum.* Of all straight lines drawn from a given point in a circle, to the circumference, the *maximum* and the *minimum* are the two parts of the diameter which pass through that point. (Euc. 7, 3.)

In isoperimetry, the object is to determine, on the one hand, in what cases the area is a *maximum,* within a given perimeter; or the capacity a *maximum,* within a given surface: and on the other hand, in what cases the perimeter is a *minimum* for a given area, or the surface a *minimum*, for a given capacity.

PROPOSITION I.

79. *An* ISOSCELES TRIANGLE *has a greater area than any scalene triangle, of equal base and perimeter.*



If ABC be an isosceles triangle whose equal sides are AC and BC; and if ABC′ be a scalene triangle on the same base AB, and having AC′ + BC′ = AC+BC; then the area of ABC is greater than that of ABC′.

Let perpendiculars be raised from each end of the base, extend AC to D, make C′D′ equal to AC′, join BD, and draw CH and C′H′ parallel to AB.

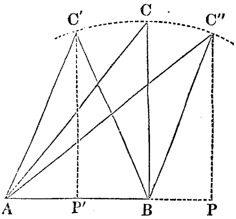
As the angle CAB=ABC, (Euc. 5, 1.) and ABD is a right angle, ABC+CBD=CAB+CDB=ABC+CDB. Therefore CBD=CDB, so that CD=CB; and by construction, C′D′=AC′. The perpendiculars of the equal right angled triangles CHD and CHB are equal; therefore, BH=$\frac{1}{2}$BD. In the same manner, AH′=$\frac{1}{2}$AD′. The line AD=AC+BC=AC′+BC′=D′C′+BC′. But D′C′+BC′>BD′. (Euc. 20, 1.) Therefore, AD>BD′; BD>AD′, (Euc. 47, 1.) and $\frac{1}{2}$ BD> $\frac{1}{2}$ AD′. But $\frac{1}{2}$BD, or BH, is the height of the isosceles triangle; (Art. 1.) and $\frac{1}{2}$AD′ or AH′, the height of the scalene triangle; and the areas of two triangles which have the same base are as their heights. (Art. 8.) Therefore the area of ABC is greater than that of ABC′. Among all triangles, then, of a given perimeter, and upon a given base, the isosceles triangle is a *maximum*.

Cor. The isosceles triangle has a *less perimeter* than any scalene triangle of the same base and area. The triangle

ABC′ being less than ABC, it is evident the perimeter of the former must be enlarged, to make its area equal to the area of the latter.

PROPOSITION II.

80. *A triangle in which two given sides make a* RIGHT ANGLE, *has a greater area than any triangle in which the same sides make an oblique angle.*

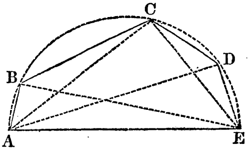


If BC, BC′ and BC″ be equal, and if BC be perpendicular to AB; then the right angled triangle ABC, has a greater area than the acute angled triangle ABC′, or the oblique angled triangle ABC″.

Let P′C′ and PC″ be perpendicular to AP. Then, as the three triangles have the same base AB, their areas are as their heights; that is, as the perpendiculars BC, P′C′, and PC″. But BC is equal to BC′, and therefore greater than P′C′. (Euc. 47. 1.) BC is also equal to BC″, and therefore greater than PC″.

PROPOSITION III.

81. *If all the sides* EXCEPT ONE *of a polygon be given, the area will be the greatest, when the given sides are so disposed that the figure may be* INSCRIBED IN A SEMICIRCLE, *of which the undetermined side is the diameter.*



If the sides AB, BC, CD, DE, be given, and if their position be such that the area, included between these and another side whose length is not determined, is a *maximum;* the figure may be inscribed in a semicircle, of which the undetermined side AE is the diameter.

Draw the lines AD, AC, EB, EC. By varying the angle at D, the triangle ADE may be enlarged or diminished, without affecting the area of the other parts of the figure. The whole area, therefore, cannot be a *maximum,* unless this triangle be a *maximum,* while the sides AD and ED are given. But if the triangle ADE be a *maximum,* under these conditions, the angle ADE is a right angle; (Art. 80.) and therefore the point D is in the circumference of a circle, of which AE is the diameter. (Euc. 31, 3.) In the same manner it may be proved, that the angles ACE and ABE are right angles, and therefore that the points C and B are in the circumference of the same circle.

The term *polygon* is used in this section to include *triangles,* and *four-sided* figures, as well as other right-lined figures.

82. The area of a polygon, inscribed in a semicircle, in the manner stated above, will not be altered by varying the *order* of the given sides.

The sides AB, BC, CD, DE, are the *chords* of so many arcs. The sum of these arcs, in whatever order they are arranged, will evidently be equal to the semicircumference. And the *segments* between the given sides and the arcs will be the same in whatever part of the circle they are situated. But the area of the polygon is equal to the area of the semicircle, diminished by the sum of these segments.

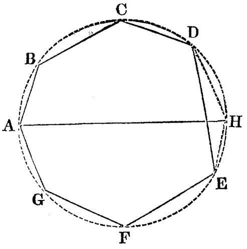
83. If a polygon, of which all the sides except one are given, be inscribed in a semicircle whose diameter is the undetermined side; a polygon having the same given sides, cannot be inscribed in any *other* semicircle which is either greater or less than this, and whose diameter is the undetermined side.

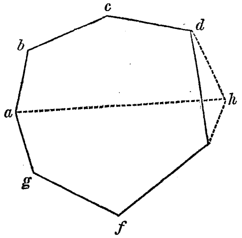
The given sides AB, BC, CD, DE, are the chords of arcs whose sum is 180 degrees. But in a larger circle, each would be the chord of a less number of degrees, and therefore the sum of the arcs would be less than 180°: and in a smaller circle, each would be the chord of a greater number of degrees, and the sum of the arcs would be greater than 180°.

PROPOSITION IV.

84. *A polygon* INSCRIBED IN A CIRCLE *has a greater area, than any polygon of equal perimeter, and the same number of sides, which cannot be inscribed in a circle.*

If in the circle ACHF, (Fig. 30.) there be inscribed a polygon ABCDEFG; and if another polygon *abcdefg* (Fig. 31.) be formed of sides which are the same in number and length, but which are so disposed, that the figure cannot be inscribed in a circle; the area of the former polygon is greater than that of the latter.





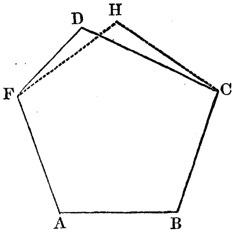
Draw the diameter AH, and the chords DH and EH. Upon *de* make the triangle *deh* equal and similar to DEH, and join *ah*. The line *ah* divides the figure *abcdhefg* into two parts, of which *one at least* cannot, by supposition, be inscribed in a semicircle of which the diameter is AH, nor in any other semicircle of which the diameter is the undetermined side. (Art. 83.) It is therefore less than the corresponding part of the figure ABCDHEFG. (Art. 81.) And the other part of *abcdhefg* is not greater than the corresponding part of ABCDHEFG. Therefore, the whole figure ABCDHEFG is greater than the whole figure *abcdhefg*. If from these there be taken the equal triangles DEH and *deh*, there will remain the polygon ABCDEFG greater than the polygon *abcdefg*.

85. A polygon of which all the sides are given in number and length, cannot be inscribed in circles of different diameters. (Art. 83.) And the area of the polygon will not be altered by changing the *order* of the sides. (Art. 82.)

PROPOSITION V.

86. *When a polygon has a greater area than any other, of the same number of sides, and of equal perimeter, the sides are* EQUAL.

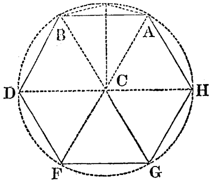
The polygon ABCDF (Fig. 29.) cannot be a *maximum*, among all polygons of the same number of sides, and of equal perimeters, unless it be equilateral. For if any two of the sides, as CD and FD, are unequal, let CH and FH be equal, and their sum the same as the sum of CD and FD. The isosceles triangle CHF is greater than the scalene triangle CDF (Art. 79.); and therefore the polygon ABCHF is greater than the polygon ABCDF; so that the latter is not a *maximum*.



PROPOSITION VI.

87. *A* REGULAR POLYGON *has a greater area than any other polygon of equal perimeter, and of the same number of sides.*

For, by the preceding article, the polygon which is a *maximum* among others of equal perimeters, and the same number of sides, is *equilateral,* and by Art. 84, it may be *inscribed in a circle.* But if a polygon inscribed in a circle is equilateral, as ABDFGH, it is also *equiangular.* For the sides of the polygon are the bases of so many isosceles triangles, whose common vertex is the centre C. The angles at these bases are all equal; and two of them, as AHC and GHC, are equal to AHG one of the angles of the polygon. The polygon, then, being equiangular, as well as equilateral, is a *regular* polygon. (Art. 1. Def. 2.)



Thus an *equilateral triangle* has a greater area, than any other triangle of equal perimeter. And a *square* has a greater area than any other four-sided figure of equal perimeter.

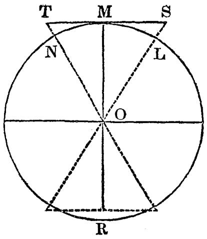
Cor. A regular polygon has a *less perimeter* than any other polygon of equal area, and the same number of sides.

For if, with a given perimeter, the regular polygon is greater than one which is not regular; it is evident the perimeter of the former must be diminished, to make its area equal to that of the latter.

Proposition VII.

88. *If a polygon be* DESCRIBED ABOUT A CIRCLE, *the areas of the two figures are as their perimeters.*

Let ST be one of the sides of a polygon, either regular or



not, which is described about the circle LNR. Join OS and OT, and to the point of contact M draw the radius OM, which will be perpendicular to ST. (Euc. 18, 3.) The triangle OST is equal to half the base ST multiplied into the radius OM. (Art. 8.) And if lines be drawn, in the same manner, from the centre of the circle, to the extremities of the several sides of the circumscribed polygon, each of the triangles thus formed will be equal to half its base multiplied into the radius of the circle. Therefore the area of the whole polygon is equal to half its perimeter multiplied into the radius: and the area of the circle is equal to half its circumference multiplied into the radius. (Art 30.) So that the two areas aie to each other as their perimeters.

Cor. 1. If different polygons are described about the same circle, their areas are to each other as their perimeters. For the area of each is equal to half its perimeter, multiplied into the radius of the inscribed circle.

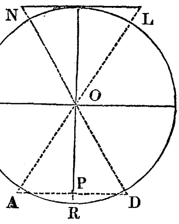
Cor. 2. The *tangent* of an arc is always greater than the arc itself. The triangle OMT is to OMN, as MT to MN. But OMT is greater than OMN, because the former includes the latter. Therefore, the tangent MT is greater than the arc MN.

PROPOSITION VIII.

89. *A* CIRCLE *has a greater area than any polygon of equal perimeter.*

If a circle and a regular polygon have the same centre, and equal perimeters; each of the sides of the polygon must fall partly *within* the circle. For the area of a *circum-*

scribing polygon is greater than the area of the circle, as the one includes the other: and therefore, by the preceding article, the *perimeter* of the former is greater than that of the latter.



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Let AD then be one side of a regular polygon, whose perimeter is equal to the circumference of the circle RLN. As this falls partly within the circle, the perpendicular OP is less than the radius OR. But the area of the polygon is equal to half its perimeter multiplied into this perpendicular (Art. 15.); and the area of the circle is equal to half its circumference multiplied into the radius. (Art. 30.) The circle then is greater than the given regular polygon; and therefore greater than any other polygon of equal perimeter. (Art. 87.)

Cor. 1. A circle has a *less perimeter*, than any polygon of equal area.

Cor. 2. Among regular polygons of a given perimeter, that which has the *greatest number of sides*, has also the *greatest area*. For the greater the number of sides, the more nearly does the perimeter of the polygon approach to a coincidence with the circumference of a circle.

PROPOSITION IX.

90. *A right* PRISM *whose bases are* REGULAR POLYGONS, *has a less surface than any other right prism of the same solidity, the same altitude, and the same number of sides.*

If the altitude of a prism is given, the area of the base is as the solidity (Art. 43.); and if the number of sides is also given, the perimeter is a *minimum* when the base is a regular polygon. (Art. 87. Cor.) But the lateral surface is as the perimeter. (Art. 47.) Of two right prisms, then, which have the same altitude, the same solidity, and the same number of sides, that whose bases are regular polygons has the least *lateral* surface, while the areas of the ends are equal.

Cor. A right prism whose bases are regular polygons has a *greater solidity,* than any other right prism of the same surface, the same altitude, and the same number of sides.

PROPOSITION X.

91. *A right* CYLINDER *has a less surface than any right prism of the same altitude and solidity.*

For if the prism and cylinder have the same altitude and solidity, the areas of their bases are equal. (Art. 64.) But the *perimeter* of the cylinder is less, than that of the prism (Art. 89. Cor. 1.); and therefore its lateral surface is less, while the areas of the ends are equal.

Cor. A right cylinder has a *greater solidity,* than any right prism of the same altitude and surface.

PROPOSITION XI.

92. *A* CUBE *has a less surface than any other right parallelopiped of the same solidity.*

A parallelopiped is a prism, any one of whose faces may be considered a base. (Art. 41. Def. I and V.) If these are not all *squares,* let one which is not a square be taken for a base. The perimeter of this may be diminished, without altering its area (Art. 87. Cor.); and therefore the surface of the solid may be diminished, without altering its altitude or solidity. (Art. 43, 47.) The same may be proved of each of the other faces which are not squares. The surface is therefore a *minimum*, when *all* the faces are squares, that is, when the solid is a *cube*.

Cor. A cube has a *greater solidity* than any other right parallelopiped of the same surface.

Proposition XII.

93. *A* CUBE *has a greater solidity than any other right parallelopiped, the sum of whose length, breadth and depth, is equal to the sum of the corresponding dimensions of the cube.*

The solidity is equal to the product of the length, breadth, and depth. If the length and breadth are unequal, the solidity may be increased, without altering the sum of the three dimensions. For the product of two factors whose sum is given, is the greatest when the factors are equal. (Euc. 27. 6.) In the same manner, if the breadth and depth are unequal, the solidity may be increased, without altering the sum of the three dimensions. Therefore, the solid cannot be a *maximum*, unless its length, breadth, and depth are equal.

Proposition XIII.

94. *If a* PRISM BE DESCRIBED ABOUT A CYLINDER, *the capacities of the two solids are as their surfaces.*

The capacities of the solids are as the *areas* of their bases, that is, as the *perimeters* of their bases. (Art. 88.) But the lateral surfaces are also as the perimeters of the bases. Therefore the *whole* surfaces are as the solidities.

Cor. The capacities of different prisms, described about the same right cylinder, are to each other as their surfaces.

PROPOSITION XIV.

95. *A right cylinder* WHOSE HEIGHT IS EQUAL TO THE DIAMETER OF ITS BASE *has a greater solidity than any other right cylinder of equal surface.*

Let C be a right cylinder whose height is equal to the diameter of its base; and C′ another right cylinder having the same surface, but a different altitude. If a square prism P be described about the former, it will be a *cube.* But a square prism P′ described about the latter will not be a cube.

Then the surfaces of C and P are as their bases (Art. 47. and 88.); which are as the bases of C′ and P′, (Sup. Euc. 7, 1.); so that,

surf C : *surf* P : : *base* C : *base* P : : *base* C′ : *base* P′ : : *surf* C′ : *surf* P′.

But the surface of C is, by supposition, equal to the surface of C′. Therefore, (Alg. 395.) the surface of P is equal to the surface of P′. And by the preceding article,

solid P : *solid* C : : *surf* P : *surf* C : : *surf* P′ : *surf* C′ : : *solid* P′ : *solid* C′.

But the solidity of P is greater than that of P′. (Art. 92. Cor.) Therefore the solidity of C is greater than that of C′.

Schol. A right cylinder whose height is equal to the diameter of its base, is that which *circumscribes a sphere.* It is also called *Archimedes' cylinder;* as he discovered the ratio of a sphere to its circumscribing cylinder; and these are the figures which were put upon his tomb.

Cor. Archimedes' cylinder has a *less surface,* than any other right cylinder of the same capacity.

PROPOSITION XV.

96. *If a* SPHERE BE CIRCUMSCRIBED *by a solid bounded by plane surfaces; the capacities of the two solids are as their surfaces.*

If planes be supposed to be drawn from the centre of the sphere, to each of the edges of the circumscribing solid, they will divide it into as many pyramids as the solid has faces. The base of each pyramid will be one of the faces; and the height will be the radius of the sphere. The capacity of the pyramid will be equal, therefore, to its base multiplied into $\frac{1}{3}$ of the radius (Art. 48.); and the capacity of the whole circumscribing solid, must be equal to its whole surface multiplied into $\frac{1}{3}$ of the radius. But the capacity of the sphere is also equal to its surface multiplied into $\frac{1}{3}$ of its radius. (Art. 70.)

Cor. The capacities of different solids circumscribing the same sphere, are as their surfaces.

PROPOSITION XVI.

97. *A* SPHERE *has a greater solidity than any regular polyedron of equal surface.*

If a sphere and a regular polyedron have the same centre, and equal surfaces; each of the faces of the polyedron must fall partly *within* the sphere. For the solidity of a *circumscribing* solid is greater than the solidity of the sphere, as the one includes the other: and therefore, by the preceding article, the *surface* of the former is greater than that of the latter.

But if the faces of the polyedron fall partly within the sphere, their perpendicular distance from the centre must be less than the radius. And therefore, if the surface of the polyedron be only equal to that of the sphere, its solidity must be less. For the solidity of the polyedron is equal to its surface multiplied into $\frac{1}{3}$ of the distance from the centre. (Art. 59.) And the solidity of the sphere is equal to its surface multiplied into $\frac{1}{3}$ of the radius.

Cor. A sphere has a *less surface* than any regular polyedron of the same capacity.

APPENDIX

GAUGING OF CASKS.

Art. 119. Gauging is a practical art, which does not admit of being treated in a very scientific manner. Casks are not commonly constructed in exact conformity with any regular mathematical figure. By most writers on the subject, however, they are considered as nearly coinciding with one of the following forms:

1. 2.	The middle frustum	of a spheroid, of a parabolic spindle.
3. 4.	The equal frustums	of a paraboloid, of a cone.

The *second* of these varieties agrees more nearly than any of the others, with the forms of casks, as they are commonly made. The first is too much curved, the third too little, and the fourth not at all, from the head to the bung.

120. Rules have already been given, for finding the capacity of each of the four varieties of casks. (Arts. 68, 110, 112, 118.) As the dimensions are taken in *inches*, these rules will give the contents in cubic inches. To abridge the computation, and adapt it to the particular measures used in gauging, the factor .7854 is divided by 282 or 231; and the quotient is used instead of .7854, for finding the capacity in ale gallons or wine gallons.

$$\text{Now } \frac{.7854}{282} = .002785, \text{ or } .0028 \text{ nearly};$$

$$\text{And } \frac{.7854}{231} = .0034$$

If then .0028 and .0034 be substituted for .7854, in the rules referred to above; the contents of the cask will be given in ale gallons and wine gallons. These numbers are to each other nearly as 9 to 11.

Problem I.

To calculate the contents of a cask, in the form of a middle frustum of a SPHEROID.

121. Add together the square of the head diameter, and twice the square of the bung diameter: multiply the sum by $\frac{1}{3}$ of the length, and the product by .0028 for ale gallons, or by .0034 for wine gallons.

If D and d = the two diameters, and l = the length;

The capacity in inches $= (2D^2 + d^2) \times \frac{1}{3}l \times .7854$. (Art. 110.)

And by substituting .0028 or .0034 for .7854, we have the capacity in ale gallons or wine gallons.

Ex. What is the capacity of a cask of the first form, whose length is 30 inches, its head diameter 18, and its bung diameter 24?

Ans. 41.3 ale gallons, or 50.2 wine gallons.

Problem II.

To calculate the contents of a cask, in the form of the middle frustum of a PARABOLIC SPINDLE.

122. Add together the square of the head diameter, and twice the square of the bung diameter, and from the sum

subtract $\frac{2}{5}$ of the square of the difference of the diameters; multiply the remainder by $\frac{1}{3}$ of the length, and the product by .0028 for ale gallons, or .0034 for wine gallons.

The capacity in inches $=(2D^2+d^2-\frac{2}{5}(D-d)^2)\times\frac{1}{3}l\times .7854$. (Art. 118.)

Ex. What is the capacity of a cask of the second form, whose length is 30 inches, its head diameter 18, and its bung diameter 24?

Ans. 40.9 ale gallons, or 49.7 wine gallons.

PROBLEM III.

To calculate the contents of a cask, in the form of two equal frustums of a PARABOLOID.

123. Add together the square of the head diameter, and the square of the bung diameter; multiply the sum by half the length, and the product by .0028 for ale gallons, or .0034 for wine gallons.

The capacity in inches $=(D^2+d^2)\times\frac{1}{2}l\times .7854$. (Art. 112 Cor.)

Ex. What is the capacity of a cask of the third form, whose dimensions are, as before, 30, 18, and 24?

Ans. 37.8 ale gallons, or 45.9 wine gallons.

PROBLEM IV.

To calculate the contents of a cask, in the form of two equal frustums of a CONE.

124. Add together the square of the head diameter, the square of the bung diameter, and the product of the two diameters; multiply the sum by $\frac{1}{3}$ of the length, and the product by .0028 for ale gallons, or .0034 for wine gallons.

The capacity in inches $=(D^2+d^2+Dd)\times\frac{1}{3}l\times .7854$. (Art. 68.)

Ex. What is the capacity of a cask of the fourth form, whose length is 30, and its diameters 18 and 24?

Ans. 37.3 ale gallons, or 45.3 wine gallons.

125. The preceding rules, though correct in theory, are not very well adapted to practice, as they suppose the form of the cask to be *known*. The two following rules, taken from Hutton's Mensuration, may be used for casks of the usual forms. For the first, *three* dimensions are required, the length, the head diameter, and the bung diameter. It is evident that no allowance is made by this, for different degrees of curvature from the head to the bung. If the cask is more or less curved than usual, the following rule is to be preferred, for which *four* dimensions are required, the head and bung diameters, and a third diameter taken in the middle between the bung and the head. For the demonstration of these rules, see Hutton's Mensuration, Part V. Sec. 2. Ch. 5 and 7.

PROBLEM V.

To calculate the contents of any common cask, from THREE *dimensions.*

126. Add together

25 times the square of the head diameter,

39 times the square of the bung diameter, and

26 times the product of the two diameters;

Multiply the sum by the length, divide the product by 90, and multiply the quotient by .0028 for ale gallons, or .0034 for wine gallons.

The capacity in inches $=(39\ D^2+25d^2+26Dd)\times\frac{l}{90}\times.7854.$

Ex. What is the capacity of a cask whose length is 30 inches, the head diameter 18, and the bung diameter 24?

Ans. 39 ale gallons, or $47\frac{1}{3}$ wine gallons.

PROBLEM VI.

To calculate the contents of a cask from FOUR *dimensions, the length, the head and bung diameters, and a diameter taken in the middle between the head and the bung.*

127. Add together the squares of the head diameter, of the bung diameter, and of double the middle diameter; multiply the sum by $\frac{1}{6}$ of the length, and the product by .0028 for ale gallons, or .0034 for wine gallons.

If D=the bung diameter, d=the head diameter, m=the middle diameter, and l=the length;

The capacity in inches$=(D^2+d^2+\overline{2m}^2)\times\frac{1}{6}l\times.7854$.

Ex. What is the capacity of a cask, whose length is 30 inches, the head diameter 18, the bung diameter 24, and the middle diameter $22\frac{1}{2}$?

Ans. 41 ale gallons, or $49\frac{2}{3}$ wine gallons.

128. In making the calculations in gauging, according to the preceding rules, the multiplications and divisions are frequently performed by means of a *Sliding Rule,* on which are placed a number of logarithmic lines, similar to those on Gunter's Scale. See Trigonometry, Sec. VI., and Note C. p. 149.

Another instrument commonly used in gauging is the *Diagonal Rod.* By this, the capacity of a cask is very expeditiously found, from a single dimension, the distance from the bung to the intersection of the opposite stave with the head; but this process is not considered sufficiently accurate for casks of a capacity exceeding 40 gallons. The measure is taken by extending the rod through the cask, from the bung to the most distant part of the head. The number of gallons corresponding to the length of the line thus found, is marked on the rod. The *logarithmic* lines on the gauging rod are to be used in the same manner, as on the sliding rule.

ULLAGE OF CASKS.

129. When a cask is *partly* filled, the whole capacity is divided, by the surface of the liquor, into two portions; the *least* of which, whether full or empty, is called the *ullage.* In finding the ullage, the cask is supposed to be in one of two positions; either *standing,* with its axis perpendicular to the horizon; or *lying,* with its axis parallel to the horizon. The rules for ullage which are *exact,* particularly those for lying casks, are too complicated for common use. The following are considered as sufficiently near approximations. See Hutton's Mensuration.

PROBLEM VII.

To calculate the ullage of a STANDING *cask.*

130. Add together the squares of the diameter at the surface of the liquor, of the diameter of the nearest end, and of double the diameter in the middle between the other two; multiply the sum by $\frac{1}{6}$ of the distance between the surface and the nearest end, and the product by .0028 for ale gallons, or .0034 for wine gallons.

If D=the diameter of the surface of the liquor,
d=the diameter of the nearest end,
m=the middle diameter, and
l=the distance between the surface and the nearest end;

$$\text{The ullage in inches}=(D^2+d^2+\overline{2m^2})\times\tfrac{1}{6}l\times.7854.$$

Ex. If the diameter at the surface of the liquor, in a standing cask, be 32 inches, the diameter of the nearest end 24, the middle diameter 29, and the distance between the surface of the liquor and the nearest end 12; what is the ullage? Ans. 27$\frac{4}{5}$ ale gallons, or 33$\frac{3}{4}$ wine gallons.

PROBLEM VIII.

To calculate the ullage of a LYING *cask.*

131. Divide the distance from the bung to the surface of the liquor, by the whole bung diameter, find the quotient in the column of heights or versed sines in a table of circular segments, take out the corresponding segment, and multiply it by the whole capacity of the cask, and the product by 1$\frac{1}{4}$ for the part which is empty.

If the cask be not half full, divide the depth of the liquor by the whole bung diameter, take out the segment, multiply, &c., for the contents of the part which is full.

Ex. If the whole capacity of a lying cask be 41 ale gallons, or 49$\frac{2}{3}$ wine gallons, the bung diameter 24 inches and the distance from the bung to the surface of the liquor 6 inches; what is the ullage?

Ans. 7$\frac{3}{4}$ ale gallons, or 9$\frac{1}{2}$ wine gallons.

NOTES.

Note A. p. 39.

The term *solidity* is used here in the customary sense, to express the magnitude of any geometrical quantity of three dimensions, length, breadth, and thickness; whether it be a solid body, or a fluid, or even a portion of empty space. This use of the word, however, is not altogether free from objection. The same term is applied to one of the general properties of matter; and also to that peculiar quality by which certain substances are distinguished from *fluids*. There seems to be an impropriety in speaking of the *solidity* of a body of *water*, or of a vessel which is *empty*. Some writers have therefore substituted the word *volume* for solidity. But the latter term, if it be properly defined, may be retained without danger of leading to mistake.

Note B. p. 76.

The following simple rule for the solidity of round timber, or of any cylinder, is nearly exact:

Multiply the length into twice the square of $\frac{1}{5}$ *of the circumference.*

If C = the circumference of a cylinder;

The area of the base $= \dfrac{C^2}{4\pi} = \dfrac{C^2}{12.566}$ But $2\left(\dfrac{C}{5}\right)^2 = \dfrac{C^2}{12.5}$.

It is common to measure *hewn timber*, by multiplying the length into the square of the *quarter-girt*. This gives exactly the solidity of a parallelopiped, if the ends are *squares.* But if the ends are parallelograms, the area of each is *less* than the square of the quarter-girt. (Euc. 27. 6.)

Timber which is *tapering* may be exactly measured by the rule for the frustum of a pyramid or cone (Art. 50, 68.); or, if the ends are not similar figures, by the rule for a prismoid. (Art. 55.) But for common purposes, it will be sufficient to multiply the length by the area of a section *in the middle* between the two ends.

A

TABLE

OF

LOGARITHMS OF NUMBERS

FROM

1 TO 10,000.

N.	Log.	N.	Log.	N.	Log.	N.	Log.
1	0.000000	26	1.414973	51	1.707570	76	1.880814
2	0.301030	27	1.431364	52	1.716003	77	1.886491
3	0.477121	28	1.447158	53	1.724276	78	1.892095
4	0.602060	29	1.462398	54	1.732394	79	1.897627
5	0.698970	30	1.477121	55	1.740363	80	1.903090
6	0.778151	31	1.491362	56	1.748188	81	1.908485
7	0.845098	32	1.505150	57	1.755875	82	1.913814
8	0.903090	33	1.518514	58	1.763428	83	1.919078
9	0.954243	34	1.531479	59	1.770852	84	1.924279
10	1.000000	35	1.544068	60	1.778151	85	1.929419
11	1.041393	36	1.556303	61	1.785330	86	1.934498
12	1.079181	37	1.568202	62	1.792392	87	1.939519
13	1.113943	38	1.579784	63	1.799341	88	1.944483
14	1.146128	39	1.591065	64	1.806180	89	1.949390
15	1.176091	40	1.602060	65	1.812913	90	1.954243
16	1.204120	41	1.612784	66	1.819544	91	1.959041
17	1.230449	42	1.623249	67	1.826075	92	1.963788
18	1.255273	43	1.633468	68	1.832509	93	1.968483
19	1.278754	44	1.643453	69	1.838849	94	1.973128
20	1.301030	45	1.653213	70	1.845098	95	1.977724
21	1.322219	46	1.662758	71	1.851258	96	1.982271
22	1.342423	47	1.672098	72	1.857333	97	1.986772
23	1.361728	48	1.681241	73	1.863323	98	1.991226
24	1.380211	49	1.690196	74	1.869232	99	1.995635
25	1.397940	50	1.698970	75	1.875061	100	2.000000

N. B. In the following table, in the last nine columns of each page, where the first or leading figures change from 9's to 0's, points or dots are introduced instead of the 0's through the rest of the line, to catch the eye, and to indicate that from thence the annexed first two figures of the Logarithm in the second column stand in the next lower line.

N.	0	1	2	3	4	5	6	7	8	9	D.
100	000000	0434	0868	1301	1734	2166	2598	3029	3461	3891	432
101	4321	4751	5181	5609	6038	6466	6894	7321	7748	8174	428
102	8600	9026	9451	9876	.300	.724	1147	1570	1993	2415	424
103	012837	3259	3680	4100	4521	4940	5360	5779	6197	6616	419
104	7033	7451	7868	8284	8700	9116	9532	9947	.361	.775	416
105	021189	1603	2016	2428	2841	3252	3664	4075	4486	4896	412
106	5306	5715	6125	6533	6942	7350	7757	8164	8571	8978	408
107	9384	9789	.195	.600	1004	1408	1812	2216	2619	3021	404
108	033424	3826	4227	4628	5029	5430	5830	6230	6629	7028	400
109	7426	7825	8223	8620	9017	9414	9811	.207	.602	.998	396
110	041393	1787	2182	2576	2969	3362	3755	4148	4540	4932	393
111	5323	5714	6105	6495	6885	7275	7664	8053	8442	8830	389
112	9218	9606	9993	.380	.766	1153	1538	1924	2309	2694	386
113	053078	3463	3846	4230	4613	4996	5378	5760	6142	6524	382
114	6905	7286	7666	8046	8426	8805	9185	9563	9942	.320	379
115	060698	1075	1452	1829	2206	2582	2958	3333	3709	4083	376
116	4458	4832	5206	5580	5953	6326	6699	7071	7443	7815	372
117	8186	8557	8928	9298	9668	..38	.407	.776	1145	1514	369
118	071882	2250	2617	2985	3352	3718	4085	4451	4816	5182	366
119	5547	5912	6276	6640	7004	7368	7731	8094	8457	8819	363
120	079181	9543	9904	.266	.626	.987	1347	1707	2067	2426	360
121	082785	3144	3503	3861	4219	4576	4934	5291	5647	6004	357
122	6360	6716	7071	7426	7781	8136	8490	8845	9198	9552	355
123	9905	.258	.611	.963	1315	1667	2018	2370	2721	3071	351
124	093422	3772	4122	4471	4820	5169	5518	5866	6215	6562	349
125	6910	7257	7604	7951	8298	8644	8990	9335	9681	..26	346
126	100371	0715	1059	1403	1747	2091	2434	2777	3119	3462	343
127	3804	4146	4487	4828	5169	5510	5851	6191	6531	6871	340
128	7210	7549	7888	8227	8565	8903	9241	9579	9916	.253	338
129	110590	0926	1263	1599	1934	2270	2605	2940	3275	3609	335
130	113943	4277	4611	4944	5278	5611	5943	6276	6608	6940	333
131	7271	7603	7934	8265	8595	8926	9256	9586	9915	.245	330
132	120574	0903	1231	1560	1888	2216	2544	2871	3198	3525	328
133	3852	4178	4504	4830	5156	5481	5806	6131	6456	6781	325
134	7105	7429	7753	8076	8399	8722	9045	9368	9690	..12	323
135	130334	0655	0977	1298	1619	1939	2260	2580	2900	3219	321
136	3539	3858	4177	4496	4814	5133	5451	5769	6086	6403	318
137	6721	7037	7354	7671	7987	8303	8618	8934	9249	9564	315
138	9879	.194	.508	.822	1136	1450	1763	2076	2389	2702	314
139	143015	3327	3639	3951	4263	4574	4885	5196	5507	5818	311
140	146128	6438	6748	7058	7367	7676	7985	8294	8603	8911	309
141	9219	9527	9835	.142	.449	.756	1063	1370	1676	1982	307
142	152288	2594	2900	3205	3510	3815	4120	4424	4728	5032	305
143	5336	5640	5943	6246	6549	6852	7154	7457	7759	8061	303
144	8362	8664	8965	9266	9567	9868	.168	.469	.769	1068	301
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147	7317	7613	7908	8203	8497	8792	9086	9380	9674	9968	295
148	170262	0555	0848	1141	1434	1726	2019	2311	2603	2895	293
149	3186	3478	3769	4060	4351	4641	4932	5222	5512	5802	291
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153	4691	4975	5259	5542	5825	6108	6391	6674	6956	7239	283
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155	190332	0612	0892	1171	1451	1730	2010	2289	2567	2846	279
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167	2716	2976	3236	3496	3755	4015	4274	4533	4792	5051	259
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185	7172	7406	7641	7875	8110	8344	8578	8812	9046	9279	234
186	9513	9746	9980	.213	.446	.679	.912	1144	1377	1609	233
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189	6462	6692	6921	7151	7380	7609	7838	8067	8296	8525	229
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191	281033	1261	1488	1715	1942	2169	2396	2622	2849	3075	227
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210	322219	2426	2633	2839	3046	3252	3458	3665	3871	4077	206
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222	6353	6549	6744	6939	7135	7330	7525	7720	7915	8110	195
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225	2183	2375	2568	2761	2954	3147	3339	3532	3724	3916	193
226	4108	4301	4493	4685	4876	5068	5260	5452	5643	5834	192
227	6026	6217	6408	6599	6790	6981	7172	7363	7554	7744	191
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229	9835	..25	.215	.404	.593	.783	.972	1161	1350	1539	189
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232	5488	5675	5862	6049	6236	6423	6610	6796	6983	7169	187
233	7356	7542	7729	7915	8101	8287	8473	8659	8845	9030	186
234	9216	9401	9587	9772	9958	.143	.328	.513	.698	.883	185
235	371068	1253	1437	1622	1806	1991	2175	2360	2544	2728	184
236	2912	3096	3280	3464	3647	3831	4015	4198	4382	4565	184
237	4748	4932	5115	5298	5481	5664	5846	6029	6212	6394	183
238	6577	6759	6942	7124	7306	7488	7670	7852	8034	8216	182
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248	4452	4627	4802	4977	5152	5326	5501	5676	5850	6025	175
249	6199	6374	6548	6722	6896	7071	7245	7419	7592	7766	174
250	397940	8114	8287	8461	8634	8808	8981	9154	9328	9501	173
251	9674	9847	..20	.192	.365	.538	.711	.883	1056	1228	173
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254	4834	5005	5176	5346	5517	5688	5858	6029	6199	6370	171
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259	3300	3467	3635	3803	3970	4137	4305	4472	4639	4806	167
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261	6641	6807	6973	7139	7306	7472	7638	7804	7970	8135	166
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265	3246	3410	3574	3737	3901	4065	4228	4392	4555	4718	164
266	4882	5045	5208	5371	5534	5697	5860	6023	6186	6349	163
267	6511	6674	6336	6999	7161	7324	7486	7648	7811	7973	162
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272	4569	4729	4888	5048	5207	5367	5526	5685	5844	6004	159
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274	7751	7909	8067	8226	8384	8542	8701	8859	9017	9175	158
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284	3318	3471	3624	3777	3930	4082	4235	4387	4540	4692	153
285	4845	4997	5150	5302	5454	5606	5758	5910	6062	6214	152
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288	9392	9543	9694	9845	9995	.146	.296	.447	.597	.748	151
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291	3893	4042	4191	4340	4490	4639	4788	4936	5085	5234	149
292	5383	5532	5680	5829	5977	6126	6274	6423	6571	6719	149
293	6868	7016	7164	7312	7460	7608	7756	7904	8052	8200	148
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295	9822	9969	.116	.263	.410	.557	.704	.851	.998	1145	147
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297	2756	2903	3049	3195	3341	3487	3633	3779	3925	4071	146
298	4216	4362	4508	4653	4799	4944	5090	5235	5381	5526	146
299	5671	5816	5962	6107	6252	6397	6542	6687	6832	6976	145
300	477121	7266	7411	7555	7700	7844	7989	8133	8278	8422	145
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302	480007	0151	0294	0438	0582	0725	0869	1012	1156	1299	144
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305	4300	4442	4585	4727	4869	5011	5153	5295	5437	5579	142
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307	7138	7280	7421	7563	7704	7845	7986	8127	8269	8410	141
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309	9958	..99	.239	.380	.520	.661	.801	.941	1081	1222	140
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312	4155	4294	4433	4572	4711	4850	4989	5128	5267	5406	139
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314	6930	7068	7206	7344	7483	7621	7759	7897	8035	8173	138
315	8311	8448	8586	8724	8862	8999	9137	9275	9412	9550	138
316	9687	9824	9962	..99	.236	.374	.511	.648	.785	.922	137
317	501059	1196	1333	1470	1607	1744	1880	2017	2154	2291	137
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320	505150	5286	5421	5557	5693	5828	5964	6099	6234	6370	136
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327	4548	4681	4813	4946	5079	5211	5344	5476	5609	5741	133
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357	2668	2790	2911	3033	3155	3276	3398	3519	3640	3762	121
358	3883	4004	4126	4247	4368	4489	4610	4731	4852	4973	121
359	5094	5215	5336	5457	5578	5699	5820	5940	6061	6182	121
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367	4666	4784	4903	5021	5139	5257	5376	5494	5612	5730	118
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369	7026	7144	7262	7379	7497	7614	7732	7849	7967	8084	118
370	568202	8319	8436	8554	8671	8788	8905	9023	9140	9257	117
371	9374	9491	9608	9725	9842	9959	..76	.193	.309	.426	117
372	570543	0660	0776	0893	1010	1126	1243	1359	1476	1592	117
373	1709	1825	1942	2058	2174	2291	2407	2523	2639	2755	116
374	2872	2988	3104	3220	3336	3452	3568	3684	3800	3915	116
375	4031	4147	4263	4379	4494	4610	4726	4841	4957	5072	116
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378	7492	7607	7722	7836	7951	8066	8181	8295	8410	8525	115
379	8639	8754	8868	8983	9097	9212	9326	9441	9555	9669	114
380	579784	9898	..12	.126	.241	.355	.469	.583	.697	.811	114
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383	3199	3312	3426	3539	3652	3765	3879	3992	4105	4218	113
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385	5461	5574	5686	5799	5912	6024	6137	6250	6362	6475	113
386	6587	6700	6812	6925	7037	7149	7262	7374	7486	7599	112
387	7711	7823	7935	8047	8160	8272	8384	8496	8608	8720	112
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389	9950	..61	.173	.284	.396	.507	.619	.730	.842	.953	112
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397	8791	8900	9009	9119	9228	9337	9446	9556	9665	9774	109
398	9883	9992	.101	.210	.319	.428	.537	.646	.755	.864	109
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410	612784	2890	2996	3102	3207	3313	3419	3525	3630	3736	106
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444	7383	7481	7579	7676	7774	7872	7969	8067	8165	8262	98
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449	2246	2343	2440	2536	2633	2730	2826	2923	3019	3116	97
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475	6694	6785	6876	6968	7059	7151	7242	7333	7424	7516	91
476	7607	7698	7789	7881	7972	8063	8154	8245	8336	8427	91
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492	1965	2053	2142	2230	2318	2406	2494	2583	2671	2759	88
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527	1811	1893	1975	2058	2140	2222	2305	2387	2469	2552	82
528	2634	2716	2798	2881	2963	3045	3127	3209	3291	3374	82
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540	732394	2474	2555	2635	2715	2796	2876	2956	3037	3117	80
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546	7193	7272	7352	7431	7511	7590	7670	7749	7829	7908	79
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552	1939	2018	2096	2175	2254	2332	2411	2489	2568	2646	79
553	2725	2804	2882	2961	3039	3118	3196	3275	3353	3431	78
554	3510	3588	3667	3745	3823	3902	3980	4058	4136	4215	78
555	4293	4371	4449	4528	4606	4684	4762	4840	4919	4997	78
556	5075	5153	5231	5309	5387	5465	5543	5621	5699	5777	78
557	5855	5933	6011	6089	6167	6245	6323	6401	6479	6556	78
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559	7412	7489	7567	7645	7722	7800	7878	7955	8033	8110	78
560	748188	8266	8343	8421	8498	8576	8653	8731	8808	8885	77
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563	750508	0586	0663	0740	0817	0894	0971	1048	1125	1202	77
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565	2048	2125	2202	2279	2356	2433	2509	2586	2663	2740	77
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567	3583	3660	3736	3813	3889	3966	4042	4119	4195	4272	77
568	4348	4425	4501	4578	4654	4730	4807	4883	4960	5036	76
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570	755875	5951	6027	6103	6180	6256	6332	6408	6484	6560	76
571	6636	6712	6788	6864	6940	7016	7092	7168	7244	7320	76
572	7396	7472	7548	7624	7700	7775	7851	7927	8003	8079	76
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591	1587	1661	1734	1808	1881	1955	2028	2102	2175	2248	73
592	2322	2395	2468	2542	2615	2688	2762	2835	2908	2981	73
593	3055	3128	3201	3274	3348	3421	3494	3567	3640	3713	73
594	3786	3860	3933	4006	4079	4152	4225	4298	4371	4444	73
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596	5246	5319	5392	5465	5538	5610	5683	5756	5829	5902	73
597	5974	6047	6120	6193	6265	6338	6411	6483	6556	6629	73
598	6701	6774	6846	6919	6992	7064	7137	7209	7282	7354	73
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605	1755	1827	1899	1971	2042	2114	2186	2258	2329	2401	72
606	2473	2544	2616	2688	2759	2831	2902	2974	3046	3117	72
607	3189	3260	3332	3403	3475	3546	3618	3689	3761	3832	71
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627	7268	7337	7406	7475	7545	7614	7683	7752	7821	7890	69
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636	3457	3525	3594	3662	3730	3798	3867	3935	4003	4071	68
637	4139	4208	4276	4344	4412	4480	4548	4616	4685	4753	68
638	4821	4889	4957	5025	5093	5161	5229	5297	5365	5433	68
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783	3762	3817	3873	3928	3984	4039	4094	4150	4205	4261	55
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798	2003	2057	2112	2166	2221	2275	2329	2384	2438	2492	54
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804	5256	5310	5364	5418	5472	5526	5580	5634	5688	5742	54
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809	7949	8002	8056	8110	8163	8217	8270	8324	8378	8431	54
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822	4872	4925	4977	5030	5083	5136	5189	5241	5294	5347	53
823	5400	5453	5505	5558	5611	5664	5716	5769	5822	5875	53
824	5927	5980	6033	6085	6138	6191	6243	6296	6349	6401	53
825	6454	6507	6559	6612	6664	6717	6770	6822	6875	6927	53
826	6980	7033	7085	7138	7190	7243	7295	7348	7400	7453	53
827	7506	7558	7611	7663	7716	7768	7820	7873	7925	7978	52
828	8030	8083	8135	8188	8240	8293	8345	8397	8450	8502	52
829	8555	8607	8659	8712	8764	8816	8869	8921	8973	9026	52
830	919078	9130	9183	9235	9287	9340	9392	9444	9496	9549	52
831	9601	9653	9706	9758	9810	9862	9914	9967	..19	..71	52
832	920123	0176	0228	0280	0332	0384	0436	0489	0541	0593	52
833	0645	0697	0749	0801	0853	0906	0958	1010	1062	1114	52
834	1166	1218	1270	1322	1374	1426	1478	1530	1582	1634	52
835	1686	1738	1790	1842	1894	1946	1998	2050	2102	2154	52
836	2206	2258	2310	2362	2414	2466	2518	2570	2622	2674	52
837	2725	2777	2829	2881	2933	2985	3037	3089	3140	3192	52
838	3244	3296	3348	3399	3451	3503	3555	3607	3658	3710	52
839	3762	3814	3865	3917	3969	4021	4072	4124	4176	4228	52
840	924279	4331	4383	4434	4486	4538	4589	4641	4693	4744	52
841	4796	4848	4899	4951	5003	5054	5106	5157	5209	5261	52
842	5312	5364	5415	5467	5518	5570	5621	5673	5725	5776	52
843	5828	5879	5931	5982	6034	6085	6137	6188	6240	6291	51
844	6342	6394	6445	6497	6548	6600	6651	6702	6754	6805	51
845	6857	6908	6959	7011	7062	7114	7165	7216	7268	7319	51
846	7370	7422	7473	7524	7576	7627	7678	7730	7781	7832	51
847	7883	7935	7986	8037	8088	8140	8191	8242	8293	8345	51
848	8396	8447	8498	8549	8601	8652	8703	8754	8805	8857	51
849	8908	8959	9010	9061	9112	9163	9215	9266	9317	9368	51
850	929419	9470	9521	9572	9623	9674	9725	9776	9827	9879	51
851	9930	9981	..32	..83	.134	.185	.236	.287	.338	.389	51
852	930440	0491	0542	0592	0643	0694	0745	0796	0847	0898	51
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855	1966	2017	2068	2118	2169	2220	2271	2322	2372	2423	51
856	2474	2524	2575	2626	2677	2727	2778	2829	2879	2930	51
857	2981	3031	3082	3133	3183	3234	3285	3335	3386	3437	51
858	3487	3538	3589	3639	3690	3740	3791	3841	3892	3943	51
859	3993	4044	4094	4145	4195	4246	4296	4347	4397	4448	51
860	934498	4549	4599	4650	4700	4751	4801	4852	4902	4953	50
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862	5507	5558	5608	5658	5709	5759	5809	5860	5910	5960	50
863	6011	6061	6111	6162	6212	6262	6313	6363	6413	6463	50
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866	7518	7568	7618	7668	7718	7769	7819	7869	7919	7969	50
867	8019	8069	8119	8169	8219	8269	8320	8370	8420	8470	50
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872	0516	0566	0616	0666	0716	0765	0815	0865	0915	0964	50
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874	1511	1561	1611	1660	1710	1760	1809	1859	1909	1958	50
875	2008	2058	2107	2157	2207	2256	2306	2355	2405	2455	50
876	2504	2554	2603	2653	2702	2752	2801	2851	2901	2950	50
877	3000	3049	3099	3148	3198	3247	3297	3346	3396	3445	49
878	3495	3544	3593	3643	3692	3742	3791	3841	3890	3939	49
879	3989	4038	4088	4137	4186	4236	4285	4335	4384	4433	49
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880	944483	4532	4581	4631	4680	4729	4779	4828	4877	4927	49
881	4976	5025	5074	5124	5173	5222	5272	5321	5370	5419	49
882	5469	5518	5567	5616	5665	5715	5764	5813	5862	5912	49
883	5961	6010	6059	6108	6157	6207	6256	6305	6354	6403	49
884	6452	6501	6551	6600	6649	6698	6747	6796	6845	6894	49
885	6943	6992	7041	7090	7140	7189	7238	7287	7336	7385	49
886	7434	7483	7532	7581	7630	7679	7728	7777	7826	7875	49
887	7924	7973	8022	8070	8119	8168	8217	8266	8315	8364	49
888	8413	8462	8511	8560	8609	8657	8706	8755	8804	8853	49
889	8902	8951	8999	9048	9097	9146	9195	9244	9292	9341	49
890	949390	9439	9488	9536	9585	9634	9683	9731	9780	9829	49
891	9878	9926	9975	..24	..73	.121	.170	.219	.267	.316	49
892	950365	0414	0462	0511	0560	0608	0657	0706	0754	0803	49
893	0851	0900	0949	0997	1046	1095	1143	1192	1240	1289	49
894	1338	1386	1435	1483	1532	1580	1629	1677	1726	1775	49
895	1823	1872	1920	1969	2017	2066	2114	2163	2211	2260	48
896	2308	2356	2405	2453	2502	2550	2599	2647	2696	2744	48
897	2792	2841	2889	2938	2986	3034	3083	3131	3180	3228	48
898	3276	3325	3373	3421	3470	3518	3566	3615	3663	3711	48
899	3760	3808	3856	3905	3953	4001	4049	4098	4146	4194	48
900	954243	4291	4339	4387	4435	4484	4532	4580	4628	4677	48
901	4725	4773	4821	4869	4918	4966	5014	5062	5110	5158	48
902	5207	5255	5303	5351	5399	5447	5495	5543	5592	5640	48
903	5688	5736	5784	5832	5880	5928	5976	6024	6072	6120	48
904	6168	6216	6265	6313	6361	6409	6457	6505	6553	6601	48
905	6649	6697	6745	6793	6840	6888	6936	6984	7032	7080	48
906	7128	7176	7224	7272	7320	7368	7416	7464	7512	7559	48
907	7607	7655	7703	7751	7799	7847	7894	7942	7990	8038	48
908	8086	8134	8181	8229	8277	8325	8373	8421	8468	8516	48
909	8564	8612	8659	8707	8755	8803	8850	8898	8946	8994	48
910	959041	9089	9137	9185	9232	9280	9328	9375	9423	9471	48
911	9518	9566	9614	9661	9709	9757	9804	9852	9900	9947	48
912	9995	..42	..90	.138	.185	.233	.280	.328	.376	.423	48
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915	1421	1469	1516	1563	1611	1658	1706	1753	1801	1848	47
916	1895	1943	1990	2038	2085	2132	2180	2227	2275	2322	47
917	2369	2417	2464	2511	2559	2606	2653	2701	2748	2795	47
918	2843	2890	2937	2985	3032	3079	3126	3174	3221	3268	47
919	3316	3363	3410	3457	3504	3552	3599	3646	3693	3741	47
920	963788	3835	3882	3929	3977	4024	4071	4118	4165	4212	47
921	4260	4307	4354	4401	4448	4495	4542	4590	4637	4684	47
922	4731	4778	4825	4872	4919	4966	5013	5061	5108	5155	47
923	5202	5249	5296	5343	5390	5437	5484	5531	5578	5625	47
924	5672	5719	5766	5813	5860	5907	5954	6001	6048	6095	47
925	6142	6189	6236	6283	6329	6376	6423	6470	6517	6564	47
926	6611	6658	6705	6752	6799	6845	6892	6939	6986	7033	47
927	7080	7127	7173	7220	7267	7314	7361	7408	7454	7501	47
928	7548	7595	7642	7688	7735	7782	7829	7875	7922	7969	47
929	8016	8062	8109	8156	8203	8249	8296	8343	8390	8436	47
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931	8950	8996	9043	9090	9136	9183	9229	9276	9323	9369	47
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933	9882	9928	9975	..21	..68	.114	.161	.207	.254	.300	47
934	970347	0393	0440	0486	0533	0579	0626	0672	0719	0765	46
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937	1740	1786	1832	1879	1925	1971	2018	2064	2110	2157	46
938	2203	2249	2295	2342	2388	2434	2481	2527	2573	2619	46
939	2666	2712	2758	2804	2851	2897	2943	2989	3035	3082	46
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942	4051	4097	4143	4189	4235	4281	4327	4374	4420	4466	46
943	4512	4558	4604	4650	4696	4742	4788	4834	4880	4926	46
944	4972	5018	5064	5110	5156	5202	5248	5294	5340	5386	46
945	5432	5478	5524	5570	5616	5662	5707	5753	5799	5845	46
946	5891	5937	5983	6029	6075	6121	6167	6212	6258	6304	46
947	6350	6396	6442	6488	6533	6579	6625	6671	6717	6763	46
948	6808	6854	6900	6946	6992	7037	7083	7129	7175	7220	46
949	7266	7312	7358	7403	7449	7495	7541	7586	7632	7678	46
950	977724	7769	7815	7861	7906	7952	7998	8043	8089	8135	46
951	8181	8226	8272	8317	8363	8409	8454	8500	8546	8591	46
952	8637	8683	8728	8774	8819	8865	8911	8956	9002	9047	46
953	9093	9138	9184	9230	9275	9321	9366	9412	9457	9503	46
954	9548	9594	9639	9685	9730	9776	9821	9867	9912	9958	46
955	980003	0049	0094	0140	0185	0231	0276	0322	0367	0412	45
956	0458	0503	0549	0594	0640	0685	0730	0776	0821	0867	45
957	0912	0957	1003	1048	1093	1139	1184	1229	1275	1320	45
958	1366	1411	1456	1501	1547	1592	1637	1683	1728	1773	45
959	1819	1864	1909	1954	2000	2045	2090	2135	2181	2226	45
960	982271	2316	2362	2407	2452	2497	2543	2588	2633	2678	45
961	2723	2769	2814	2859	2904	2949	2994	3040	3085	3130	45
962	3175	3220	3265	3310	3356	3401	3446	3491	3536	3581	45
963	3626	3671	3716	3762	3807	3852	3897	3942	3987	4032	45
964	4077	4122	4167	4212	4257	4302	4347	4392	4437	4482	45
965	4527	4572	4617	4662	4707	4752	4797	4842	4887	4932	45
966	4977	5022	5067	5112	5157	5202	5247	5292	5337	5382	45
967	5426	5471	5516	5561	5606	5651	5696	5741	5786	5830	45
968	5875	5920	5965	6010	6055	6100	6144	6189	6234	6279	45
969	6324	6369	6413	6458	6503	6548	6593	6637	6682	6727	45
970	986772	6817	6861	6906	6951	6996	7040	7085	7130	7175	45
971	7219	7264	7309	7353	7398	7443	7488	7532	7577	7622	45
972	7666	7711	7756	7800	7845	7890	7934	7979	8024	8068	45
973	8113	8157	8202	8247	8291	8336	8381	8425	8470	8514	45
974	8559	8604	8648	8693	8737	8782	8826	8871	8916	8960	45
975	9005	9049	9094	9138	9183	9227	9272	9316	9361	9405	45
976	9450	9494	9539	9583	9628	9672	9717	9761	9806	9850	44
977	9895	9939	9983	..28	..72	.117	.161	.206	.250	.294	44
978	990339	0383	0428	0472	0516	0561	0605	0650	0694	0738	44
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980	991226	1270	1315	1359	1403	1448	1492	1536	1580	1625	44
981	1669	1713	1758	1802	1846	1890	1935	1979	2023	2067	44
982	2111	2156	2200	2244	2288	2333	2377	2421	2465	2509	44
983	2554	2598	2642	2686	2730	2774	2819	2863	2907	2951	44
984	2995	3039	3083	3127	3172	3216	3260	3304	3348	3392	44
985	3436	3480	3524	3568	3613	3657	3701	3745	3789	3833	44
986	3877	3921	3965	4009	4053	4097	4141	4185	4229	4273	44
987	4317	4361	4405	4449	4493	4537	4581	4625	4669	4713	44
988	4757	4801	4845	4889	4933	4977	5021	5065	5108	5152	44
989	5196	5240	5284	5328	5372	5416	5460	5504	5547	5591	44
990	995635	5679	5723	5767	5811	5854	5898	5942	5986	6030	44
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997	8695	8739	8782	8826	8869	8913	8956	9000	9043	9087	44
998	9131	9174	9218	9261	9305	9348	9392	9435	9479	9522	44
999	9565	9609	9652	9696	9739	9783	9826	9870	9913	9957	43
N.	0	1	2	3	4	5	6	7	8	9	D.

SINES AND TANGENTS,

FOR EVERY

DEGREE AND MINUTE

OF THE QUADRANT.

N.B. The minutes in the left-hand column of each page, increasing downwards, belong to the degrees at the top; and those increasing upwards, in the right-hand column, belong to the degrees below.

(0 Degree.)

M.	Sine	D	Cosine.	D.	Tang.	D.	Cotang.	
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2	764756	293485	000000	00	764756	293483	235244	58
3	940847	208231	000000	00	940847	208231	059153	57
4	7.065786	161517	000000	00	7.065786	161517	12.934214	56
5	162696	131968	000000	00	162696	131969	837304	55
6	241877	111575	9.999999	01	241878	111578	758122	54
7	308824	96653	999999	01	308825	96653	691175	53
8	366816	85254	999999	01	366817	85254	633183	52
9	417968	76263	999999	01	417970	76263	582030	51
10	463725	68988	999998	01	463727	68988	536273	50
11	7.505118	62981	9.999998	01	7.505120	62981	12.494880	49
12	542906	57936	999997	01	542909	57939	457091	48
13	577668	53641	999997	01	577672	53642	422328	47
14	609853	49938	999996	01	609857	49939	390143	46
15	639816	46714	999996	01	639820	46715	360180	45
16	667845	43881	999995	01	667849	43882	332151	44
17	694173	41372	999995	01	694179	41373	305821	43
18	718997	39135	999994	01	719003	39136	280997	42
19	742477	37127	999993	01	742484	37128	257516	41
20	764754	35315	999993	01	764761	35136	235239	40
21	7.785943	33672	9.999992	01	7.785951	33673	12.214049	39
22	806146	32175	999991	01	806155	32176	193845	38
23	825451	30805	999990	01	825460	30806	174540	37
24	843934	29547	999989	02	843944	29549	156056	36
25	861662	28388	999988	02	861674	28390	138326	35
26	878695	27317	999988	02	878708	27318	121292	34
27	895085	26323	999987	02	895099	26325	104901	33
28	910879	25399	999986	02	910894	25401	089106	32
29	926119	24538	999985	02	926134	24540	073866	31
30	940842	23733	999983	02	940858	23735	059142	30
31	7.955082	22980	9.999982	02	7.955100	22981	12.044900	29
32	968870	22273	999981	02	968889	22275	031111	28
33	982233	21608	999980	02	982253	21610	017747	27
34	995198	20981	999979	02	995219	20983	004781	26
35	8.007787	20390	999977	02	8.007809	20392	11.992191	25
36	020021	19831	999976	02	020045	19333	979955	24
37	031919	19302	999975	02	031945	19305	968055	23
38	043501	18801	999973	02	043527	18803	956473	22
39	054781	18325	999972	02	054809	18327	945191	21
40	065776	17872	999971	02	065806	17874	934194	20
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43	097183	16639	999966	02	097217	16642	902783	17
44	107167	16265	999964	03	107202	16268	892797	16
45	116926	15908	999963	03	116963	15910	883037	15
46	126471	15566	999961	03	126510	15568	873490	14
47	135810	15238	999959	03	135851	15241	864149	13
48	144953	14924	999958	03	144996	14927	855004	12
49	153907	14622	999956	03	153952	14627	846048	11
50	162681	14333	999954	03	162727	14336	837273	10
51	8.171280	14054	9.999952	03	8.171328	14057	11.828672	9
52	179713	13786	999950	03	179763	13790	820237	8
53	187985	13529	999948	03	188036	13532	811964	7
54	196102	13280	·999946	03	196156	13284	803844	6
55	204070	13041	999944	03	204126	13044	795874	5
56	211895	12810	999942	04	211953	12814	788047	4
57	219581	12587	999940	04	219641	12590	780359	3
58	227134	12372	999938	04	227195	12376	772805	2
59	234557	12164	999936	04	234621	12168	765379	1
60	241855	11963	999934	04	241921	11967	758079	0
	Cosine		Sine		Cotang.		Tang.	M.

89 Degrees.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	8.241855	11963	9.999934	04	8.241921	11967	11.758079	60
1	249033	11768	999932	04	249102	11772	750898	59
2	256094	11580	999929	04	256165	11584	743835	58
3	263042	11398	999927	04	263115	11402	736885	57
4	269881	11221	999925	04	269956	11225	730044	56
5	276614	11050	999922	04	276691	11054	723309	55
6	283243	10883	999920	04	283323	10887	716677	54
7	289773	10721	999918	04	289856	10726	710144	53
8	296207	10565	999915	04	296292	10570	703708	52
9	302546	10413	999913	04	302634	10418	697366	51
10	308794	10266	999910	04	308884	10270	691116	50
11	8.314954	10122	9.999907	04	8.315046	10126	11.684954	49
12	321027	9982	999905	04	321122	9987	678878	48
13	327016	9847	999902	04	327114	9851	672886	47
14	332924	9714	999899	05	333025	9719	666975	46
15	338753	9586	999897	05	338856	9590	661144	45
16	344504	9460	999894	05	344610	9465	655390	44
17	350181	9338	999891	05	350289	9343	649711	43
18	355783	9219	999888	05	355895	9224	644105	42
19	361315	9103	999885	05	361430	9108	638570	41
20	366777	8990	999882	05	366895	8995	633105	40
21	8.372171	8880	9.999879	05	8.372292	8885	11.627708	39
22	377499	8772	999876	05	377622	8777	622378	38
23	382762	8667	999873	05	382889	8672	617111	37
24	387962	8564	999870	05	388092	8570	611908	36
25	393101	8464	999867	05	393234	8470	606766	35
26	398179	8366	999864	05	398315	8371	601685	34
27	403199	8271	999861	05	403338	8276	596662	33
28	408161	8177	999858	05	408304	8182	591696	32
29	413068	8086	999854	05	413213	8091	586787	31
30	417919	7996	999851	06	418068	8002	581932	30
31	8.422717	7909	9.999848	06	8.422869	7914	11.577131	29
32	427462	7823	999844	06	427618	7830	572382	28
33	432156	7740	999841	06	432315	7745	567685	27
34	436800	7657	999838	06	436962	7663	563038	26
35	441394	7577	999834	06	441560	7583	558440	25
36	445941	7499	999831	06	446110	7505	553890	24
37	450440	7422	999827	06	450613	7428	549387	23
38	454893	7346	999823	06	455070	7352	544930	22
39	459301	7273	999820	06	459481	7279	540519	21
40	463665	7200	999816	06	463849	7206	536151	20
41	8.467985	7129	9.999812	06	8.468172	7135	11.531828	19
42	472263	7060	999809	06	472454	7066	527546	18
43	476498	6991	999805	06	476693	6998	523307	17
44	480693	6924	999801	06	480892	6931	519108	16
45	484848	6859	999797	07	485050	6865	514950	15
46	488963	6794	999793	07	489170	6801	510830	14
47	493040	6731	999790	07	493250	6738	506750	13
48	497078	6669	999786	07	497293	6676	502707	12
49	501080	6608	999782	07	501298	6615	498702	11
50	505045	6548	999778	07	505267	6555	494733	10
51	8.508974	6489	9.999774	07	8.509200	6496	11.490800	9
52	512867	6431	999769	07	513098	6439	486902	8
53	516726	6375	999765	07	516961	6382	483039	7
54	520551	6319	999761	07	520790	6326	479210	6
55	524343	6264	999757	07	524586	6272	475414	5
56	528102	6211	999753	07	528349	6218	471651	4
57	531828	6158	999748	07	532080	6165	467920	3
58	535523	6106	999744	07	535779	6113	464221	2
59	539186	6055	999740	07	539447	6062	460553	1
60	542819	6004	999735	07	543084	6012	456916	0
	Cosine		Sine		Cotang.		Tang.	M.

89 Degrees.

(2 Degrees.)

M.	Sine	D.	Cosine.	D.	Tang.	D.	Cotang.	
0	8.542819	6004	9.999735	07	8.543084	6012	11.456916	60
1	546422	5955	999731	07	546691	5962	453309	59
2	549995	5906	999726	07	550268	5914	449732	58
3	553539	5858	999722	08	553817	5866	446183	57
4	557054	5811	999717	08	557336	5819	442664	56
5	560540	5765	999713	08	560828	5773	439172	55
6	563999	5719	999708	08	564291	5727	435709	54
7	567431	5674	999704	08	567727	5682	432273	53
8	570836	5630	999699	08	571137	5638	428863	52
9	574214	5587	999694	08	574520	5595	425480	51
10	577566	5544	999689	08	577877	5552	422123	50
11	8.580892	5502	9.999685	08	8.581208	5510	11.418792	49
12	584193	5460	999680	08	584514	5468	415486	48
13	587469	5419	999675	08	587795	5427	412205	47
14	590721	5379	999670	08	591051	5387	408949	46
15	593948	5339	999665	08	594283	5347	405717	45
16	597152	5300	999660	08	597492	5308	402508	44
17	600332	5261	999655	08	600677	5270	399323	43
18	603489	5223	999650	08	603839	5232	396161	42
19	606623	5186	999645	09	606978	5194	393022	41
20	609734	5149	999640	09	610094	5158	389906	40
21	8.612823	5112	9.999635	09	8.613189	5121	11.386811	39
22	615891	5076	999629	09	616262	5085	383738	38
23	618937	5041	999624	09	619313	5050	380687	37
24	621962	5006	999619	09	622343	5015	377657	36
25	624965	4972	999614	09	625352	4981	374648	35
26	627948	4938	999608	09	628340	4947	371660	34
27	630911	4904	999603	09	631308	4913	368692	33
28	633854	4871	999597	09	634256	4880	365744	32
29	636776	4839	999592	09	637184	4848	362816	31
30	639680	4806	999586	09	640093	4816	359907	30
31	8.642563	4775	9.999581	09	8.642982	4784	11.357018	29
32	645428	4743	999575	09	645853	4753	354147	28
33	648274	4712	999570	09	648704	4722	351296	27
34	651102	4682	999564	09	651537	4691	348463	26
35	653911	4652	999558	10	654352	4661	345648	25
36	656702	4622	999553	10	657149	4631	342851	24
37	659475	4592	999547	10	659928	4602	340072	23
38	662230	4563	999541	10	662689	4573	337311	22
39	664968	4535	999535	10	665433	4544	334567	21
40	667689	4506	999529	10	668160	4526	331840	20
41	8.670393	4479	9.999524	10	8.670870	4488	11.329130	19
42	673080	4451	999518	10	673563	4461	326437	18
43	675751	4424	999512	10	676239	4434	323761	17
44	678405	4397	999506	10	678900	4417	321100	16
45	681043	4370	999500	10	681544	4380	318456	15
46	683665	4344	999493	10	684172	4354	315828	14
47	686272	4318	999487	10	686784	4328	313216	13
48	688863	4292	999481	10	689381	4303	310619	12
49	691438	4267	999475	10	691963	4277	308037	11
50	693998	4242	999469	10	694529	4252	305471	10
51	8.696543	4217	9.999463	11	8.697081	4228	11.302919	9
52	699073	4192	999456	11	699617	4203	300383	8
53	701589	4168	999450	11	702139	4179	297861	7
54	704090	4144	999443	11	704646	4155	295354	6
55	706577	4121	999437	11	707140	4132	292860	5
56	709049	4097	999431	11	709618	4108	290382	4
57	711507	4074	999424	11	712083	4085	287917	3
58	713952	4051	999418	11	714534	4062	285465	2
59	716383	4029	999411	11	716972	4040	283028	1
60	718800	4006	999404	11	719396	4017	280604	0
	Cosine		Sine		Cotang.		Tang.	M

87 Degrees.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	8.718800	4006	9.999404	11	8.719396	4017	11.280604	60
1	721204	3984	999398	11	721806	3995	278194	59
2	723595	3962	999391	11	724204	3974	275796	58
3	725972	3941	999384	11	726588	3952	273412	57
4	728337	3919	999378	11	728959	3930	271041	56
5	730688	3898	999371	11	731317	3909	268683	55
6	733027	3877	999364	12	733663	3889	266337	54
7	735354	3857	999357	12	735996	3868	264004	53
8	737667	3836	999350	12	738317	3848	261683	52
9	739969	3816	999343	12	740626	3827	259374	51
10	742259	3796	999336	12	742922	3807	257078	50
11	8.744536	3776	9.999329	12	8.745207	3787	11.254793	49
12	746802	3756	999322	12	747479	3768	252521	48
13	749055	3737	999315	12	749740	3749	250260	47
14	751297	3717	999308	12	751989	3729	248011	46
15	753528	3698	999301	12	754227	3710	245773	45
16	755747	3679	999294	12	756453	3692	243547	44
17	757955	3661	999286	12	758668	3673	241332	43
18	760151	3642	999279	12	760872	3655	239128	42
19	762337	3624	999272	12	763065	3636	236935	41
20	764511	3606	999265	12	765246	3618	234754	40
21	8.766675	3588	9.999257	12	8.767417	3600	11.232583	39
22	768828	3570	999250	13	769578	3583	230422	38
23	770970	3553	999242	13	771727	3565	228273	37
24	773101	3535	999235	13	773866	3548	226134	36
25	775223	3518	999227	13	775995	3531	224005	35
26	777333	3501	999220	13	778114	3514	221886	34
27	779434	3484	999212	13	780222	3497	219778	33
28	781524	3467	999205	13	782320	3480	217680	32
29	783605	3451	999197	13	784408	3464	215592	31
30	785675	3431	999189	13	786486	3447	213514	30
31	8.787736	3418	9.999181	13	8.788554	3431	11.211446	29
32	789787	3402	999174	13	790613	3414	209387	28
33	791828	3386	999166	13	792662	3399	207338	27
34	793859	3370	999158	13	794701	3383	205299	26
35	795881	3354	999150	13	796731	3368	203269	25
36	797894	3339	999142	13	798752	3352	201248	24
37	799897	3323	999134	13	800763	3337	199237	23
38	801892	3308	999126	13	802765	3322	197235	22
39	803876	3293	999118	13	804758	3307	195242	21
40	805852	3278	999110	13	806742	3292	193258	20
41	8.807819	3263	9.999102	13	8.808717	3278	11.191283	19
42	809777	3249	999094	14	810683	3262	189317	18
43	811726	3234	999086	14	812641	3248	187359	17
44	813667	3219	999077	14	814589	3233	185411	16
45	815599	3205	999069	14	816529	3219	183471	15
46	817522	3191	999061	14	818461	3205	181539	14
47	819436	3177	999053	14	820384	3191	179616	13
48	821343	3163	999044	14	822298	3177	177702	12
49	823240	3149	999036	14	824205	3163	175795	11
50	825130	3135	999027	14	826103	3150	173897	10
51	8.827011	3122	9.999019	14	8.827992	3136	11.172008	9
52	828884	3108	999010	14	829874	3123	170126	8
53	830749	3095	999002	14	831748	3110	168252	7
54	832607	3082	998993	14	833613	3096	166387	6
55	834456	3069	998984	14	835471	3083	164529	5
56	836297	3056	998976	14	837321	3070	162679	4
57	838130	3043	998967	15	839163	3057	160837	3
58	839956	3030	998958	15	840998	3045	159002	2
59	841774	3017	998950	15	842825	3032	157175	1
60	843585	3000	998941	15	844644	3019	155356	0
	Cosine		Sine		Cotang.		Tang.	M

86 Degrees.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	8.843585	3005	9.998941	15	8.844644	3019	11.155356	60
1	845387	2992	998932	15	846455	3007	153545	59
2	847183	2980	998923	15	848260	2995	151740	58
3	848971	2967	998914	15	850057	2982	149943	57
4	850751	2955	998905	15	851846	2970	148154	56
5	852525	2943	998896	15	853628	2958	146372	55
6	854291	2931	998887	15	855403	2946	144597	54
7	856049	2919	998878	15	857171	2935	142829	53
8	857801	2907	998869	15	858932	2923	141068	52
9	859546	2896	998860	15	860686	2911	139314	51
10	861283	2884	998851	15	862433	2900	137567	50
11	8.863014	2873	9.998841	15	8.864173	2888	11.135827	49
12	864738	2861	998832	15	865906	2877	134094	48
13	866455	2850	998823	16	867632	2866	132368	47
14	868165	2839	998813	16	869351	2854	130649	46
15	869868	2828	998804	16	871064	2843	128936	45
16	871565	2817	998795	16	872770	2832	127230	44
17	873255	2806	998785	16	874469	2821	125531	43
18	874938	2795	998776	16	876162	2811	123838	42
19	876615	2785	998766	16	877849	2800	122151	41
20	878285	2773	998757	16	879529	2789	120471	40
21	8.879949	2763	9.998747	16	8.881202	2779	11.118798	39
22	881607	2752	998738	16	882869	2768	117131	38
23	883258	2742	998728	16	884530	2758	115470	37
24	884903	2731	998718	16	886185	2747	113815	36
25	886542	2721	998708	16	887833	2737	112167	35
26	888174	2711	998699	16	889476	2727	110524	34
27	889801	2700	998689	16	891112	2717	108888	33
28	891421	2690	998679	16	892742	2707	107258	32
29	893035	2680	998669	17	894366	2697	105634	31
30	894643	2670	998659	17	895984	2687	104016	30
31	8.896246	2660	9.998649	17	8.897596	2677	11.102404	29
32	897842	2651	998639	17	899203	2667	100797	28
33	899432	2641	998629	17	900803	2658	099197	27
34	901017	3631	998619	17	902398	2648	097602	26
35	902596	2622	998609	17	903987	2638	096013	25
36	904169	2612	998599	17	905570	2629	094430	24
37	905736	2603	998589	17	907147	2620	092853	23
38	907297	2593	998578	17	908719	2610	091281	22
39	908853	2584	998568	17	910285	2601	089715	21
40	910404	2575	998558	17	911846	2592	088154	20
41	8.911949	2566	9.998548	17	8.913401	2583	11.086599	19
42	913488	2556	998537	17	914951	2574	085049	18
43	915022	2547	998527	17	916495	2565	083505	17
44	916550	2538	998516	18	918034	2556	081966	16
45	918073	2529	998506	18	919568	2547	080432	15
46	919591	2520	998495	18	921096	2538	078904	14
47	921103	2512	998485	18	922619	2530	077381	13
48	922610	2503	998474	18	924136	2521	075864	12
49	924112	2494	998464	18	925649	2512	074351	11
50	925609	2486	998453	18	927156	2503	072844	10
51	8.927100	2477	9.998442	18	8.928658	2495	11.071342	9
52	928587	2469	998431	18	930155	2486	069845	8
53	930068	2460	998421	18	931647	2478	068353	7
54	931544	2452	998410	18	933134	2470	066866	6
55	933015	2443	998399	18	934616	2461	065384	5
56	934481	2435	998388	18	936093	2453	063907	4
57	935942	2427	998377	18	937565	2445	062435	3
58	937398	2419	998366	18	939032	2437	060968	2
59	938850	2411	998355	18	940494	2430	059506	1
60	940296	2403	998344	18	941952	2421	058048	0
	Cosine		Sine		Cotang.		Tang.	M.

85 Degrees.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	8.940296	2403	9.998344	19	8.941952	2421	11.058048	60
1	941738	2394	998333	19	943404	2413	056596	59
2	943174	2387	998322	19	944852	2405	055148	58
3	944606	2379	998311	19	946295	2397	053705	57
4	946034	2371	998300	19	947734	2390	052266	56
5	947456	2363	998289	19	949168	2382	050832	55
6	948874	2355	998277	19	950597	2374	049403	54
7	950287	2348	998266	19	952021	2366	047979	53
8	951696	2340	998255	19	953441	2360	046559	52
9	953100	2332	998243	19	954856	2351	045144	51
10	954499	2325	998232	19	956267	2344	043733	50
11	8.955894	2317	9.998220	19	8.957674	2337	11.042326	49
12	957284	2310	998209	19	959075	2329	040925	48
13	958670	2302	998197	19	960473	2323	039527	47
14	960052	2295	998186	19	961866	2314	038134	46
15	961429	2288	998174	19	963255	2307	036745	45
16	962801	2280	998163	19	964639	2300	035361	44
17	964170	2273	998151	19	966019	2293	033981	43
18	965534	2266	998139	20	967394	2286	032606	42
19	966893	2259	998128	20	968766	2279	031234	41
20	968249	2252	998116	20	970133	2271	029867	40
21	8.969600	2244	9.998104	20	8.971496	2265	11.028504	39
22	970947	2238	998092	20	972855	2257	027145	38
23	972289	2231	998080	20	974209	2251	025791	37
24	973628	2224	998068	20	975560	2244	024440	36
25	974962	2217	998056	20	976906	2237	023094	35
26	976293	2210	998044	20	978248	2230	021752	34
27	977619	2203	998032	20	979586	2223	020414	33
28	978941	2197	998020	20	980921	2217	019079	32
29	980259	2190	998008	20	982251	2210	017749	31
30	981573	2183	997996	20	983577	2204	016423	30
31	8.982883	2177	9.997984	20	8.984899	2197	11.015101	29
32	984189	2170	997972	20	986217	2191	013783	28
33	985491	2163	997959	20	987532	2184	012468	27
34	986789	2157	997947	20	988842	2178	011158	26
35	988083	2150	997935	21	990149	2171	009851	25
36	989374	2144	997922	21	991451	2165	008549	24
37	990660	2138	997910	21	992750	2158	007250	23
38	991943	2131	997897	21	994045	2152	005955	22
39	993222	2125	997885	21	995337	2146	004663	21
40	994497	2119	997872	21	996624	2140	003376	20
41	8.995768	2112	9.997860	21	8.997908	2134	11.002092	19
42	997036	2106	997847	21	999188	2127	000812	18
43	998299	2100	997835	21	9.000465	2121	10.999535	17
44	999560	2094	997822	21	001738	2115	998262	16
45	9.000816	2087	997809	21	003007	2109	996993	15
46	002069	2082	997797	21	004272	2103	995728	14
47	003318	2076	997784	21	005534	2097	994466	13
48	004563	2070	997771	21	006792	2091	993208	12
49	005805	2064	997758	21	008047	2085	991953	11
50	007044	2058	997745	21	009298	2080	990702	10
51	9.008278	2052	9.997732	21	9.010546	2074	10.989454	9
52	009510	2046	997719	21	011790	2068	988210	8
53	010737	2040	997706	21	013031	2062	986969	7
54	011962	2034	997693	22	014268	2056	985732	6
55	013182	2029	997680	22	015502	2051	984498	5
56	014400	2023	997667	22	016732	2045	983268	4
57	015613	2017	997654	22	017959	2040	982041	3
58	016824	2012	997641	22	019183	2033	980817	2
59	018031	2006	997628	22	020403	2028	979597	1
60	019235	2000	997614	22	021620	2023	978380	0
	Cosine		Sine		Cotang.		Tang.	M.

84 Degrees.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.019235	2000	9.997614	22	9.021620	2023	10.978380	60
1	020435	1995	997601	22	022834	2017	977166	59
2	021632	1989	997588	22	024044	2011	975956	58
3	022825	1984	997574	22	025251	2006	974749	57
4	024016	1978	997561	22	026455	2000	973545	56
5	025203	1973	997547	22	027655	1995	972345	55
6	026386	1967	997534	23	028852	1990	971148	54
7	027567	1962	997520	23	030046	1985	969954	53
8	028744	1957	997507	23	031237	1979	968763	52
9	029918	1951	997493	23	032425	1974	967575	51
10	031089	1947	997480	23	033609	1969	966391	50
11	9.032257	1941	9.997466	23	9.034791	1964	10.965209	49
12	033421	1936	997452	23	035969	1958	964031	48
13	034582	1930	997439	23	037144	1953	962856	47
14	035741	1925	997425	23	038316	1948	961684	46
15	036896	1920	997411	23	039485	1943	960515	45
16	038048	1915	997397	23	040651	1938	959349	44
17	039197	1910	997383	23	041813	1933	958187	43
18	040342	1905	997369	23	042973	1928	957027	42
19	041485	1899	997355	23	044130	1923	955870	41
20	042625	1894	997341	23	045284	1918	954716	40
21	9.043762	1889	9.997327	24	9.046434	1913	10.953566	39
22	044895	1884	997313	24	047582	1908	952418	38
23	046026	1879	997299	24	048727	1903	951273	37
24	047154	1875	997285	24	049869	1898	950131	36
25	048279	1870	997271	24	051008	1893	948992	35
26	049400	1865	997257	24	052144	1889	947856	34
27	050519	1860	997242	24	053277	1884	946723	33
28	051635	1855	997228	24	054407	1879	945593	32
29	052749	1850	997214	24	055535	1874	944465	31
30	053859	1845	997199	24	056659	1870	943341	30
31	9.054966	1841	9.997185	24	9.057781	1865	10.942219	29
32	056071	1836	997170	24	058900	1869	941100	28
33	057172	1831	997156	24	060016	1855	939984	27
34	058271	1827	997141	24	061130	1851	938870	26
35	059367	1822	997127	24	062240	1846	937760	25
36	060460	1817	997112	24	063348	1842	936652	24
37	061551	1813	997098	24	064453	1837	935547	23
38	062639	1808	997083	25	065556	1833	934444	22
39	063724	1804	997068	25	066655	1828	933345	21
40	064806	1799	997053	25	067752	1824	932248	20
41	9.065885	1794	9.997039	25	9.068846	1819	10.931154	19
42	066962	1790	997024	25	069938	1815	930062	18
43	068036	1786	997009	25	071027	1810	928973	17
44	069107	1781	996994	25	072113	1806	927887	16
45	070176	1777	996979	25	073197	1802	926803	15
46	071242	1772	996964	25	074278	1797	925722	14
47	072306	1768	996949	25	075356	1793	924644	13
48	073366	1763	996934	25	076432	1789	923568	12
49	074424	1759	996919	25	077505	1784	922495	11
50	075480	1755	996904	25	078576	1780	921424	10
51	9.076533	1750	9.996889	25	9.079644	1776	10.920356	9
52	077583	1746	996874	25	080710	1772	919290	8
53	078631	1742	996858	25	081773	1767	918227	7
54	079676	1738	996843	25	082833	1763	917167	6
55	080719	1733	996828	25	083891	1759	916109	5
56	081759	1729	996812	26	084947	1755	915053	4
57	082797	1725	996797	26	086000	1751	914000	3
58	083832	1721	996782	26	087050	1747	912950	2
59	084864	1717	996766	26	088098	1743	911902	1
60	085894	1713	996751	26	089144	1738	910856	0
	Cosine		Sine		Cotang.		Tang.	M.

83 Degrees.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.085894	1713	9.996751	26	9.089144	1738	10.910856	60
1	086922	1709	996735	26	090187	1734	909813	59
2	087947	1704	996720	26	091228	1730	908772	58
3	088970	1700	996704	26	092266	1727	907734	57
4	089990	1696	996688	26	093302	1722	906698	56
5	091008	1692	996673	26	094336	1719	905664	55
6	092024	1688	996657	26	095367	1715	904633	54
7	093037	1684	996641	26	096395	1711	903605	53
8	094047	1680	996625	26	097422	1707	902578	52
9	095056	1676	996610	26	098446	1703	901554	51
10	096062	1673	996594	26	099468	1699	900532	50
11	9.097065	1668	9.996578	27	9.100487	1695	10.899513	49
12	098066	1665	996562	27	101504	1691	898496	48
13	099065	1661	996546	27	102519	1687	897481	47
14	100062	1657	996530	27	103532	1684	896468	46
15	101056	1653	996514	27	104542	1680	895458	45
16	102048	1649	996498	27	105550	1676	894450	44
17	103037	1645	996482	27	106556	1672	893444	43
18	104025	1641	996465	27	107559	1669	892441	42
19	105010	1638	996449	27	108560	1665	891440	41
20	105992	1634	996433	27	109559	1661	890441	40
21	9.106973	1630	9.996417	27	9.110556	1658	10.889444	39
22	107951	1627	996400	27	111551	1654	888449	38
23	108927	1623	996384	27	112543	1650	887457	37
24	109901	1619	996368	27	113533	1646	886467	36
25	110873	1616	996351	27	114521	1643	885479	35
26	111842	1612	996335	27	115507	1639	884493	34
27	112809	1608	996318	27	116491	1636	883509	33
28	113774	1605	996302	28	117472	1632	882528	32
29	114737	1601	996285	28	118452	1629	881548	31
30	115698	1597	996269	28	119429	1625	880571	30
31	9.116656	1594	9.996252	28	9.120404	1622	10.879596	29
32	117613	1590	996235	28	121377	1618	878623	28
33	118567	1587	996219	28	122348	1615	877652	27
34	119519	1583	996202	28	123317	1611	876683	26
35	120469	1580	996185	28	124284	1607	875716	25
36	121417	1576	996168	28	125249	1604	874751	24
37	122362	1573	996151	28	126211	1601	873789	23
38	123306	1569	996134	28	127172	1597	872828	22
39	124248	1566	996117	28	128130	1594	871870	21
40	125187	1562	996100	28	129087	1591	870913	20
41	9.126125	1559	9.996083	29	9.130041	1587	10.869959	19
42	127060	1556	996066	29	130994	1584	869006	18
43	127993	1552	996049	29	131944	1581	868056	17
44	128925	1549	996032	29	132893	1577	867107	16
45	129854	1545	996015	29	133839	1574	866161	15
46	130781	1542	995998	29	134784	1571	865216	14
47	131706	1539	995980	29	135726	1567	864274	13
48	132630	1535	995963	29	136667	1564	863333	12
49	133551	1532	995946	29	137605	1561	862395	11
50	134470	1529	995928	29	138542	1558	861458	10
51	9.135387	1525	9.995911	29	9.139476	1555	10.860524	9
52	136303	1522	995894	29	140409	1551	859591	8
53	137216	1519	995876	29	141340	1548	858660	7
54	138128	1516	995859	29	142269	1545	857731	6
55	139037	1512	995841	29	143196	1542	856804	5
56	139944	1509	995823	29	144121	1539	855879	4
57	140850	1506	995806	29	145044	1535	854956	3
58	141754	1503	995788	29	145966	1532	854034	2
59	142655	1500	995771	29	146885	1529	853115	1
60	143555	1496	995753	29	147803	1526	852197	0
	Cosine		Sine		Cotang.		Tang.	M.

82 Degrees.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.143555	1496	9.995753	30	9.147803	1526	10.852197	60
1	144453	1493	995735	30	148718	1523	851282	59
2	145349	1490	995717	30	149632	1520	850368	58
3	146243	1487	995699	30	150544	1517	849456	57
4	147136	1484	995681	30	151454	1514	848546	56
5	148026	1481	995664	30	152363	1511	847637	55
6	148915	1478	995646	30	153269	1508	846731	54
7	149802	1475	995628	30	154174	1505	845826	53
8	150686	1472	995610	30	155077	1502	844923	52
9	151569	1469	995591	30	155978	1499	844022	51
10	152451	1466	995573	30	156877	1496	843123	50
11	9.153330	1463	9.995555	30	9.157775	1493	10.842225	49
12	154208	1460	995537	30	158671	1490	841329	48
13	155083	1457	995519	30	159565	1487	840435	47
14	155957	1454	995501	31	160457	1484	839543	46
15	156830	1451	995482	31	161347	1481	838653	45
16	157700	1448	995464	31	162236	1479	837764	44
17	158569	1445	995446	31	163123	1476	836877	43
18	159435	1442	995427	31	164008	1473	835992	42
19	160301	1439	995409	31	164892	1470	835108	41
20	161164	1436	995390	31	165774	1467	834226	40
21	9.162025	1433	9.995372	31	9.166654	1464	10.833346	39
22	162885	1430	995353	31	167532	1461	832468	38
23	163743	1427	995334	31	168409	1458	831591	37
24	164600	1424	995316	31	169284	1455	830716	36
25	165454	1422	995297	31	170157	1453	829843	35
26	166307	1419	995278	31	171029	1450	828971	34
27	167159	1416	995260	31	171899	1447	828101	33
28	168008	1413	995241	32	172767	1444	827233	32
29	168856	1410	995222	32	173634	1442	826366	31
30	169702	1407	995203	32	174499	1439	825501	30
31	9.170547	1405	9.995184	32	9.175362	1436	10.824638	29
32	171389	1402	995165	32	176224	1433	823776	28
33	172230	1399	995146	32	177084	1431	822916	27
34	173070	1396	995127	32	177942	1428	822058	26
35	173908	1394	995108	32	178799	1425	821201	25
36	174744	1391	995089	32	179655	1423	820345	24
37	175578	1388	995070	32	180508	1420	819492	23
38	176411	1386	995051	32	181360	1417	818640	22
39	177242	1383	995032	32	182211	1415	817789	21
40	178072	1380	995013	32	183059	1412	816941	20
41	9.178900	1377	9.994993	32	9.183907	1409	10.816093	19
42	179726	1374	994974	32	184752	1407	815248	18
43	180551	1372	994955	32	185597	1404	814403	17
44	181374	1369	994935	32	186439	1402	813561	16
45	182196	1366	994916	33	187280	1399	812720	15
46	183016	1364	994896	33	188120	1396	811880	14
47	183834	1361	994877	33	188958	1393	811042	13
48	184651	1359	994857	33	189794	1391	810206	12
49	185466	1356	994838	33	190629	1389	809371	11
50	186280	1353	994818	33	191462	1386	808538	10
51	9.187092	1351	9.994798	33	9.192294	1384	10.807706	9
52	187903	1348	994779	33	193124	1381	806876	8
53	188712	1346	994759	33	193953	1379	806047	7
54	189519	1343	994739	33	194780	1376	805220	6
55	190325	1341	994719	33	195606	1374	804394	5
56	191130	1338	994700	33	196430	1371	803570	4
57	191933	1336	994680	33	197253	1369	802747	3
58	192734	1333	994660	33	198074	1366	801926	2
59	193534	1330	994640	33	198894	1364	801106	1
60	194332	1328	994620	33	199713	1361	800287	0
	Cosine		Sine		Cotang.		Tang.	M.

81 Degrees.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.194332	1328	9.994620	33	9.199713	1361	10.800287	60
1	195129	1326	994600	33	200529	1359	799471	59
2	195925	1323	994580	33	201345	1356	798655	58
3	196719	1321	994560	34	202159	1354	797841	57
4	197511	1318	994540	34	202971	1352	797029	56
5	198302	1316	994519	34	203782	1349	796218	55
6	199091	1313	994499	34	204592	1347	795408	54
7	199879	1311	994479	34	205400	1345	794600	53
8	200666	1308	994459	34	206207	1342	793793	52
9	201451	1306	994438	34	207013	1340	792987	51
10	202234	1304	994418	34	207817	1338	792183	50
11	9.203017	1301	9.994397	34	9.208619	1335	10.791381	49
12	203797	1299	994377	34	209420	1333	790580	48
13	204577	1296	994357	34	210220	1331	789780	47
14	205354	1294	994336	34	211018	1328	788982	46
15	206131	1292	994316	34	211815	1326	788185	45
16	206906	1289	994295	34	212611	1324	787389	44
17	207679	1287	994274	35	213405	1321	786595	43
18	208452	1285	994254	35	214198	1319	785802	42
19	209222	1282	994233	35	214989	1317	785011	41
20	209992	1280	994212	35	215780	1315	784220	40
21	9.210760	1278	9.994191	35	9.216568	1312	10.783432	39
22	211526	1275	994171	35	217356	1310	782644	38
23	212291	1273	994150	35	218142	1308	781858	37
24	213055	1271	994129	35	218926	1305	781074	36
25	213818	1268	994108	35	219710	1303	780290	35
26	214579	1266	994087	35	220492	1301	779508	34
27	215338	1264	994066	35	221272	1299	778728	33
28	216097	1261	994045	35	222052	1297	777948	32
29	216854	1259	994024	35	222830	1294	777170	31
30	217609	1257	994003	35	223606	1292	776394	30
31	9.218363	1255	9.993981	35	9.224382	1290	10.775618	29
32	219116	1253	993960	35	225156	1288	774844	28
33	219868	1250	993939	35	225929	1286	774071	27
34	220618	1248	993918	35	226700	1284	773300	26
35	221367	1246	993896	36	227471	1281	772529	25
36	222115	1244	993875	36	228239	1279	771761	24
37	222861	1242	993854	36	229007	1277	770993	23
38	223606	1239	993832	36	229773	1275	770227	22
39	224349	1237	993811	36	230539	1273	769461	21
40	225092	1235	993789	36	231302	1271	768698	20
41	9.225833	1233	9.993768	36	9.232065	1269	10.767935	19
42	226573	1231	993746	36	232826	1267	767174	18
43	227311	1228	993725	36	233586	1265	766414	17
44	228048	1226	993703	36	234345	1262	765655	16
45	228784	1224	993681	36	235103	1260	764897	15
46	229518	1222	993660	36	235859	1258	764141	14
47	230252	1220	993638	36	236614	1256	763386	13
48	230984	1218	993616	36	237368	1254	762632	12
49	231714	1216	993594	37	238120	1252	761880	11
50	232444	1214	993572	37	238872	1250	761128	10
51	9.233172	1212	9.993550	37	9.239622	1248	10.760378	9
52	233899	1209	993528	37	240371	1246	759629	8
53	234625	1207	993506	37	241118	1244	758882	7
54	235349	1205	993484	37	241865	1242	758135	6
55	236073	1203	993462	37	242610	1240	757390	5
56	236795	1201	993440	37	243354	1238	756646	4
57	237515	1199	993418	37	244097	1236	755903	3
58	238235	1197	993396	37	244839	1234	755161	2
59	238953	1195	993374	37	245579	1232	754421	1
60	239670	1193	993351	37	246319	1230	753681	0
	Cosine		Sine		Cotang.		Tang.	M.

85 Degrees.

M.	Sine	D.	Cosine	D.	Tang:	D.	Cotang.	
0	9.239670	1193	9.993351	37	9.246319	1230	10.753681	60
1	240386	1191	993329	37	247057	1228	752943	59
2	241101	1189	993307	37	247794	1226	752206	58
3	241814	1187	993285	37	248530	1224	751470	57
4	242526	1185	993262	37	249264	1222	750736	56
5	243237	1183	993240	37	249998	1220	750002	55
6	243947	1181	993217	38	250730	1218	749270	54
7	244656	1179	993195	38	251461	1217	748539	53
8	245363	1177	993172	38	252191	1215	747809	52
9	246069	1175	993149	38	252920	1213	747080	51
10	246775	1173	993127	38	253648	1211	746352	50
11	9.247478	1171	9.993104	38	9.254374	1209	10.745626	49
12	248181	1169	993081	38	255100	1207	744900	48
13	248883	1167	993059	38	255824	1205	744176	47
14	249583	1165	993036	38	256547	1203	743453	46
15	250282	1163	993013	38	257269	1201	742731	45
16	250980	1161	992990	38	257990	1200	742010	44
17	251677	1159	992967	38	258710	1198	741290	43
18	252373	1158	992944	38	259429	1196	740571	42
19	253067	1156	992921	38	260146	1194	739854	41
20	253761	1154	992898	38	260863	1192	739137	40
21	9.254453	1152	9.992875	38	9.261578	1190	10.738422	39
22	255144	1150	992852	38	262292	1189	737708	38
23	255834	1148	992829	39	263005	1187	736995	37
24	256523	1146	992806	39	263717	1185	736283	36
25	257211	1144	992783	39	264428	1183	735572	35
26	257898	1142	992759	39	265138	1181	734862	34
27	258583	1141	992736	39	265847	1179	734153	33
28	259268	1139	992713	39	266555	1178	733445	32
29	259951	1137	992690	39	267261	1176	732739	31
30	260633	1135	992666	39	267967	1174	732033	30
31	9.261314	1133	9.992643	39	9.268671	1172	10.731329	29
32	261994	1131	992619	39	269375	1170	730625	28
33	262673	1130	992596	39	270077	1169	729923	27
34	263351	1128	992572	39	270779	1167	729221	26
35	264027	1126	992549	39	271479	1165	728521	25
36	264703	1124	992525	39	272178	1164	727822	24
37	265377	1122	992501	39	272876	1162	727124	23
38	266051	1120	992478	40	273573	1160	726427	22
39	266723	1119	992454	40	274269	1158	725731	21
40	267395	1117	992430	40	274964	1157	725036	20
41	9.268065	1115	9.992406	40	9.275658	1155	10.724342	19
42	268734	1113	992382	40	276351	1153	723649	18
43	269402	1111	992359	40	277043	1151	722957	17
44	270069	1110	992335	40	277734	1150	722266	16
45	270735	1108	992311	40	278424	1148	721576	15
46	271400	1106	992287	40	279113	1147	720887	14
47	272064	1105	992263	40	279801	1145	720199	13
48	272726	1103	992239	40	280488	1143	719512	12
49	273388	1101	992214	40	281174	1141	718826	11
50	274049	1099	992190	40	281858	1140	718142	10
51	9.274708	1098	9.992166	40	9.282542	1138	10.717458	9
52	275367	1096	992142	40	283225	1136	716775	8
53	276024	1094	992117	41	283907	1135	716093	7
54	276681	1092	992093	41	284588	1133	715412	6
55	277337	1091	992069	41	285268	1131	714732	5
56	277991	1089	992044	41	285947	1130	714053	4
57	278644	1087	992020	41	286624	1128	713376	3
58	279297	1086	991996	41	287301	1126	712699	2
59	279948	1084	991971	41	287977	1125	712023	1
60	280599	1082	991947	41	288652	1123	711348	0
	Cosine		Sine		Cotang.		Tang.	M.

79 Degrees.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.280599	1082	9.991947	41	9.288652	1123	10.711348	60
1	281248	1081	991922	41	289326	1122	710674	59
2	281897	1079	991897	41	289999	1120	710001	58
3	282544	1077	991873	41	290671	1118	709329	57
4	283190	1076	991848	41	291342	1117	708658	56
5	283836	1074	991823	41	292013	1115	707987	55
6	284480	1072	991799	41	292682	1114	707318	54
7	285124	1071	991774	42	293350	1112	706650	53
8	285766	1069	991749	42	294017	1111	705983	52
9	286408	1067	991724	42	294684	1109	705316	51
10	287048	1066	991699	42	295349	1107	704651	50
11	9.287687	1064	9.991674	42	9.296013	1106	10.703987	49
12	288326	1063	991649	42	296677	1104	703323	48
13	288964	1061	991624	42	297339	1103	702661	47
14	289600	1059	991599	42	298001	1101	701999	46
15	290236	1058	991574	42	298662	1100	701338	45
16	290870	1056	991549	42	299322	1098	700678	44
17	291504	1054	991524	42	299980	1096	700020	43
18	292137	1053	991498	42	300638	1095	699362	42
19	292768	1051	991473	42	301295	1093	698705	41
20	293399	1050	991448	42	301951	1092	698049	40
21	9.294029	1048	9.991422	42	9.302607	1090	10.697393	39
22	294658	1046	991397	42	303261	1089	696739	38
23	295286	1045	991372	43	303914	1087	696086	37
24	295913	1043	991346	43	304567	1086	695433	36
25	296539	1042	991321	43	305218	1084	694782	35
26	297164	1040	991295	43	305869	1083	694131	34
27	297788	1039	991270	43	306519	1081	693481	33
28	298412	1037	991244	43	307168	1080	692832	32
29	299034	1036	991218	43	307815	1078	692185	31
30	299655	1034	991193	43	308463	1077	691537	30
31	9.300276	1032	9.991167	43	9.309109	1075	10.690891	29
32	300895	1031	991141	43	309754	1074	690246	28
33	301514	1029	991115	43	310398	1073	689602	27
34	302132	1028	991090	43	311042	1071	688958	26
35	302748	1026	991064	43	311685	1070	688315	25
36	303364	1025	991038	43	312327	1068	687673	24
37	303979	1023	991012	43	312967	1067	687033	23
38	304593	1022	990986	43	313608	1065	686392	22
39	305207	1020	990960	43	314247	1064	685753	21
40	305819	1019	990934	44	314885	1062	685115	20
41	9.306430	1017	9.990908	44	9.315523	1061	10.684477	19
42	307041	1016	990882	44	316159	1060	683841	18
43	307650	1014	990855	44	316795	1058	683205	17
44	308259	1013	990829	44	317430	1057	682570	16
45	308867	1011	990803	44	318064	1055	681936	15
46	309474	1010	990777	44	318697	1054	681303	14
47	310080	1008	990750	44	319329	1053	680671	13
48	310685	1007	990724	44	319961	1051	680039	12
49	311289	1005	990697	44	320592	1050	679408	11
50	311893	1004	990671	44	321222	1048	678778	10
51	9.312495	1003	9.990644	44	9.321851	1047	10.678149	9
52	313097	1001	990618	44	322479	1045	677521	8
53	313698	1000	990591	44	323106	1044	676894	7
54	314297	998	990565	44	323733	1043	676267	6
55	314897	997	990538	44	324358	1041	675642	5
56	315495	996	990511	45	324983	1040	675017	4
57	316092	994	990485	45	325607	1039	674393	3
58	316689	993	990458	45	326231	1037	673769	2
59	317284	991	990431	45	326853	1036	673147	1
60	317879	990	990404	45	327475	1035	672525	0
	Cosine		Sine		Cotang.		Tang.	M.

78 Degrees.

(12 Degrees.)

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.317879	990	9.990404	45	9.327474	1035	10.672526	60
1	318473	988	990378	45	328095	1033	671905	59
2	319066	987	990351	45	328715	1032	671285	58
3	319658	986	990324	45	329334	1030	670666	57
4	320249	984	990297	45	329953	1029	670047	56
5	320840	983	990270	45	330570	1028	669430	55
6	321430	982	990243	45	331187	1026	668813	54
7	322019	980	990215	45	331803	1025	668197	53
8	322607	979	990188	45	332418	1024	667582	52
9	323194	977	990161	45	333033	1023	666967	51
10	323780	976	990134	45	333646	1021	666354	50
11	9.324366	975	9.990107	46	9.334259	1020	10.665741	49
12	324950	973	990079	46	334871	1019	665129	48
13	325534	972	990052	46	335482	1017	664518	47
14	326117	970	990025	46	336093	1016	663907	46
15	326700	969	989997	46	336702	1015	663298	45
16	327281	968	989970	46	337311	1013	662689	44
17	327862	966	989942	46	337919	1012	662081	43
18	328442	965	989915	46	338527	1011	661473	42
19	329021	964	989887	46	339133	1010	660867	41
20	329599	962	989860	46	339739	1008	660261	40
21	9.330176	961	9.989832	46	9.340344	1007	10.659656	39
22	330753	960	989804	46	340948	1006	659052	38
23	331329	958	989777	46	341552	1004	658448	37
24	331903	957	989749	47	342155	1003	657845	36
25	332478	956	989721	47	342757	1002	657243	35
26	333051	954	989693	47	343358	1000	656642	34
27	333624	953	989665	47	343958	999	656042	33
28	334195	952	989637	47	344558	998	655442	32
29	334766	950	989609	47	345157	997	654843	31
30	335337	949	989582	47	345755	996	654245	30
31	9.335906	948	9.989553	47	9.346353	994	10.653647	29
32	336475	946	989525	47	346949	993	653051	28
33	337043	945	989497	47	347545	992	652455	27
34	337610	944	989469	47	348141	991	651859	26
35	338176	943	989441	47	348735	990	651265	25
36	338742	941	989413	47	349329	988	650671	24
37	339306	940	989384	47	349922	987	650078	23
38	339871	939	989356	47	350514	986	649486	22
39	340434	937	989328	47	351106	985	648894	21
40	340996	936	989300	47	351697	983	648303	20
41	9.341558	935	9.989271	47	9.352287	982	10.647713	19
42	342119	934	989243	47	352876	981	647124	18
43	342679	932	989214	47	353465	980	646535	17
44	343239	931	989186	47	354053	979	645947	16
45	343797	930	989157	47	354640	977	645360	15
46	344355	929	989128	48	355227	976	644773	14
47	344912	927	989100	48	355813	975	644187	13
48	345469	926	989071	48	356398	974	643602	12
49	346024	925	989042	48	356982	973	643018	11
50	346579	924	989014	48	357566	971	642434	10
51	9.347134	922	9.988985	48	9.358149	970	10.641851	9
52	347687	921	988956	48	358731	969	641269	8
53	348240	920	988927	48	359313	968	640687	7
54	348792	919	988898	48	359893	967	640107	6
55	349343	917	988869	48	360474	966	639526	5
56	349893	916	988840	48	361053	965	638947	4
57	350443	915	988811	49	361632	963	638368	3
58	350992	914	988782	49	362210	962	637790	2
59	351540	913	988753	49	362787	961	637213	1
60	352088	911	988724	49	363364	960	636636	0
	Cosine		Sine		Cotang.		Tang.	M.

77 Degrees.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.352088	911	9.988724	49	9.363364	960	10.636636	60
1	352635	910	988695	49	363940	959	636060	59
2	353181	909	988666	49	364515	958	635485	58
3	353726	908	988636	49	365090	957	634910	57
4	354271	907	988607	49	365664	955	634336	56
5	354815	905	988578	49	366237	954	633763	55
6	355358	904	988548	49	366810	953	633190	54
7	355901	903	988519	49	367382	952	632618	53
8	356443	902	988489	49	367953	951	632047	52
9	356984	901	988460	49	368524	950	631476	51
10	357524	899	988430	49	369094	949	630906	50
11	9.358064	898	9.988401	49	9.369663	948	10.630337	49
12	358603	897	988371	49	370232	946	629768	48
13	359141	896	988342	49	370799	945	629201	47
14	359678	895	988312	50	371367	944	628633	46
15	360215	893	988282	50	371933	943	628067	45
16	360752	892	988252	50	372499	942	627501	44
17	361287	891	988223	50	373064	941	626936	43
18	361822	890	988193	50	373629	940	626371	42
19	362356	889	988163	50	374193	939	625807	41
20	362889	888	988133	50	374756	938	625244	40
21	9.363422	887	9.988103	50	9.375319	937	10.624681	39
22	363954	885	988073	50	375881	935	624119	38
23	364485	884	988043	50	376442	934	623558	37
24	365016	883	988013	50	377003	933	622997	36
25	365546	882	987983	50	377563	932	622437	35
26	366075	881	987953	50	378122	931	621878	34
27	366604	880	987922	50	378681	930	621319	33
28	367131	879	987892	50	379239	929	620761	32
29	367659	877	987862	50	379797	928	620203	31
30	368185	876	987832	51	380354	927	619646	30
31	9.368711	875	9.987801	51	9.380910	926	10.619090	29
32	369236	874	987771	51	381466	925	618534	28
33	369761	873	987740	51	382020	924	617980	27
34	370285	872	987710	51	382575	923	617425	26
35	370808	871	987679	51	383129	922	616871	25
36	371330	870	987649	51	383682	921	616318	24
37	371852	869	987618	51	384234	920	615766	23
38	372373	867	987588	51	384786	919	615214	22
39	372894	866	987557	51	385337	918	614663	21
40	373414	865	987526	51	385888	917	614112	20
41	9.373933	864	9.987496	51	9.386438	915	10.613562	19
42	374452	863	987465	51	386987	914	613013	18
43	374970	862	987434	51	387536	913	612464	17
44	375487	861	987403	52	388084	912	611916	16
45	376003	860	987372	52	388631	911	611369	15
46	376519	859	987341	52	389178	910	610822	14
47	377035	858	987310	52	389724	909	610276	13
48	377549	857	987279	52	390270	908	609730	12
49	378063	856	987248	52	390815	907	609185	11
50	378577	854	987217	52	391360	906	608640	10
51	9.379089	853	9.987186	52	9.391903	905	10.608097	9
52	379601	852	987155	52	392447	904	607553	8
53	380113	851	987124	52	392989	903	607011	7
54	380624	850	987092	52	393531	902	606469	6
55	381134	849	987061	52	394073	901	605927	5
56	381643	848	987030	52	394614	900	605386	4
57	382152	847	986998	52	395154	899	604846	3
58	382661	846	986967	52	395694	898	604306	2
59	383168	845	986936	52	396233	897	603767	1
60	383675	844	986904	52	396771	896	603229	0
	Cosine		Sine		Cotang.		Tang.	M.

76 Degrees.

M.	Sine	D	Cosine	D.	Tang.	D.	Cotang.	
0	9.383675	844	9.986904	52	9.396771	896	10.603229	60
1	384182	843	986873	53	397309	896	602691	59
2	384687	842	986841	53	397846	895	602154	58
3	385192	841	986809	53	398383	894	601617	57
4	385697	840	986778	53	398919	893	601081	56
5	386201	839	986746	53	399455	892	600545	55
6	386704	838	986714	53	399990	891	600010	54
7	387207	837	986683	53	400524	890	599476	53
8	387709	836	986651	53	401058	889	598942	52
9	388210	835	986619	53	401591	888	598409	51
10	388711	834	986587	53	402124	887	597876	50
11	9.389211	833	9.986555	53	9.402656	886	10.597344	49
12	389711	832	986523	53	403187	885	596813	48
13	390210	831	986491	53	403718	884	596282	47
14	390708	830	986459	53	404249	883	595751	46
15	391206	828	986427	53	404778	882	595222	45
16	391703	827	986395	53	405308	881	594692	44
17	392199	826	986363	54	405836	880	594164	43
18	392695	825	986331	54	406364	879	593636	42
19	393191	824	986299	54	406892	878	593108	41
20	393685	823	986266	54	407419	877	592581	40
21	9.394179	822	9.986234	54	9.407945	876	10.592055	39
22	394673	821	986202	54	408471	875	591529	38
23	395166	820	986169	54	408997	874	591003	37
24	395658	819	986137	54	409521	874	590479	36
25	396150	818	986104	54	410045	873	589955	35
26	396641	817	986072	54	410569	872	589431	34
27	397132	817	986039	54	411092	871	588908	33
28	397621	816	986007	54	411615	870	588385	32
29	398111	815	985974	54	412137	869	587863	31
30	398600	814	985942	54	412658	868	587342	30
31	9.399088	813	9.985909	55	9.413179	867	10.586821	29
32	399575	812	985876	55	413699	866	586301	28
33	400062	811	985843	55	414219	865	585781	27
34	400549	810	985811	55	414738	864	585262	26
35	401035	809	985778	55	415257	864	584743	25
36	401520	808	985745	55	415775	863	584225	24
37	402005	807	985712	55	416293	862	583707	23
38	402489	806	985679	55	416810	861	583190	22
39	402972	805	985646	55	417326	860	582674	21
40	403455	804	985613	55	417842	859	582158	20
41	9.403938	803	9.985580	55	9.418358	858	10.581642	19
42	404420	802	985547	55	418873	857	581127	18
43	404901	801	985514	55	419387	856	580613	17
44	405382	800	985480	55	419901	855	580099	16
45	405862	799	985447	55	420415	855	579585	15
46	406341	798	985414	56	420927	854	579073	14
47	406820	797	985380	56	421440	853	578560	13
48	407299	796	985347	56	421952	852	578048	12
49	407777	795	985314	56	422463	851	577537	11
50	408254	794	985280	56	422974	850	577026	10
51	9.408731	794	9.985247	56	9.423484	849	10.576516	9
52	409207	793	985213	56	423993	848	576007	8
53	409682	792	985180	56	424503	848	575497	7
54	410157	791	985146	56	425011	847	574989	6
55	410632	790	985113	56	425519	846	574481	5
56	411106	789	985079	56	426027	845	573973	4
57	411579	788	985045	56	426534	844	573466	3
58	412052	787	985011	56	427041	843	572959	2
59	412524	786	984978	56	427547	843	572453	1
60	412996	785	984944	56	428052	842	571948	0
	Cosine		Sine		Cotang.		Tang.	M.

75 Degrees.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.412996	785	9.984944	57	9.428052	842	10.571948	60
1	413467	784	984910	57	428557	841	571443	59
2	413938	783	984876	57	429062	840	570938	58
3	414408	783	984842	57	429566	839	570434	57
4	414878	782	984808	57	430070	838	569930	56
5	415347	781	984774	57	430573	838	569427	55
6	415815	780	984740	57	431075	837	568925	54
7	416283	779	984706	57	431577	836	568423	53
8	416751	778	984672	57	432079	835	567921	52
9	417217	777	984637	57	432580	834	567420	51
10	417684	776	984603	57	433080	833	566920	50
11	9.418150	775	9.984569	57	9.433580	832	10.566420	49
12	418615	774	984535	57	434080	832	565920	48
13	419079	773	984500	57	434579	831	565421	47
14	419544	773	984466	57	435078	830	564922	46
15	420007	772	984432	58	435576	829	564424	45
16	420470	771	984397	58	436073	828	563927	44
17	420933	770	984363	58	436570	828	563430	43
18	421395	769	984328	58	437067	827	562933	42
19	421857	768	984294	58	437563	826	562437	41
20	422318	767	984259	58	438059	825	561941	40
21	9.422778	767	9.984224	58	9.438554	824	10.561446	39
22	423238	766	984190	58	439048	823	560952	38
23	423697	765	984155	58	439543	823	560457	37
24	424156	764	984120	58	440036	822	559964	36
25	424615	763	984085	58	440529	821	559471	35
26	425073	762	984050	58	441022	820	558978	34
27	425530	761	984015	58	441514	819	558486	33
28	425987	760	983981	58	442006	819	557994	32
29	426443	760	983946	58	442497	818	557503	31
30	426899	759	983911	58	442988	817	557012	30
31	9.427354	758	9.983875	58	9.443479	816	10.556521	29
32	427809	757	983840	59	443968	816	556032	28
33	428263	756	983805	59	444458	815	555542	27
34	428717	755	983770	59	444947	814	555053	26
35	429170	754	983735	59	445435	813	554565	25
36	429623	753	983700	59	445923	812	554077	24
37	430075	752	983664	59	446411	812	553589	23
38	430527	752	983629	59	446898	811	553102	22
39	430978	751	983594	59	447384	810	552616	21
40	431429	750	983558	59	447870	809	552130	20
41	9.431879	749	9.983523	59	9.448356	809	10.551644	19
42	432329	749	983487	59	448841	808	551159	18
43	432778	748	983452	59	449326	807	550674	17
44	433226	747	983416	59	449810	806	550190	16
45	433675	746	983381	59	450294	806	549706	15
46	434122	745	983345	59	450777	805	549223	14
47	434569	744	983309	59	451260	804	548740	13
48	435016	744	983273	60	451743	803	548257	12
49	435462	743	983238	60	452225	802	547775	11
50	435908	742	983202	60	452706	802	547294	10
51	9.436353	741	9.983166	60	9.453187	801	10.546813	9
52	436798	740	983130	60	453668	800	546332	8
53	437242	740	983094	60	454148	799	545852	7
54	437686	739	983058	60	454628	799	545372	6
55	438129	738	983022	60	455107	798	544893	5
56	438572	737	982986	60	455586	797	544414	4
57	439014	736	982950	60	456064	796	543936	3
58	439456	736	982914	60	456542	796	543458	2
59	439897	735	982878	60	457019	795	542981	1
60	440338	734	982842	60	457496	794	542504	0
	Cosine		Sine		Cotang.		Tang.	M.

74 Degrees.

M	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.440338	734	9.982842	60	9.457496	794	10.542504	60
1	440778	733	982805	60	457973	793	542027	59
2	441218	732	982769	61	458449	793	541551	58
3	441658	731	982733	61	458925	792	541075	57
4	442096	731	982696	61	459400	791	540600	56
5	442535	730	982660	61	459875	790	540125	55
6	442973	729	982624	61	460349	790	539651	54
7	443410	728	982587	61	460823	789	539177	53
8	443847	727	982551	61	461297	788	538703	52
9	444284	727	982514	61	461770	788	538230	51
10	444720	726	982477	61	462242	787	537758	50
11	9.445155	725	9.982441	61	9.462714	786	10.537286	49
12	445590	724	982404	61	463186	785	536814	48
13	446025	723	982367	61	463658	785	536342	47
14	446459	723	982331	61	464129	784	535871	46
15	446893	722	982294	61	464599	783	535401	45
16	447326	721	982257	61	465069	783	534931	44
17	447759	720	982220	62	465539	782	534461	43
18	448191	720	982183	62	466008	781	533992	42
19	448623	719	982146	62	466476	780	533524	41
20	449054	718	982109	62	466945	780	533055	40
21	9.449485	717	9.982072	62	9.467413	779	10.532587	39
22	449915	716	982035	62	467880	778	532120	38
23	450345	716	981998	62	468347	778	531653	37
24	450775	715	981961	62	468814	777	531186	36
25	451204	714	981924	62	469280	776	530720	35
26	451632	713	981886	62	469746	775	530254	34
27	452060	713	981849	62	470211	775	529789	33
28	452488	712	981812	62	470676	774	529324	32
29	452915	711	981774	62	471141	773	528859	31
30	453342	710	981737	62	471605	773	528395	30
31	9.453768	710	9.981699	63	9.472068	772	10.527932	29
32	454194	709	981662	63	472532	771	527468	28
33	454619	708	981625	63	472995	771	527005	27
34	455044	707	981587	63	473457	770	526543	26
35	455469	707	981549	63	473919	769	526081	25
36	455893	706	981512	63	474381	769	525619	24
37	456316	705	981474	63	474842	768	525158	23
38	456739	704	981436	63	475303	767	524697	22
39	457162	704	981399	63	475763	767	524237	21
40	457584	703	981361	63	476223	766	523777	20
41	9.458006	702	9.981323	63	9.476683	765	10.523317	19
42	458427	701	981285	63	477142	765	522858	18
43	458848	701	981247	63	477601	764	522399	17
44	459268	700	981209	63	478059	763	521941	16
45	459688	699	981171	63	478517	763	521483	15
46	460108	698	981133	64	478975	762	521025	14
47	460527	698	981095	64	479432	761	520568	13
48	460946	697	981057	64	479889	761	520111	12
49	461364	696	981019	64	480345	760	519655	11
50	461782	695	980981	64	480801	759	519199	10
51	9.462199	695	9.980942	64	9.481257	759	10.518743	9
52	462616	694	980904	64	481712	758	518288	8
53	463032	693	980866	64	482167	757	517833	7
54	463448	693	980827	64	482621	757	517379	6
55	463864	692	980789	64	483075	756	516925	5
56	464279	691	980750	64	483529	755	516471	4
57	464694	690	980712	64	483982	755	516018	3
58	465108	690	980673	64	484435	754	515565	2
59	465522	689	980635	64	484887	753	515113	1
60	465935	688	980596	64	485339	753	514661	0
	Cosine		Sine		Cotang.		Tang.	M.

73 Degrees.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.465935	688	9.980596	64	9.485339	755	10.514661	60
1	466348	688	980558	64	485791	752	514209	59
2	466761	687	980519	65	486242	751	513758	58
3	467173	686	980480	65	486693	751	513307	57
4	467585	685	980442	65	487143	750	512857	56
5	467996	685	980403	65	487593	749	512407	55
6	468407	684	980364	65	488043	749	511957	54
7	468817	683	980325	65	488492	748	511508	53
8	469227	683	980286	65	488941	747	511059	52
9	469637	682	980247	65	489390	747	510610	51
10	470046	681	980208	65	489838	746	510162	50
11	9.470455	680	9.980169	65	9.490286	746	10.509714	49
12	470863	680	980130	65	490733	745	509267	48
13	471271	679	980091	65	491180	744	508820	47
14	471679	678	980052	65	491627	744	508373	46
15	472086	678	980012	65	492073	743	507927	45
16	472492	677	979973	65	492519	743	507481	44
17	472898	676	979934	66	492965	742	507035	43
18	473304	676	979895	66	493410	741	506590	42
19	473710	675	979855	66	493854	740	506146	41
20	474115	674	979816	66	494299	740	505701	40
21	9.474519	674	9.979776	66	9.494743	740	10.505257	39
22	474923	673	979737	66	495186	739	504814	38
23	475327	672	979697	66	495630	738	504370	37
24	475730	672	979658	66	496073	737	503927	36
25	476133	671	979618	66	496515	737	503485	35
26	476536	670	979579	66	496957	736	503043	34
27	476938	669	979539	66	497399	736	502601	33
28	477340	669	979499	66	497841	735	502159	32
29	477741	668	979459	66	498282	734	501718	31
30	478142	667	979420	66	498722	734	501278	30
31	9.478542	667	9.979380	66	9.499163	733	10.500837	29
32	478942	666	979340	66	499603	733	500397	28
33	479342	665	979300	67	500042	732	499958	27
34	479741	665	979260	67	500481	731	499519	26
35	480140	664	979220	67	500920	731	499080	25
36	480539	663	979180	67	501359	730	498641	24
37	480937	663	979140	67	501797	730	498203	23
38	481334	662	979100	67	502235	729	497765	22
39	481731	661	979059	67	502672	728	497328	21
40	482128	661	979019	67	503109	728	496891	20
41	9.482525	660	9.978979	67	9.503546	727	10.496454	19
42	482921	659	978939	67	503982	727	496018	18
43	483316	659	978898	67	504418	726	495582	17
44	483712	658	978858	67	504854	725	495146	16
45	484107	657	978817	67	505289	725	494711	15
46	484501	657	978777	67	505724	724	494276	14
47	484895	656	978736	67	506159	724	493841	13
48	485289	655	978696	68	506593	723	493407	12
49	485682	655	978655	68	507027	722	492973	11
50	486075	654	978615	68	507460	722	492540	10
51	9.486467	653	9.978574	68	9.507893	721	10.492107	9
52	486860	653	978533	68	508326	721	491674	8
53	487251	652	978493	68	508759	720	491241	7
54	487643	651	978452	68	509191	719	490809	6
55	488034	651	978411	68	509622	719	490378	5
56	488424	650	978370	68	510054	718	489946	4
57	488814	650	978329	68	510485	718	489515	3
58	489204	649	978288	68	510916	717	489084	2
59	489593	648	978247	68	511346	716	488654	1
60	489982	648	978206	68	511776	716	488224	0
	Cosine		Sine		Cotang.		Tang.	M.

72 Degrees.

M.	Sine	D	Cosine	D.	Tang.	D.	Cotang.	
0	9.489982	648	9.978206	68	9.511776	716	10.488224	60
1	490371	648	978165	68	512206	716	487794	59
2	490759	647	978124	68	512635	715	487365	58
3	491147	646	978083	69	513064	714	486936	57
4	491535	646	978042	69	513493	714	486507	56
5	491922	645	978001	69	513921	713	486079	55
6	492308	644	977959	69	514349	713	485651	54
7	492695	644	977918	69	514777	712	485223	53
8	493081	643	977877	69	515204	712	484796	52
9	493466	642	977835	69	515631	711	484369	51
10	493851	642	977794	69	516057	710	483943	50
11	9.494236	641	9.977752	69	9.516484	710	10.483516	49
12	494621	641	977711	69	516910	709	483090	48
13	495005	640	977669	69	517335	709	482665	47
14	495388	639	977628	69	517761	708	482239	46
15	495772	639	977586	69	518185	708	481815	45
16	496154	638	977544	70	518610	707	481390	44
17	496537	637	977503	70	519034	706	480966	43
18	496919	637	977461	70	519458	706	480542	42
19	497301	636	977419	70	519882	705	480118	41
20	497682	636	977377	70	520305	705	479695	40
21	9.498064	635	9.977335	70	9.520728	704	10.479272	39
22	498444	634	977293	70	521151	703	478849	38
23	498825	634	977251	70	521573	703	478427	37
24	499204	633	977209	70	521995	703	478005	36
25	499584	632	977167	70	522417	702	477583	35
26	499963	632	977125	70	522838	702	477162	34
27	500342	631	977083	70	523259	701	476741	33
28	500721	631	977041	70	523680	701	476320	32
29	501099	630	976999	70	524100	700	475900	31
30	501476	629	976957	70	524520	699	475480	30
31	9.501854	629	9.976914	70	9.524939	699	10.475061	29
32	502231	628	976872	71	525359	698	474641	28
33	502607	628	976830	71	525778	698	474222	27
34	502984	627	976787	71	526197	697	473803	26
35	503360	626	976745	71	526615	697	473385	25
36	503735	626	976702	71	527033	696	472967	24
37	504110	625	976660	71	527451	696	472549	23
38	504485	625	976617	71	527868	695	472132	22
39	504860	624	976574	71	528285	695	471715	21
40	505234	623	976532	71	528702	694	471298	20
41	9.505608	623	9.976489	71	9.529119	693	10.470881	19
42	505981	622	976446	71	529535	693	470465	18
43	506354	622	976404	71	529950	693	470050	17
44	506727	621	976361	71	530366	692	469634	16
45	507099	620	976318	71	530781	691	469219	15
46	507471	620	976275	71	531196	691	468804	14
47	507843	619	976232	72	531611	690	468389	13
48	508214	619	976189	72	532025	690	467975	12
49	508585	618	976146	72	532439	689	467561	11
50	508956	618	976103	72	532853	689	467147	10
51	9.509326	617	9.976060	72	9.533266	688	10.466734	9
52	509696	616	976017	72	533679	688	466321	8
53	510065	616	975974	72	534092	687	465908	7
54	510434	615	975930	72	534504	687	465496	6
55	510803	615	975887	72	534916	686	465084	5
56	511172	614	975844	72	535328	686	464672	4
57	511540	613	975800	72	535739	685	464261	3
58	511907	613	975757	72	536150	685	463850	2
59	512275	612	975714	72	536561	684	463439	1
60	512642	612	975670	72	536972	684	463028	0
	Cosine		Sine		Cotang.		**Tang.**	**M.**

71 Degrees.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.512642	612	9.975670	73	9.536972	684	10.463028	60
1	513009	611	975627	73	537382	683	462618	59
2	513375	611	975583	73	537792	683	462208	58
3	513741	610	975539	73	538202	682	461798	57
4	514107	609	975496	73	538611	682	461389	56
5	514472	609	975452	73	539020	681	460980	55
6	514837	608	975408	73	539429	681	460571	54
7	515202	608	975365	73	539837	680	460163	53
8	515566	607	975321	73	540245	680	459755	52
9	515930	607	975277	73	540653	679	459347	51
10	516294	606	975233	73	541061	679	458939	50
11	9.516657	605	9.975189	73	9.541468	678	10.458532	49
12	517020	605	975145	73	541875	678	458125	48
13	517382	604	975101	73	542281	677	457719	47
14	517745	604	975057	73	542688	677	457312	46
15	518107	603	975013	73	543094	676	456906	45
16	518468	603	974969	74	543499	676	456501	44
17	518829	602	974925	74	543905	675	456095	43
18	519190	601	974880	74	544310	675	455690	42
19	519551	601	974836	74	544715	674	455285	41
20	519911	600	974792	74	545119	674	454881	40
21	9.520271	600	9.974748	74	9.545524	673	10.454476	39
22	520631	599	974703	74	545928	673	454072	38
23	520990	599	974659	74	546331	672	453669	37
24	521349	598	974614	74	546735	672	453265	36
25	521707	598	974570	74	547138	671	452862	35
26	522066	597	974525	74	547540	671	452460	34
27	522424	596	974481	74	547943	670	452057	33
28	522781	596	974436	74	548345	670	451655	32
29	523138	595	974391	74	548747	669	451253	31
30	523495	595	974347	75	549149	669	450851	30
31	9.523852	594	9.974302	75	9.549550	668	10.450450	29
32	524208	594	974257	75	549951	668	450049	28
33	524564	593	974212	75	550352	667	449648	27
34	524920	593	974167	75	550752	667	449248	26
35	525275	592	974122	75	551152	666	448848	25
36	525630	591	974077	75	551552	666	448448	24
37	525984	591	974032	75	551952	665	448048	23
38	526339	590	973987	75	552351	665	447649	22
39	526693	590	973942	75	552750	665	447250	21
40	527046	589	973897	75	553149	664	446851	20
41	9.527400	589	9.973852	75	9.553548	664	10.446452	19
42	527753	588	973807	75	553946	663	446054	18
43	528105	588	973761	75	554344	663	445656	17
44	528458	587	973716	76	554741	662	445259	16
45	528810	587	973671	76	555139	662	444861	15
46	529161	586	973625	76	555536	661	444464	14
47	529513	586	973580	76	555933	661	444067	13
48	529864	585	973535	76	556329	660	443671	12
49	530215	585	973489	76	556725	660	443275	11
50	530565	584	973444	76	557121	659	442879	10
51	9.530915	584	9.973398	76	9.557517	659	10.442483	9
52	531265	583	973352	76	557913	659	442087	8
53	531614	582	973307	76	558308	658	441692	7
54	531963	582	973261	76	558702	658	441298	6
55	532312	581	973215	76	559097	657	440903	5
56	532661	581	973169	76	559491	657	440509	4
57	533009	580	973124	76	559885	656	440115	3
58	533357	580	973078	76	560279	656	439721	2
59	533704	579	973032	77	560673	655	439327	1
60	534052	578	972986	77	561066	655	438934	0
	Cosine		Sine		Cotang.		Tang.	M.

70 Degrees.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.534052	578	9.972986	77	9.561066	655	10.438934	60
1	534399	577	972940	77	561459	654	438541	59
2	534745	577	972894	77	561851	654	438149	58
3	535092	577	972848	77	562244	653	437756	57
4	535438	576	972802	77	562636	653	437364	56
5	535783	576	972755	77	563028	653	436972	55
6	536129	575	972709	77	563419	652	436581	54
7	536474	574	972663	77	563811	652	436189	53
8	536818	574	972617	77	564202	651	435798	52
9	537163	573	972570	77	564592	651	435408	51
10	537507	573	972524	77	564983	650	435017	50
11	9.537851	572	9.972478	77	9.565373	650	10.434627	49
12	538194	572	972431	78	565763	649	434237	48
13	838538	571	972385	78	566153	649	433847	47
14	538880	571	972338	78	566542	649	433458	46
15	539223	570	972291	78	566932	648	433068	45
16	539565	570	972245	78	567320	648	432680	44
17	539907	569	972198	78	567709	647	432291	43
18	540249	569	972151	78	568098	647	431902	42
19	540590	568	972105	78	568486	646	431514	41
20	540931	568	972058	78	568873	646	431127	40
21	9.541272	567	9.972011	78	9.569261	645	10.430739	39
22	541613	567	971964	78	569648	645	430352	38
23	541953	566	971917	78	570035	645	429965	37
24	542293	566	971870	78	570422	644	429578	36
25	542632	565	971823	78	570809	644	429191	35
26	542971	565	971776	78	571195	643	428805	34
27	543310	564	971729	79	571581	643	428419	33
28	543649	564	971682	79	571967	642	428033	32
29	543987	563	971635	79	572352	642	427648	31
30	544325	563	971588	79	572738	642	427262	30
31	9.544663	562	9.971540	79	9.573123	641	10.426877	29
32	545000	562	971493	79	573507	641	426493	28
33	545338	561	971446	79	573892	640	426108	27
34	545674	561	971398	79	574276	640	425724	26
35	546011	560	971351	79	574660	639	425340	25
36	546347	560	971303	79	575044	639	424956	24
37	546683	559	971256	79	575427	639	424573	23
38	547019	559	971208	79	575810	638	424190	22
39	547354	558	971161	79	576193	638	423807	21
40	547689	558	971113	79	576576	637	423424	20
41	9.548024	557	9.971066	80	9.576958	637	10.423041	19
42	548359	557	971018	80	577341	636	422659	18
43	548693	556	970970	80	577723	636	422277	17
44	549027	556	970922	80	578104	636	421896	16
45	549360	555	970874	80	578486	635	421514	15
46	549693	555	970827	80	578867	635	421133	14
47	550026	554	970779	80	579248	634	420752	13
48	550359	554	970731	80	579629	634	420371	12
49	550692	553	970683	80	580009	634	419991	11
50	551024	553	970635	80	580389	633	419611	10
51	9 551356	552	9.970586	80	9.580769	633	10.419231	9
52	551687	552	970538	80	581149	632	418851	8
53	552018	552	970490	80	581528	632	418472	7
54	552349	551	970442	80	581907	632	418093	6
55	552680	551	970394	80	582286	631	417714	5
56	553010	550	970345	81	582665	631	417335	4
57	553341	550	970297	81	583043	630	416957	3
58	553670	549	970249	81	583422	630	416578	2
59	554000	549	970200	81	583800	629	416200	1
60	554329	548	970152	81	584177	629	415823	0
	Cosine		Sine		Cotang.		Tang.	M.

69 Degrees.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.554329	548	9.970152	81	9.584177	629	10.415823	60
1	554658	548	970103	81	584555	629	415445	59
2	554987	547	970055	81	584932	628	415068	58
3	555315	547	970006	81	585309	628	414691	57
4	555643	546	969957	81	585686	627	414314	56
5	555971	546	969909	81	586062	627	413938	55
6	556299	545	969860	81	586439	627	413561	54
7	556626	545	969811	81	586815	626	413185	53
8	556953	544	969762	81	587190	626	412810	52
9	557280	544	969714	81	587566	625	412434	51
10	557606	543	969665	81	587941	625	412059	50
11	9.557932	543	9.969616	82	9.588316	625	10.411684	49
12	558258	543	969567	82	588691	624	411309	48
13	558583	542	969518	82	589066	624	410934	47
14	558909	542	969469	82	589440	623	410560	46
15	559234	541	969420	82	589814	623	410186	45
16	559558	541	969370	82	590188	623	409812	44
17	559883	540	969321	82	590562	622	409438	43
18	560207	540	969272	82	590935	622	409065	42
19	560531	539	969223	82	591308	622	408692	41
20	560855	539	969173	82	591681	621	408319	40
21	9.561178	538	9.969124	82	9.592054	621	10.407946	39
22	561501	538	969075	82	592426	620	407574	38
23	561824	537	969025	82	592798	620	407202	37
24	562146	537	968976	82	593170	619	406829	36
25	562468	536	968926	83	593542	619	406458	35
26	562790	536	968877	83	593914	618	406086	34
27	563112	536	968827	83	594285	618	405715	33
28	563433	535	968777	83	594656	618	405344	32
29	563755	535	968728	83	595027	617	404973	31
30	564075	534	968678	83	595398	617	404602	30
31	9.564396	534	9.968628	83	9.595768	617	10.404232	29
32	564716	533	968578	83	596138	616	403862	28
33	565036	533	968528	83	596508	616	403492	27
34	565356	532	968479	83	596878	616	403122	26
35	565676	532	968429	83	597247	615	402753	25
36	565995	531	968379	83	597616	615	402384	24
37	566314	531	968329	83	597985	615	402015	23
38	566632	531	968278	83	598354	614	401646	22
39	566951	530	968228	84	598722	614	401278	21
40	567269	530	968178	84	599091	613	400909	20
41	9.567587	529	9.968128	84	9.599459	613	10.400541	19
42	567904	529	968078	84	599827	613	400173	18
43	568222	528	968027	84	600194	612	399806	17
44	568539	528	967977	84	600562	612	399438	16
45	568856	528	967927	84	600929	611	399071	15
46	569172	527	967876	84	601296	611	398704	14
47	569488	527	967826	84	601662	611	398338	13
48	569804	526	967775	84	602029	610	397971	12
49	570120	526	967725	84	602395	610	397605	11
50	570435	525	967674	84	602761	610	397239	10
51	9.570751	525	9.967624	84	9.603127	609	10.396873	9
52	571066	524	967573	84	603493	609	396507	8
53	571380	524	967522	85	603858	609	396142	7
54	571695	523	967471	85	604223	608	395777	6
55	572009	523	967421	85	604588	608	395412	5
56	572323	523	967370	85	604953	607	395047	4
57	572636	522	967319	85	605317	607	394683	3
58	572950	522	967268	85	605682	607	394318	2
59	573263	521	967217	85	606046	606	393954	1
60	573575	521	967166	85	606410	606	393590	0
	Cosine		Sine		Cotang.		Tang.	M.

68 Degrees.

(22 Degrees.)

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang	
0	9.573575	521	9.967166	85	9.606410	606	10.393590	60
1	573888	520	967115	85	606773	606	393227	59
2	574200	520	967064	85	607137	605	392863	58
3	574512	519	967013	85	607500	605	392500	57
4	574824	519	966961	85	607863	604	392137	56
5	575136	519	966910	85	608225	604	391775	55
6	575447	518	966859	85	608588	604	391412	54
7	575758	518	966808	85	608950	603	391050	53
8	576069	517	966756	86	609312	603	390688	52
9	576379	517	966705	86	609674	603	390326	51
10	576689	516	966653	86	610036	602	389964	50
11	9.576999	516	9.966602	86	9.610397	602	10.389603	49
12	577309	516	966550	86	610759	602	389241	48
13	577618	515	966499	86	611120	601	388880	47
14	577927	515	966447	86	611480	601	388520	46
15	578236	514	966395	86	611841	601	388159	45
16	578545	514	966344	86	612201	600	387799	44
17	578853	513	966292	86	612561	600	387439	43
18	579162	513	966240	86	612921	600	387079	42
19	579470	513	966188	86	613281	599	386719	41
20	579777	512	966136	86	613641	599	386359	40
21	9.580085	512	9.966085	87	9.614000	598	10.386000	39
22	580392	511	966033	87	614359	598	385641	38
23	580699	511	965981	87	614718	598	385282	37
24	581005	511	965928	87	615077	597	384923	36
25	581312	510	965876	87	615435	597	384565	35
26	581618	510	965824	87	615793	597	384207	34
27	581924	509	965772	87	616151	596	383849	33
28	582229	509	965720	87	616509	596	383491	32
29	582535	509	965668	87	616867	596	383133	31
30	582840	508	965615	87	617224	595	382776	30
31	9.583145	508	9.965563	87	9.617582	595	10.382418	29
32	583449	507	965511	87	617939	595	382061	28
33	583754	507	965458	87	618295	594	381705	27
34	584058	506	965406	87	618652	594	381348	26
35	584361	506	965353	88	619008	594	380992	25
36	584665	506	965301	88	619364	593	380636	24
37	584968	505	965248	88	619721	593	380279	23
38	585272	505	965195	88	620076	593	379924	22
39	585574	504	965143	88	620432	592	379568	21
40	585877	504	965090	88	620787	592	379213	20
41	9.586179	503	9.965037	88	9.621142	592	10.378858	19
42	586482	503	964984	88	621497	591	378503	18
43	586783	503	964931	88	621852	591	378148	17
44	587085	502	964879	88	622207	590	377793	16
45	587386	502	964826	88	622561	590	377439	15
46	587688	501	964773	88	622915	590	377085	14
47	587989	501	964719	88	623269	589	376731	13
48	588289	501	964666	89	623623	589	376377	12
49	588590	500	964613	89	623976	589	376024	11
50	588890	500	964560	89	624330	588	375670	10
51	9.589190	499	9.964507	89	9.624683	588	10.375317	9
52	589489	499	964454	89	625036	588	374964	8
53	589789	499	964400	89	625388	587	374612	7
54	590088	498	964347	89	625741	587	374259	6
55	590387	498	964294	89	626093	587	373907	5
56	590686	497	964240	89	626445	586	373555	4
57	590984	497	964187	89	626797	586	373203	3
58	591282	497	964133	89	627149	586	372851	2
59	591580	496	964080	89	627501	585	372499	1
60	591878	496	964026	89	627852	585	372148	0
	Cosine		Sine		Cotang.		Tang.	M.

67 Degrees.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.591878	496	9.964026	89	9.627852	585	10.372148	60
1	592176	495	963972	89	628203	585	371797	59
2	592473	495	963919	89	628554	585	371446	58
3	592770	495	963865	90	628905	584	371095	57
4	593067	494	963811	90	629255	584	370745	56
5	593363	494	963757	90	629606	583	370394	55
6	593659	493	963704	90	629956	583	370044	54
7	593955	493	963650	90	630306	583	369694	53
8	594251	493	963596	90	630656	583	369344	52
9	594547	492	963542	90	631005	582	368995	51
10	594842	492	963488	90	631355	582	368645	50
11	9.595137	491	9.963434	90	9.631704	582	10.368296	49
12	595432	491	963379	90	632053	581	367947	48
13	595727	491	963325	90	632401	581	367599	47
14	596021	490	963271	90	632750	581	367250	46
15	596315	490	963217	90	633098	580	366902	45
16	596609	489	963163	90	633447	580	366553	44
17	596903	489	963108	91	633795	580	366205	43
18	597196	489	963054	91	634143	579	365857	42
19	597490	488	962999	91	634490	579	365510	41
20	597783	488	962945	91	634838	579	365162	40
21	9.598075	487	9.962890	91	9.635185	578	10.364815	39
22	598368	487	962836	91	635532	578	364468	38
23	598660	487	962781	91	635879	578	364121	37
24	598952	486	962727	91	636226	577	363774	36
25	599244	486	962672	91	636572	577	363428	35
26	599536	485	962617	91	636919	577	363081	34
27	599827	485	962562	91	637265	577	362735	33
28	600118	485	962508	91	637611	576	362389	32
29	600409	484	962453	91	637956	576	362044	31
30	600700	484	962398	92	638302	576	361698	30
31	9.600990	484	9.962343	92	9.638647	575	10.361353	29
32	601280	483	962288	92	638992	575	361008	28
33	601570	483	962233	92	639337	575	360663	27
34	601860	482	962178	92	639682	574	360318	26
35	602150	482	962123	92	640027	574	359973	25
36	602439	482	962067	92	640371	574	359629	24
37	602728	481	962012	92	640716	573	359284	23
38	603017	481	961957	92	641060	573	358940	22
39	603305	481	961902	92	641404	573	358596	21
40	603594	480	961846	92	641747	572	358253	20
41	9.603882	480	9.961791	92	9.642091	572	10.357909	19
42	604170	479	961735	92	642434	572	357566	18
43	604457	479	961680	92	642777	572	357223	17
44	604745	479	961624	93	643120	571	356880	16
45	605032	478	961569	93	643463	571	356537	15
46	605319	478	961513	93	643806	571	356194	14
47	605606	478	961458	93	644148	570	355852	13
48	605892	477	961402	93	644490	570	355510	12
49	606179	477	961346	93	644832	570	355168	11
50	606465	476	961290	93	645174	569	354826	10
51	9.606751	476	9.961235	93	9.645516	569	10.354484	9
52	607036	476	961179	93	645857	569	354143	8
53	607322	475	961123	93	646199	569	353801	7
54	607607	475	961067	93	646540	568	353460	6
55	607892	474	961011	93	646881	568	353119	5
56	608177	474	960955	93	647222	568	352778	4
57	608461	474	960899	93	647562	567	352438	3
58	608745	473	960843	94	647903	567	352097	2
59	609029	473	960786	94	648243	567	351757	1
60	609313	473	960730	94	648583	566	351417	0
	Cosine		Sine		Cotang.		Tang.	M.

66 Degrees.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.609313	473	9.960730	94	9.648583	566	10.351417	60
1	609597	472	960674	94	648923	566	351077	59
2	609880	472	960618	94	649263	566	350737	58
3	610164	472	960561	94	649602	566	350398	57
4	610447	471	960505	94	649942	565	350058	56
5	610729	471	960448	94	650281	565	349719	55
6	611012	470	960392	94	650620	565	349380	54
7	611294	470	960335	94	650959	564	349041	53
8	611576	470	960279	94	651297	564	648703	52
9	611858	469	960222	94	651636	564	348364	51
10	612140	469	960165	94	651974	563	348026	50
11	9.612421	469	9.960109	95	9.652312	563	10.347688	49
12	612702	468	960052	95	652650	563	347350	48
13	612983	468	959995	95	652988	563	347012	47
14	613264	467	959938	95	653326	562	346674	46
15	613545	467	959882	95	653663	562	346337	45
16	613825	467	959825	95	654000	562	346000	44
17	614105	466	959768	95	654337	561	345663	43
18	614385	466	959711	95	654674	561	345326	42
19	614665	466	959654	95	655011	561	344989	41
20	614944	465	959596	95	655348	561	344652	40
21	9.615223	465	9.959539	95	9.655684	560	10.344316	39
22	615502	465	959482	95	656020	560	343980	38
23	615781	464	959425	95	656356	560	343644	37
24	616060	464	959368	95	656692	559	343308	36
25	616338	464	959310	96	657028	559	342972	35
26	616616	463	959253	96	657364	559	342636	34
27	616894	463	959195	96	657699	559	342301	33
28	617172	462	959138	96	658034	558	341966	32
29	617450	462	959081	96	658369	558	341631	31
30	617727	462	959023	96	658704	558	341296	30
31	9.618004	461	9.958965	96	9.659039	558	10.340961	29
32	618281	461	958908	96	659373	557	340627	28
33	618558	461	958850	96	659708	557	340292	27
34	618834	460	958792	96	660042	557	339958	26
35	619110	460	958734	96	660376	557	339624	25
36	619386	460	958677	96	660710	556	339290	24
37	619662	459	958619	96	661043	556	338957	23
38	619938	459	958561	96	661377	556	338623	22
39	620213	459	958503	97	661710	555	338290	21
40	620488	458	958445	97	662043	555	337957	20
41	9.620763	458	9.958387	97	9.662376	555	10.337624	19
42	621038	457	958329	97	662709	554	337291	18
43	621313	457	958271	97	663042	554	336958	17
44	621587	457	958213	97	663375	554	336625	16
45	621861	456	958154	97	663707	554	336293	15
46	622135	456	958096	97	664039	553	335961	14
47	622409	456	958038	97	664371	553	335629	13
48	622682	455	957979	97	664703	553	335297	12
49	622956	455	957921	97	665035	553	334965	11
50	623229	455	957863	97	665366	552	334634	10
51	9.623502	454	9.957804	97	9.665697	552	10.334303	9
52	623774	454	957746	98	666029	552	333971	8
53	624047	454	957687	98	666360	551	333640	7
54	624319	453	957628	98	666691	551	333309	6
55	624591	453	957570	98	667021	551	332979	5
56	624863	453	957511	98	667352	551	332648	4
57	625135	452	957452	98	667682	550	332318	3
58	625406	452	957393	98	668013	550	331987	2
59	625677	452	957335	98	668343	550	331657	1
60	625948	451	957276	98	668672	550	331328	0
	Cosine		Sine		Cotang.		Tang.	M.

65 Degrees.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.625948	451	9.957276	98	9.668673	550	10.331327	60
1	626219	451	957217	98	669002	549	330998	59
2	626490	451	957158	98	669332	549	330668	58
3	626760	450	957099	98	669661	549	330339	57
4	627030	450	957040	98	669991	548	330009	56
5	627300	450	956981	98	670320	548	329680	55
6	627570	449	956921	99	670649	548	329351	54
7	627840	449	956862	99	670977	548	329023	53
8	628109	449	956803	99	671306	547	328694	52
9	628378	448	956744	99	671634	547	328366	51
10	628647	448	956684	99	671963	547	328037	50
11	9.628916	447	9.956625	99	9.672291	547	10.327709	49
12	629185	447	956566	99	672619	546	327381	48
13	629453	447	956506	99	672947	546	327053	47
14	629721	446	956447	99	673274	546	326726	46
15	629989	446	956387	99	673602	546	326398	45
16	630257	446	956327	99	673929	545	326071	44
17	630524	446	956268	99	674257	545	325743	43
18	630792	445	956208	100	674584	545	325416	42
19	631059	445	956148	100	674910	544	325090	41
20	631326	445	956089	100	675237	544	324763	40
21	9.631593	444	9.956029	100	9.675564	544	10.324436	39
22	631859	444	955969	100	675890	544	324110	38
23	632125	444	955909	100	676216	543	323784	37
24	632392	443	955849	100	676543	543	323457	36
25	632658	443	955789	100	676869	543	323131	35
26	632923	443	955729	100	677194	543	322806	34
27	633189	442	955669	100	677520	542	322480	33
28	633454	442	955609	100	677846	542	322154	32
29	633719	442	955548	100	678171	542	321829	31
30	633984	441	955488	100	678496	542	321504	30
31	9.634249	441	9.955428	101	9.678821	541	10.321179	29
32	634514	440	955368	101	679146	541	320854	28
33	634778	440	955307	101	679471	541	320529	27
34	635042	440	955247	101	679795	541	320205	26
35	635306	439	955186	101	680120	540	319880	25
36	635570	439	955126	101	680444	540	319556	24
37	635834	439	955065	101	680768	540	319232	23
38	636097	438	955005	101	681092	540	318908	22
39	636360	438	954944	101	681416	539	318584	21
40	636623	438	954883	101	681740	539	318260	20
41	9.636886	437	9.954823	101	9.682063	539	10.317937	19
42	637148	437	954762	101	682387	539	317613	18
43	637411	437	954701	101	682710	538	317290	17
44	637673	437	954640	101	683033	538	316967	16
45	637935	436	954579	101	683356	538	316644	15
46	638197	436	954518	102	683679	538	316321	14
47	638458	436	954457	102	684001	537	315999	13
48	638720	435	954396	102	684324	537	315676	12
49	638981	435	954335	102	684646	537	315354	11
50	639242	435	954274	102	684968	537	315032	10
51	9.639503	434	9.954213	102	9.685290	536	10.314710	9
52	639764	434	954152	102	685672	536	314388	8
53	640024	434	954090	102	685934	536	314066	7
54	640284	433	954029	102	686255	536	313745	6
55	640544	433	953968	102	686577	535	313423	5
56	640804	433	953906	102	686898	535	313102	4
57	641064	432	953845	102	687219	535	312781	3
58	641324	432	953783	102	687540	535	312460	2
59	641584	432	953722	103	687861	534	312139	1
60	641842	431	953660	103	688182	534	311818	0
	Cosine		Sine		Cotang.		Tang.	M.

64 Degrees.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.641842	431	9.953660	103	9.688182	534	10.311818	60
1	642101	431	953599	103	688502	534	311498	59
2	642360	431	953537	103	688823	534	311177	58
3	642618	430	953475	103	689143	533	310857	57
4	642877	430	953413	103	689463	533	310537	56
5	643135	430	953352	103	689783	533	310217	55
6	643393	430	953290	103	690103	533	309897	54
7	643650	429	953228	103	690423	533	309577	53
8	643908	429	953166	103	690742	532	309258	52
9	644165	429	953104	103	691062	532	308938	51
10	644423	428	953042	103	691381	532	308619	50
11	9.644680	428	9.952980	104	9.691700	531	10.308300	49
12	644936	428	952918	104	692019	531	307981	48
13	645193	427	952855	104	692338	531	307662	47
14	645450	427	952793	104	692656	531	307344	46
15	645706	427	952731	104	692975	531	307025	45
16	645962	426	952669	104	693293	530	306707	44
17	646218	426	952606	104	693612	530	306388	43
18	646474	426	952544	104	693930	530	306070	42
19	646729	425	952481	104	694248	530	305752	41
20	646984	425	952419	104	694566	529	305434	40
21	9.647240	425	9.952356	104	9.694883	529	10.305117	39
22	647494	424	952294	104	695201	529	304799	38
23	647749	424	952231	104	695518	529	304482	37
24	648004	424	952168	105	695836	529	304164	36
25	648258	424	952106	105	696153	528	303847	35
26	648512	423	952043	105	696470	528	303530	34
27	648766	423	951980	105	696787	528	303213	33
28	649020	423	951917	105	697103	528	302897	32
29	649274	422	951854	105	697420	527	302580	31
30	649527	422	951791	105	697736	527	302264	30
31	9.649781	422	9.951728	105	9.698053	527	10.301947	29
32	650034	422	951665	105	698369	527	301631	28
33	650287	421	951602	105	698685	526	301315	27
34	650539	421	951539	105	699001	526	300999	26
35	650792	421	951476	105	699316	526	300684	25
36	651044	420	951412	105	699632	526	300368	24
37	651297	420	951349	106	699947	526	300053	23
38	651549	420	951286	106	700263	525	299737	22
39	651800	419	951222	106	700578	525	299422	21
40	652052	419	951159	106	700893	525	299107	20
41	9.652304	419	9.951096	106	9.701208	524	10.298792	19
42	652555	418	951032	106	701523	524	298477	18
43	652806	418	950968	106	701837	524	298163	17
44	653057	418	950905	106	702152	524	297848	16
45	653308	418	950841	106	702466	524	297534	15
46	653558	417	950778	106	702780	523	297220	14
47	653808	417	950714	106	703095	523	296905	13
48	654059	417	950650	106	703409	523	296591	12
49	654309	416	950586	106	703723	523	296277	11
50	654558	416	950522	107	704036	522	295964	10
51	9.654808	416	9.950458	107	9.704350	522	10.295650	9
52	655058	416	950394	107	704663	522	295337	8
53	655307	415	950330	107	704977	522	295023	7
54	655556	415	950266	107	705290	522	294710	6
55	655805	415	950202	107	705603	521	294397	5
56	656054	414	950138	107	705916	521	294084	4
57	656302	414	950074	107	706228	521	293772	3
58	656551	414	950010	107	706541	521	293459	2
59	656799	413	949945	107	706854	521	293146	1
60	657047	413	949881	107	707166	520	292834	0
	Cosine		Sine		Cotang.		Tang.	M.

63 Degrees.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.657047	413	9.949881	107	9.707166	520	10.292834	60
1	657295	413	949816	107	707478	520	292522	59
2	657542	412	949752	107	707790	520	292210	58
3	657790	412	949688	108	708102	520	291898	57
4	658037	412	949623	108	708414	519	291586	56
5	658284	412	949558	108	708726	519	291274	55
6	658531	411	949494	108	709037	519	290963	54
7	658778	411	949429	108	709349	519	290651	53
8	659025	411	949364	108	709660	519	290340	52
9	659271	410	949300	108	709971	518	290029	51
10	659517	410	949235	108	710282	518	289718	50
11	9.659763	410	9.949170	108	9.710593	518	10.289407	49
12	660009	409	949105	108	710904	518	289096	48
13	660255	409	949040	108	711215	818	288785	47
14	660501	409	948975	108	711525	517	288475	46
15	660746	409	948910	108	711836	517	288164	45
16	660991	408	948845	108	712146	517	287854	44
17	661236	408	948780	109	712456	517	287544	43
18	661481	408	948715	109	712766	516	287234	42
19	661726	407	948650	109	713076	516	286924	41
20	661970	407	948584	109	713386	516	286614	40
21	9.662214	407	9.948519	109	9.713696	516	10.286304	39
22	662459	407	948454	109	714005	516	285995	38
23	662703	406	948388	109	714314	515	285686	37
24	662946	406	948323	109	714624	515	285376	36
25	663190	406	948257	109	714933	515	285067	35
26	663433	405	948192	109	715242	515	284758	34
27	663677	405	948126	109	715551	514	284449	33
28	663920	405	948060	109	715860	514	284140	32
29	664163	405	947995	110	716168	514	283832	31
30	664406	404	947929	110	716477	514	283523	30
31	9.664648	404	9.947863	110	9.716785	514	10.283215	29
32	664891	404	947797	110	717093	513	282907	28
33	665133	403	947731	110	717401	513	282599	27
34	665375	403	947665	110	717709	513	282291	26
35	665617	403	947600	110	718017	513	281983	25
36	665859	402	947533	110	718325	513	281675	24
37	666100	402	947467	110	718633	512	281367	23
38	666342	402	947401	110	718940	512	281060	22
39	666583	402	947335	110	719248	512	280752	21
40	666824	401	947269	110	719555	512	280445	20
41	9.667065	401	9.947203	110	9.719862	512	10.280138	19
42	667305	401	947136	111	720169	511	279831	18
43	667546	401	947070	111	720476	511	279524	17
44	667786	400	947004	111	720783	511	279217	16
45	668027	400	946937	111	721089	511	278911	15
46	668267	400	946871	111	721396	511	278604	14
47	668506	399	946804	111	721702	510	278298	13
48	668746	399	946738	111	722009	510	277991	12
49	668986	399	946671	111	722315	510	277685	11
50	669225	399	946604	111	722621	510	277379	10
51	9.669464	398	9.946538	111	9.722927	510	10.277073	9
52	669703	398	946471	111	723232	509	276768	8
53	669942	398	946404	111	723538	509	276462	7
54	670181	397	946337	111	723844	509	276156	6
55	670419	397	946270	112	724149	509	275851	5
56	670658	397	946203	112	724454	509	275546	4
57	670896	397	946136	112	724759	508	275241	3
58	671134	396	946069	112	725065	508	274935	2
59	671372	396	946002	112	725369	508	274631	1
60	671609	396	945935	112	725674	508	274326	0
	Cosine		Sine		Cotang.		Tang.	M.

62 Degrees.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.671609	396	9.945935	112	9.725674	508	10.274326	60
1	671847	395	945868	112	725979	508	274021	59
2	672084	395	945800	112	726284	507	273716	58
3	672321	395	945733	112	726588	507	273412	57
4	672558	395	945666	112	726892	507	273108	56
5	672795	394	945598	112	727197	507	272803	55
6	673032	394	945531	112	727501	507	272499	54
7	673268	394	945464	113	727805	506	272195	53
8	673505	394	945396	113	728109	506	271891	52
9	673741	393	945328	113	728412	506	271588	51
10	673977	393	945261	113	728716	506	271284	50
11	9.674213	393	9.945193	113	9.729020	506	10.270980	49
12	674448	392	945125	113	729323	505	270677	48
13	674684	392	945058	113	729626	505	270374	47
14	674919	392	944990	113	729929	505	270071	46
15	675155	392	944922	113	730233	505	269767	45
16	675390	391	944854	113	730535	505	269465	44
17	675624	391	944786	113	730838	504	269162	43
18	675859	391	944718	113	731141	504	268859	42
19	676094	391	944650	113	731444	504	268556	41
20	676328	390	944582	114	731746	504	268254	40
21	9.676562	390	9.944514	114	9.732048	504	10.267952	39
22	676796	390	944446	114	732351	503	267649	38
23	677030	390	944377	114	732653	503	267347	37
24	677264	389	944309	114	732955	503	267045	36
25	677498	389	944241	114	733257	503	266743	35
26	677731	389	944172	114	733558	503	266442	34
27	677964	388	944104	114	733860	502	266140	33
28	678197	388	944036	114	734162	502	265838	32
29	678430	388	943967	114	734463	502	265537	31
30	678663	388	943899	114	734764	502	265236	30
31	9.678895	387	9.943830	114	9.735066	502	10.264934	29
32	679128	387	943761	114	735367	502	264633	28
33	679360	387	943693	115	735668	501	264332	27
34	679592	387	943624	115	735969	501	264031	26
35	679824	386	943555	115	736269	501	263731	25
36	680056	386	943486	115	736570	501	263430	24
37	680288	386	943417	115	736871	501	263129	23
38	680519	385	943348	115	737171	500	262829	22
39	680750	385	943279	115	737471	500	262529	21
40	680982	385	943210	115	737771	500	262229	20
41	9.681213	385	9.943141	115	9.738071	500	10.261929	19
42	681443	384	943072	115	738371	500	261629	18
43	681674	384	943003	115	738671	499	261329	17
44	681905	384	942934	115	738971	499	261029	16
45	682135	384	942864	115	739271	499	260729	15
46	682365	383	942795	116	739570	499	260430	14
47	682595	383	942726	116	739870	499	260130	13
48	682825	383	942656	116	740169	499	259831	12
49	683055	383	942587	116	740468	498	259532	11
50	683284	382	942517	116	740767	498	259233	10
51	9.683514	382	9.942448	116	9.741066	498	10.258934	9
52	683743	382	942378	116	741365	498	258635	8
53	683972	382	942308	116	741664	498	258336	7
54	684201	381	942239	116	741962	497	258038	6
55	684430	381	942169	116	742261	497	257739	5
56	684658	381	942099	116	742559	497	257441	4
57	684887	380	942029	116	742858	497	257142	3
58	685115	380	941959	116	743156	497	256844	2
59	685343	380	941889	117	743454	497	256546	1
60	685571	380	941819	117	743752	496	256248	0
	Cosine		Sine		Cotang.		Tang.	M.

61 Degrees.

(29 Degrees.)

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.685571	380	9.941819	117	9.743752	496	10.256248	60
1	685799	379	941749	117	744050	496	255950	59
2	686027	379	941679	117	744348	496	255652	58
3	686254	379	941609	117	744645	496	255355	57
4	686482	379	941539	117	744943	496	255057	56
5	686709	378	941469	117	745240	496	254760	55
6	686936	378	941398	117	745538	495	254462	54
7	687163	378	941328	117	745835	495	254165	53
8	687389	378	941258	117	746132	495	253868	52
9	687616	377	941187	117	746429	495	253571	51
10	687843	377	941117	117	746726	495	253274	50
11	9.688069	377	9.941046	118	9.747023	494	10.252977	49
12	688295	377	940975	118	747319	494	252681	48
13	688521	376	940905	118	747616	494	252384	47
14	688747	376	940834	118	747913	494	252087	46
15	688972	376	940763	118	748209	494	251791	45
16	689198	376	940693	118	748505	493	251495	44
17	689423	375	940622	118	748801	493	251199	43
18	689648	375	940551	118	749097	493	250903	42
19	689873	375	940480	118	749393	493	250607	41
20	690098	375	940409	118	749689	493	250311	40
21	9.690323	374	9.940338	118	9.749985	493	10.250015	39
22	690548	374	940267	118	750281	492	249719	38
23	690772	374	940196	118	750576	492	249424	37
24	690996	374	940125	119	750872	492	249128	36
25	691220	373	940054	119	751167	492	248833	35
26	691444	373	939982	119	751462	492	248538	34
27	691668	373	939911	119	751757	492	248243	33
28	691892	373	939840	119	752052	491	247948	32
29	692115	372	939768	119	752347	491	247653	31
30	692339	372	939697	119	752642	491	247358	30
31	9.692562	372	9.939625	119	9.752937	491	10.247063	29
32	692785	371	939554	119	753231	491	246769	28
33	693008	371	939482	119	753526	491	246474	27
34	693231	371	939410	119	753820	490	246180	26
35	693453	371	939339	119	754115	490	245885	25
36	693676	370	939267	120	754409	490	245591	24
37	693898	370	939195	120	754703	490	245297	23
38	694120	370	939123	120	754997	490	245003	22
39	694342	370	939052	120	755291	490	244709	21
40	694564	369	938980	120	755585	489	244415	20
41	9.694786	369	9.938908	120	9.755878	489	10.244122	19
42	695007	369	938836	120	756172	489	243828	18
43	695229	369	938763	120	756465	489	243535	17
44	695450	368	938691	120	756759	489	243241	16
45	695671	368	938619	120	757052	489	242948	15
46	695892	368	938547	120	757345	488	242655	14
47	696113	368	638475	120	757638	488	242362	13
48	696334	367	938402	121	757931	488	242069	12
49	696554	367	938330	121	758224	488	241776	11
50	696775	367	938258	121	758517	488	241483	10
51	9.696995	367	9.938185	121	9.758810	488	10.241190	9
52	697215	366	938113	121	759102	487	240898	8
53	697435	366	938040	121	759395	487	240605	7
54	697654	366	937967	121	759687	487	240313	6
55	697874	366	937895	121	759979	487	240021	5
56	698094	365	937822	121	760272	487	239728	4
57	698313	365	937749	121	760564	487	239436	3
58	698532	365	937676	121	760856	486	239144	2
59	698751	365	937604	121	761148	486	238852	1
60	698970	364	937531	121	761439	486	238561	0
	Cosine		Sine		Cotang.		Tang.	M.

60 Degrees.

M.	Sine	D	Cosine	D.	Tang.	D.	Cotang.	
0	9.698970	364	9.937531	121	9.761439	486	10.238561	60
1	699189	364	937458	122	761731	486	238269	59
2	699407	364	937385	122	762023	486	237977	58
3	699626	364	937312	122	762314	486	237686	57
4	699844	363	937238	122	762606	485	237394	56
5	700062	363	937165	122	762897	485	237103	55
6	700280	363	937092	122	763188	485	236812	54
7	700498	363	937019	122	763479	485	236521	53
8	700716	363	936946	122	763770	485	236230	52
9	700933	362	936872	122	764061	485	235939	51
10	701151	362	936799	122	764352	484	235648	50
11	9.701368	362	9.936725	122	9.764643	484	10.235357	49
12	701585	362	936652	123	764933	484	235067	48
13	701802	361	936578	123	765224	484	234776	47
14	702019	361	936505	123	765514	484	234486	46
15	702236	361	936431	123	765805	484	234195	45
16	702452	361	936357	123	766095	484	233905	44
17	702669	360	936284	123	766385	483	233615	43
18	702885	360	936210	123	766675	483	233325	42
19	703101	360	936136	123	766965	483	233035	41
20	703317	360	936062	123	767255	483	232745	40
21	9.703533	359	9.935988	123	9.767545	483	10.232455	39
22	703749	359	935914	123	767834	483	232166	38
23	703964	359	935840	123	768124	482	231876	37
24	704179	359	935766	124	768413	482	231587	36
25	704395	359	935692	124	768703	482	231297	35
26	704610	358	935618	124	768992	482	231008	34
27	704825	358	935543	124	769281	482	230719	33
28	705040	358	935469	124	769570	482	230430	32
29	705254	358	935395	124	769860	481	230140	31
30	705469	357	935320	124	770148	481	229852	30
31	9.705683	357	9.935246	124	9.770437	481	10.229563	29
32	705898	357	935171	124	770726	481	229274	28
33	706112	357	935097	124	771015	481	228985	27
34	706326	356	935022	124	771303	481	228697	26
35	706539	356	934948	124	771592	481	228408	25
36	706753	356	934873	124	771880	480	228120	24
37	706967	356	934798	125	772168	480	227832	23
38	707180	355	934723	125	772457	480	227543	22
39	707393	355	934649	125	772745	480	227255	21
40	707606	355	934574	125	773033	480	226967	20
41	9.707819	355	9.934499	125	9.773321	480	10.226679	19
42	708032	354	934424	125	773608	479	226392	18
43	708245	354	934349	125	773896	479	226104	17
44	708458	354	934274	125	774184	479	225816	16
45	708670	354	934199	125	774471	479	225529	15
46	708882	353	934123	125	774759	479	225241	14
47	709094	353	934048	125	775046	479	224954	13
48	709306	353	933973	125	775333	479	224667	12
49	709518	353	933898	126	775621	478	224379	11
50	709730	353	933822	126	775908	478	224092	10
51	9.709941	352	9.933747	126	9.776195	478	10.223805	9
52	710153	352	933671	126	776482	478	223518	8
53	710364	352	933596	126	776769	478	223231	7
54	710575	352	933520	126	777055	478	222945	6
55	710786	351	933445	126	777342	478	222658	5
56	710997	351	933369	126	777628	477	222372	4
57	711208	351	933293	126	777915	477	222085	3
58	711419	351	933217	126	778201	477	221799	2
59	711629	350	933141	126	778487	477	221512	1
60	711839	350	933066	126	778774	477	221226	0
	Cosine		Sine		Cotang.		Tang.	M.

59 Degrees.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.711839	350	9.933066	126	9.778774	477	10.221226	60
1	712050	350	932990	127	779060	477	220940	59
2	712260	350	932914	127	779346	476	220654	58
3	712469	349	932838	127	779632	476	220368	57
4	712679	349	932762	127	779918	476	220082	56
5	712889	349	932685	127	780203	476	219797	55
6	713098	349	932609	127	780489	476	219511	54
7	713308	349	932533	127	780775	476	219225	53
8	713517	348	932457	127	781060	476	218940	52
9	713726	348	932380	127	781346	475	218654	51
10	713935	348	932304	127	781631	475	218369	50
11	9.714144	348	9.932228	127	9.781916	475	10.218084	49
12	714352	347	932151	127	782201	475	217799	48
13	714561	347	932075	128	782486	475	217514	47
14	714769	347	931998	128	782771	475	217229	46
15	714978	347	931921	128	783056	475	216944	45
16	715186	347	931845	128	783341	475	216659	44
17	715394	346	931768	128	783626	474	216374	43
18	715602	346	931691	128	783910	474	216090	42
19	715809	346	931614	128	784195	474	215805	41
20	716017	346	931537	128	784479	474	215521	40
21	9.716224	345	9.931460	128	9.784764	474	10.215236	39
22	716432	345	931383	128	785048	474	214952	38
23	716639	345	931306	128	785332	473	214668	37
24	716846	345	931229	129	785616	473	214384	36
25	717053	345	931152	129	785900	473	214100	35
26	717259	344	931075	129	786184	473	213816	34
27	717466	344	930998	129	786468	473	213532	33
28	717673	344	930921	129	786752	473	213248	32
29	717879	344	930843	129	787036	473	212964	31
30	718085	343	930766	129	787319	472	212681	30
31	9.718291	343	9.930688	129	9.787603	472	10.212397	29
32	718497	343	930611	129	787886	472	212114	28
33	718703	343	930533	129	788170	472	211830	27
34	718909	343	930456	129	788453	472	211547	26
35	719114	342	930378	129	788736	472	211264	25
36	719320	342	930300	130	789019	472	210981	24
37	719525	342	930223	130	789302	471	210698	23
38	719730	342	930145	130	789585	471	210415	22
39	719935	341	930067	130	789868	471	210132	21
40	720140	341	929989	130	790151	471	209849	20
41	9.720345	341	9.929911	130	9.790433	471	10.209567	19
42	720549	341	929833	130	790716	471	209284	18
43	720754	340	929755	130	790999	471	209001	17
44	720958	340	929677	130	791281	471	208719	16
45	721162	340	929599	130	791563	470	208437	15
46	721366	340	929521	130	791846	470	208154	14
47	721570	340	929442	130	792128	470	207872	13
48	721774	339	929364	131	792410	470	207590	12
49	721978	339	929286	131	792692	470	207308	11
50	722181	339	929207	131	792974	470	207026	10
51	9.722385	339	9.929129	131	9.793256	470	10.206744	9
52	722588	339	929050	131	793538	469	206462	8
53	722791	338	928972	131	793819	469	206181	7
54	722994	338	928893	131	794101	469	205899	6
55	723197	338	928815	131	794383	469	205617	5
56	723400	338	928736	131	794664	469	205336	4
57	723603	337	928657	131	794945	469	205055	3
58	723805	337	928578	131	795227	469	204773	2
59	724007	337	928499	131	795508	468	204492	1
60	724210	337	928420	131	795789	468	204211	0
	Cosine		Sine		Cotang.		Tang.	M.

58 Degrees.

M.	Sine	D	Cosine	D.	Tang.	D.	Cotang.	
0	9.724210	337	9.928420	132	9.795789	468	10.204211	60
1	724412	337	928342	132	796070	468	203930	59
2	724614	336	928263	132	796351	468	203649	58
3	724816	336	928183	132	796632	468	203368	57
4	725017	336	928104	132	796913	468	203087	56
5	725219	336	928025	132	797194	468	202806	55
6	725420	335	927946	132	797475	468	202525	54
7	725622	335	927867	132	797755	468	202245	53
8	725823	335	927787	132	798036	467	201964	52
9	726024	335	927708	132	798316	467	201684	51
10	726225	335	927629	132	798596	467	201404	50
11	9.726426	334	9.927549	132	9.798877	467	10.201123	49
12	726626	334	927470	133	799157	467	200843	48
13	726827	334	927390	133	799437	467	200563	47
14	727027	334	927310	133	799717	467	200283	46
15	727228	334	927231	133	799997	466	200003	45
16	727428	333	927151	133	800277	466	199723	44
17	727628	333	927071	133	800557	466	199443	43
18	727828	333	926991	133	800836	466	199164	42
19	728027	333	926911	133	801116	466	198884	41
20	728227	333	926831	133	801396	466	198604	40
21	9.728427	332	9.926751	133	9.801675	466	10.198325	39
22	728626	332	926671	133	801955	466	198045	38
23	728825	332	926591	133	802234	465	197766	37
24	729024	332	926511	134	802513	465	197487	36
25	729223	331	926431	134	802792	465	197208	35
26	729422	331	926351	134	803072	465	196928	34
27	729621	331	926270	134	803351	465	196649	33
28	729820	331	926190	134	803630	465	196370	32
29	730018	330	926110	134	803908	465	196092	31
30	730216	330	926029	134	804187	465	195813	30
31	9.730415	330	9.925949	134	9.804466	464	10.195534	29
32	730613	330	925868	134	804745	464	195255	28
33	730811	330	925788	134	805023	464	194977	27
34	731009	329	925707	134	805302	464	194698	26
35	731206	329	925626	134	805580	464	194420	25
36	731404	329	925545	135	805859	464	194141	24
37	731602	329	925465	135	806137	464	193863	23
38	731799	329	925384	135	806415	463	193585	22
39	731996	328	925303	135	806693	463	193307	21
40	732193	328	925222	135	806971	463	193029	20
41	9.732390	328	9.925141	135	9.807249	463	10.192751	19
42	732587	328	925060	135	807527	463	192473	18
43	732784	328	924979	135	807805	463	192195	17
44	732980	327	924897	135	808083	463	191917	16
45	733177	327	924816	135	808361	463	191639	15
46	733373	327	924735	136	808638	462	191362	14
47	733569	327	924654	136	808916	462	191084	13
48	733765	327	924572	136	809193	462	190807	12
49	733961	326	924491	136	809471	462	190529	11
50	734157	326	924409	136	809748	462	190252	10
51	9.734353	326	9.924328	136	9.810025	462	10.189975	9
52	734549	326	924246	136	810302	462	189698	8
53	734744	325	924164	136	810580	462	189420	7
54	734939	325	924083	136	810857	462	189143	6
55	735135	325	924001	136	811134	461	188866	5
56	735330	325	923919	136	811410	461	188590	4
57	735525	325	923837	136	811687	461	188313	3
58	735719	324	923755	137	811964	461	188036	2
59	735914	324	923673	137	812241	461	187759	1
60	736109	324	923591	137	812517	461	187483	0
	Cosine		Sine		Cotang.		Tang.	M.

57 Degrees.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang	
0	9.736109	324	9.923591	137	9.812517	461	10.187482	60
1	736303	324	923509	137	812794	461	187206	59
2	736498	324	923427	137	813070	461	186930	58
3	736692	323	923345	137	813347	460	186653	57
4	736886	323	923263	137	813623	460	186377	56
5	737080	323	923181	137	813899	460	186101	55
6	737274	323	923098	137	814175	460	185825	54
7	737467	323	923016	137	814452	460	185548	53
8	737661	322	922933	137	814728	460	185272	52
9	737855	322	922851	137	815004	460	184996	51
10	738048	322	922768	138	815279	460	184721	50
11	9.738241	322	9.922686	138	9.815555	459	10.184445	49
12	738434	322	922603	138	815831	459	184169	48
13	738627	321	922520	138	816107	459	183893	47
14	738820	321	922438	138	816382	459	183618	46
15	739013	321	922355	138	816658	459	183342	45
16	739206	321	922272	138	816933	459	183067	44
17	739398	321	922189	138	817209	459	182791	43
18	739590	320	922106	138	817484	459	182516	42
19	739783	320	922023	138	817759	459	182241	41
20	739975	320	921940	138	818035	458	181965	40
21	9.740167	320	9.921857	139	9.818310	458	10.181690	39
22	740359	320	921774	139	818585	458	181415	38
23	740550	319	921691	139	818860	458	181140	37
24	740742	319	921607	139	819135	458	180865	36
25	740934	319	921524	139	819410	458	180590	35
26	741125	319	921441	139	819684	458	180316	34
27	741316	319	921357	139	819959	458	180041	33
28	741508	318	921274	139	820234	458	179766	32
29	741699	318	921190	139	820508	457	179492	31
30	741889	318	921107	139	820783	457	179217	30
31	9.742080	318	9.921023	139	9.821057	457	10.178943	29
32	742271	318	920939	140	821332	457	178668	28
33	742462	317	920856	140	821606	457	178394	27
34	742652	317	920772	140	821880	457	178120	26
35	742842	317	920688	140	822154	357	177846	25
36	743033	317	920604	140	822429	457	177571	24
37	743223	317	920520	140	822703	457	177297	23
38	743413	316	920436	140	822977	456	177023	22
39	743602	316	920352	140	823250	456	176750	21
40	743792	316	920268	140	823524	456	176476	20
41	9.743982	316	9.920184	140	9.823798	456	10.176202	19
42	744171	316	920099	140	824072	456	175928	18
43	744361	315	920015	140	824345	456	175655	17
44	744550	315	919931	141	824619	456	175381	16
45	744739	315	919846	141	824893	456	175107	15
46	744928	315	919762	141	825166	456	174834	14
47	745117	315	919677	141	825439	455	174561	13
48	745306	314	919593	141	825713	455	174287	12
49	745494	314	919508	141	825986	455	174014	11
50	745683	314	919424	141	826259	455	173741	10
51	9.745871	314	9.919339	141	9.826532	455	10.173468	9
52	746059	314	919254	141	826805	455	173195	8
53	746248	313	919169	141	827078	455	172922	7
54	746436	313	919085	141	827351	455	172649	6
55	746624	313	919000	141	827624	455	172376	5
56	746812	313	918915	142	827897	454	172103	4
57	746999	313	918830	142	828170	454	171830	3
58	747187	312	918745	142	828442	454	171558	2
59	747374	312	918659	142	828715	454	171285	1
60	747562	312	918574	142	828987	454	171013	0
	Cosine		Sine		Cotang.		Tang.	M.

56 Degrees.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.747562	312	9.918574	142	9.828987	454	10.171013	60
1	747749	312	918489	142	829260	454	170740	59
2	747936	312	918404	142	829532	454	170468	58
3	748123	311	918318	142	829805	454	170195	57
4	748310	311	918233	142	830077	454	169923	56
5	748497	311	918147	142	830349	453	169651	55
6	748683	311	918062	142	830621	453	169379	54
7	748870	311	917976	143	830893	453	169107	53
8	749056	310	917891	143	831165	453	168835	52
9	749243	310	917805	143	831437	453	168563	51
10	749429	310	917719	143	831709	453	168291	50
11	9.749615	310	9.917634	143	9.831981	453	10.168019	49
12	749801	310	917548	143	832253	453	167747	48
13	749987	309	917462	143	832525	453	167475	47
14	750172	309	917376	143	832796	453	167204	46
15	750358	309	917290	143	833068	452	166932	45
16	750543	309	917204	143	833339	452	166661	44
17	750729	309	917118	144	833611	452	166389	43
18	750914	308	917032	144	833882	452	166118	42
19	751099	308	916946	144	834154	452	165846	41
20	751284	308	916859	144	834425	452	165575	40
21	9.751469	308	9.916773	144	9.834696	452	10.165304	39
22	751654	308	916687	144	834967	452	165033	38
23	751839	308	916600	144	835238	452	164762	37
24	752023	307	916514	144	835509	452	164491	36
25	752208	307	916427	144	835780	451	164220	35
26	752392	307	916341	144	836051	451	163949	34
27	752576	307	916254	144	836322	451	163678	33
28	752760	307	916167	145	836593	451	163407	32
29	752944	306	916081	145	836864	451	163136	31
30	753128	306	915994	145	837134	451	162866	30
31	9.753312	306	9.915907	145	9.837405	451	10.162595	29
32	753495	306	915820	145	837675	451	162325	28
33	753679	306	915733	145	837946	451	162054	27
34	753862	305	915646	145	838216	451	161784	26
35	754046	305	915559	145	838487	450	161513	25
36	754229	305	915472	145	838757	450	161243	24
37	754412	305	915385	145	839027	450	160973	23
38	754595	305	915297	145	839297	450	160703	22
39	754778	304	915210	145	839568	450	160432	21
40	754960	304	915123	146	839838	450	160162	20
41	9.755143	304	9.915035	146	9.840108	450	10.159892	19
42	755326	304	914948	146	840378	450	159622	18
43	755508	304	914860	146	840647	450	159353	17
44	755690	304	914773	146	840917	449	159083	16
45	755872	303	914685	146	841187	449	158813	15
46	756054	303	914598	146	841457	449	158543	14
47	756236	303	914510	146	841726	449	158274	13
48	756418	303	914422	146	841996	449	158004	12
49	756600	303	914334	146	842266	449	157734	11
50	756782	302	914246	147	842535	449	157465	10
51	9.756963	302	9.914158	147	9.842805	449	10.157195	9
52	757144	302	914070	147	843074	419	156926	8
53	757326	302	913982	147	843343	449	156657	7
54	757507	302	913894	147	843612	449	156388	6
55	757688	301	913806	147	843882	448	156118	5
56	757869	301	913718	147	844151	448	155849	4
57	758050	301	913630	147	844420	448	155580	3
58	758230	301	913541	147	844689	448	155311	2
59	758411	301	913453	147	844958	448	155042	1
60	758591	301	913365	147	845227	448	154773	0
	Cosine		Sine		Cotang.		Tang.	M.

55 Degrees.

(35 Degrees.)

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.758591	301	9.913365	147	9.845227	448	10.154773	60
1	758772	300	913276	147	845496	448	154504	59
2	758952	300	913187	148	845764	448	154236	58
3	759132	300	913099	148	846033	448	153967	57
4	759312	300	913010	148	846302	448	153698	56
5	759492	300	912922	148	846570	447	153430	55
6	759672	299	912833	148	846839	447	153161	54
7	759852	299	912744	148	847107	447	152893	53
8	760031	299	912655	148	847376	447	152624	52
9	760211	299	912566	148	847644	447	152356	51
10	760390	289	912477	148	847913	447	152087	50
11	9.760569	298	9.912388	148	9.848181	447	10.151819	49
12	760748	298	912299	149	848449	447	151551	48
13	760927	298	912210	149	848717	447	151283	47
14	761106	298	912121	149	848986	447	151014	46
15	761285	298	912031	149	849254	447	150746	45
16	761464	298	911942	149	849522	447	150478	44
17	761642	297	911853	149	849790	446	150210	43
18	761821	297	911763	149	850058	446	149942	42
19	761999	297	911674	149	850325	446	149675	41
20	762177	297	911584	149	850593	446	149407	40
21	9.762356	297	9.911495	149	9.850861	446	10.149139	39
22	762534	296	911405	149	851129	446	148871	38
23	762712	296	911315	150	851396	446	148604	37
24	762889	296	911226	150	851664	446	148336	36
25	763067	296	911136	150	851931	446	148069	35
26	763245	296	911046	150	852199	446	147801	34
27	763422	296	910956	150	852466	446	147534	33
28	763600	295	910866	150	852733	445	147267	32
29	763777	295	910776	150	853001	445	146999	31
30	763954	295	910686	150	853268	445	146732	30
31	9.764131	295	9.910596	150	9.853535	445	10.146465	29
32	764308	295	910506	150	853802	445	146198	28
33	764485	294	910415	150	854069	445	145931	27
34	764662	294	910325	151	854336	445	145664	26
35	764838	294	910235	151	854603	445	145397	25
36	765015	294	910144	151	854870	445	145130	24
37	765191	294	910054	151	855137	445	144863	23
38	765367	294	909963	151	855404	445	144596	22
39	765544	293	909873	151	855671	444	144329	21
40	765720	293	909782	151	855938	444	144062	20
41	9.765896	293	9.909691	151	9.856204	444	10.143796	19
42	766072	293	909601	151	856471	444	143529	18
43	766247	293	909510	151	856737	444	143263	17
44	766423	293	909419	151	857004	444	142996	16
45	765598	292	909328	152	857270	444	142730	15
46	766774	292	909237	152	857537	444	142463	14
47	766949	292	909146	152	857803	444	142197	13
48	767124	292	909055	152	858069	444	141931	12
49	767300	292	908964	152	858336	444	141664	11
50	767475	291	908873	152	858602	443	141398	10
51	9.767649	291	9.908781	152	9.858868	443	10.141132	9
52	767824	291	908690	152	859134	443	140866	8
53	767999	291	908599	152	859400	443	140600	7
54	768173	291	908507	152	859666	443	140334	6
55	768348	290	908416	153	859932	443	140068	5
56	768522	290	908324	153	860198	443	139802	4
57	768697	290	908233	153	860464	443	139536	3
58	768871	290	908141	153	860730	443	139270	2
59	769045	290	908049	153	860995	443	139005	1
60	769219	290	907958	153	861261	443	138739	0
	Cosine		Sine		Cotang.		Tang.	M.

54 Degrees.

M.	Sine	D	Cosine	D.	Tang.	D.	Cotang.	
0	9.769219	290	9.907958	153	9.861261	443	10.138739	60
1	769393	289	907866	153	861527	443	138473	59
2	769566	289	907774	153	861792	442	138208	58
3	769740	289	907682	153	862058	442	137942	57
4	769913	289	907590	153	862323	442	137677	56
5	770087	289	907498	153	862589	442	137411	55
6	770260	288	907406	153	862854	442	137146	54
7	770433	288	907314	154	863119	442	136881	53
8	770606	288	907222	154	863385	442	136615	52
9	770779	288	907129	154	863650	442	136350	51
10	770952	288	907037	154	863915	442	136085	50
11	9.771125	288	9.906945	154	9.864180	442	10.135820	49
12	771298	287	906852	154	864445	442	135555	48
13	771470	287	906760	154	864710	442	135290	47
14	771643	287	906667	154	864975	441	135025	46
15	771815	287	906575	154	865240	441	134760	45
16	771987	287	906482	154	865505	441	134495	44
17	772159	287	906389	155	865770	441	134230	43
18	772331	286	906296	155	866035	441	133965	42
19	772503	286	906204	155	866300	441	133700	41
20	772675	286	906111	155	866564	441	133436	40
21	9.772847	286	9.906018	155	9.866829	441	10.133171	39
22	773018	286	905925	155	867094	441	132906	38
23	773190	286	905832	155	867358	441	132642	37
24	773361	285	905739	155	867623	441	132377	36
25	773533	285	905645	155	867887	441	132113	35
26	773704	285	905552	155	868152	440	131848	34
27	773875	285	905459	155	868416	440	131584	33
28	774046	285	905366	156	868680	440	131320	32
29	774217	285	905272	156	868945	440	131055	31
30	774388	284	905179	156	869209	440	130791	30
31	9.774558	284	9.905085	156	9.869473	440	10.130527	29
32	774729	284	904992	156	869737	440	130263	28
33	774899	284	904898	156	870001	440	129999	27
34	775070	284	904804	156	870265	440	129735	26
35	775240	284	904711	156	870529	440	129471	25
36	775410	283	904617	156	870793	440	129207	24
37	775580	283	904523	156	871057	440	128943	23
38	775750	283	904429	157	871321	440	128679	22
39	775920	283	904335	157	871585	440	128415	21
40	776090	283	904241	157	871849	439	128151	20
41	9.776259	283	9.904147	157	9.872112	439	10.127888	19
42	776429	282	904053	157	872376	439	127624	18
43	776598	282	903959	157	872640	439	127360	17
44	776768	282	903864	157	872903	439	127097	16
45	776937	282	903770	157	873167	439	126833	15
46	777106	282	903676	157	873430	439	126570	14
47	777275	281	903581	157	873694	439	126306	13
48	777444	281	903487	157	873957	439	126043	12
49	777613	281	903392	158	874220	439	125780	11
50	777781	281	903298	158	874484	439	125516	10
51	9.777950	281	9.903203	158	9.874747	439	10.125253	9
52	778119	281	903108	158	875010	439	124990	8
53	778287	280	903014	158	875273	438	124727	7
54	778455	280	902919	158	875536	438	124464	6
55	778624	280	902824	158	875800	438	124200	5
56	778792	280	902729	158	876063	438	123937	4
57	778960	280	902634	158	876326	438	123674	3
58	779128	280	902539	159	876589	438	123411	2
59	779295	279	902444	159	876851	438	123149	1
60	779463	279	902349	159	877114	438	122886	0
	Cosine		Sine		Cotang.		Tang.	M.

53 Degrees.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang	
0	9.779463	279	9.902349	159	9.877114	438	10.122886	60
1	779631	279	902253	159	877377	438	122623	59
2	779798	279	902158	159	877640	438	122360	58
3	779966	279	902063	159	877903	438	122097	57
4	780133	279	901967	159	878165	438	121835	56
5	780300	278	901872	159	878428	438	121572	55
6	780467	278	901776	159	878691	438	121309	54
7	780634	278	901681	159	878953	437	121047	53
8	780801	278	901585	159	879216	437	120784	52
9	780968	278	901490	159	879478	437	120522	51
10	781134	278	901394	160	879741	437	120259	50
11	9.781301	277	9.901298	160	9.880003	437	10.119997	49
12	781468	277	901202	160	880265	437	119735	48
13	781634	277	901106	160	880528	437	119472	47
14	781800	277	901010	160	880790	437	119210	46
15	781966	277	900914	160	881052	437	118948	45
16	782132	277	900818	160	881314	437	118686	44
17	782298	276	900722	160	881576	437	118424	43
18	782464	276	900626	160	881839	437	118161	42
19	782630	276	900529	160	882101	437	117899	41
20	782796	276	900433	161	882363	436	117637	40
21	9.782961	276	9.900337	161	9.882625	436	10.117375	39
22	783127	276	900240	161	882887	436	117113	38
23	783292	275	900144	161	883148	436	116852	37
24	783458	275	900047	161	883410	436	116590	36
25	783623	275	899951	161	883672	436	116328	35
26	783788	275	899854	161	883934	436	116066	34
27	783953	275	899757	161	884196	436	115804	33
28	784118	275	899660	161	884457	436	115543	32
29	784282	274	899564	161	884719	436	115281	31
30	784447	274	899467	162	884980	436	115020	30
31	9.784612	274	9.899370	162	9.885242	436	10.114758	29
32	784776	274	899273	162	885503	436	114497	28
33	784941	274	899176	162	885765	436	114235	27
34	785105	274	899078	162	886026	436	113974	26
35	785269	273	898981	162	886288	436	113712	25
36	785433	273	898884	162	886549	435	113451	24
37	785597	273	898787	162	886810	435	113190	23
38	785761	273	898689	162	887072	435	112928	22
39	785925	273	898592	162	887333	435	112667	21
40	786089	273	898494	163	887594	435	112406	20
41	9.786252	272	9.898397	163	9.887855	435	10.112145	19
42	786416	272	898299	163	888116	435	111884	18
43	786579	272	898202	163	888377	435	111623	17
44	786742	272	898104	163	888639	435	111361	16
45	786906	272	898006	163	888900	435	111100	15
46	787069	272	897908	163	889160	435	110840	14
47	787232	271	897810	163	889421	435	110579	13
48	787395	271	897712	163	889682	435	110318	12
49	787557	271	897614	163	889943	435	110057	11
50	787720	271	897516	163	890204	434	109796	10
51	9.787883	271	9.897418	164	9.890465	434	10.109535	9
52	788045	271	897320	164	890725	434	109275	8
53	788208	271	897222	164	890986	434	109014	7
54	788370	270	897123	164	891247	434	108753	6
55	788532	270	897025	164	891507	434	108493	5
56	788694	270	896926	164	891768	434	108232	4
57	788856	270	896828	164	892028	434	107972	3
58	789018	270	896729	164	892289	434	107711	2
59	789180	270	896631	164	892549	434	107451	1
60	789342	269	896532	164	892810	434	107190	0
	Cosine		Sine		Cotang.		Tang.	M.

52 Degrees.

(38 Degrees.)

M.	Sine	D	Cosine	D.	Tang.	D.	Cotang.	
0	9.789342	269	9.896532	164	9.892810	434	10.107190	60
1	789504	269	896433	165	893070	434	106930	59
2	789665	269	896335	165	893331	434	106669	58
3	789827	269	896236	165	893591	434	106409	57
4	789988	269	896137	165	893851	434	106149	56
5	790149	269	896038	165	894111	434	105889	55
6	790310	268	895939	165	894371	434	105629	54
7	790471	268	895840	165	894632	433	105368	53
8	790632	268	895741	165	894892	433	105108	52
9	790793	268	895641	165	895152	433	104848	51
10	790954	268	895542	165	895412	433	104588	50
11	9.791115	268	9.895443	166	9.895672	433	10.104328	49
12	791275	267	895343	166	895932	433	104068	48
13	791436	267	895244	166	896192	433	103808	47
14	791596	267	895145	166	896452	433	103548	46
15	791757	267	895045	166	896712	433	103288	45
16	791917	267	894945	166	896971	433	103029	44
17	792077	267	894846	166	897231	433	102769	43
18	792237	266	894746	166	897491	433	102509	42
19	792397	266	894646	166	897751	433	102249	41
20	792557	266	894546	166	898010	433	101990	40
21	9.792716	266	9.894446	167	9.898270	433	10.101730	39
22	792876	266	894346	167	898530	433	101470	38
23	793035	266	894246	167	898789	433	101211	37
24	793195	265	894146	167	899049	432	100951	36
25	793354	265	894046	167	899308	432	100692	35
26	793514	265	893946	167	899568	432	100432	34
27	793673	265	893846	167	899827	432	100173	33
28	793832	265	893745	167	900086	432	099914	32
29	793991	265	893645	167	900346	432	099654	31
30	794150	264	893544	167	900605	432	099395	30
31	9.794308	264	9.893444	168	9.900864	432	10.099136	29
32	794467	264	893343	168	901124	432	098876	28
33	794626	264	893243	168	901383	432	098617	27
34	794784	264	893142	168	901642	432	098358	26
35	794942	264	893041	168	901901	432	098099	25
36	795101	264	892940	168	902160	432	097840	24
37	795259	263	892839	168	902419	432	097581	23
38	795417	263	892739	168	902679	432	097321	22
39	795575	263	892638	168	902938	432	097062	21
40	795733	263	892536	168	903197	431	096803	20
41	9.795891	263	9.892435	169	9.903455	431	10.096545	19
42	796049	263	892334	169	903714	431	096286	18
43	796206	263	892233	169	903973	431	096027	17
44	796364	262	892132	169	904232	431	095768	16
45	796521	262	892030	169	904491	431	095509	15
46	796679	262	891929	169	904750	431	095250	14
47	796836	262	891827	169	905008	431	094992	13
48	796993	262	891726	169	905267	431	094733	12
49	797150	261	891624	169	905526	431	094474	11
50	797307	261	891523	170	905784	431	094216	10
51	9.797464	261	9.891421	170	9.906043	431	10.093957	9
52	797621	261	891319	170	906302	431	093698	8
53	797777	261	891217	170	906560	431	093440	7
54	797934	261	891115	170	906819	431	093181	6
55	798091	261	891013	170	907077	431	092923	5
56	798247	261	890911	170	907336	431	092664	4
57	798403	260	890809	170	907594	431	092406	3
58	798560	260	890707	170	907852	431	092148	2
59	798716	260	890605	170	908111	430	091889	1
60	798872	260	890503	170	908369	430	091631	0
	Cosine		Sine		Cotang.		Tang.	M.

51 Degrees.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang	
0	9.798872	260	9.890503	170	9.908369	430	10.091631	60
1	799028	260	890400	171	908628	430	091372	59
2	799184	260	890298	171	908886	430	091114	58
3	799339	259	890195	171	909144	430	090856	57
4	799495	259	890093	171	909402	430	090598	56
5	799651	259	889990	171	909660	430	090340	55
6	799806	259	889888	171	909918	430	090082	54
7	799962	259	889785	171	910177	430	089823	53
8	800117	259	889682	171	910435	430	089565	52
9	800272	258	889579	171	910693	430	089307	51
10	800427	258	889477	171	910951	430	089049	50
11	9.800582	258	9.889374	172	9.911209	430	10.088791	49
12	800737	258	889271	172	911467	430	088533	48
13	800892	258	889168	172	911724	430	088276	47
14	801047	258	889064	172	911982	430	088018	46
15	801201	258	888961	172	912240	430	087760	45
16	801356	257	888858	172	912498	430	087502	44
17	801511	257	888755	172	912756	430	087244	43
18	801665	257	888651	172	913014	429	086986	42
19	801819	257	888548	172	913271	429	086729	41
20	801973	257	888444	173	913529	429	086471	40
21	9.802128	257	9.888341	173	9.913787	429	10.086213	39
22	802282	256	888237	173	914044	429	085956	38
23	802436	256	888134	173	914302	429	085698	37
24	802589	256	888030	173	914560	429	085440	36
25	802743	256	887926	173	914817	429	085183	35
26	802897	256	887822	173	915075	429	084925	34
27	803050	256	887718	173	915332	429	084668	33
28	803204	256	887614	173	915590	429	084410	32
29	803357	255	887510	173	915847	429	084153	31
30	803511	255	887406	174	916104	429	083896	30
31	9.803664	255	9.887302	174	9.916362	429	10.083638	29
32	803817	255	887198	174	916619	429	083381	28
33	803970	255	887093	174	916877	429	083123	27
34	804123	255	886989	174	917134	429	082866	26
35	804276	254	886885	174	917391	429	082609	25
36	804428	254	886780	174	917648	429	082352	24
37	804581	254	886676	174	917905	429	082095	23
38	804734	254	886571	174	918163	428	081837	22
39	804886	254	886466	174	918420	428	081580	21
40	805039	254	886362	175	918677	428	081323	20
41	9.805191	254	9.886257	175	9.918934	428	10.081066	19
42	805343	253	886152	175	919191	428	080809	18
43	805495	253	886047	175	919448	428	080552	17
44	805647	253	885942	175	919705	428	080295	16
45	805799	253	885837	175	919962	428	080038	15
46	805951	253	885732	175	920219	428	079781	14
47	806103	253	885627	175	920476	428	079524	13
48	806254	253	885522	175	920733	428	079267	12
49	806406	252	885416	175	920990	428	079010	11
50	806557	252	885311	176	921247	428	078753	10
51	9.806709	252	9.885205	176	9.921503	428	10.078497	9
52	806860	252	885100	176	921760	428	078240	8
53	807011	252	884994	176	922017	428	077983	7
54	807163	252	884889	176	922274	428	077726	6
55	807314	252	884783	176	922530	428	077470	5
56	807465	251	884677	176	922787	428	077213	4
57	807615	251	884572	176	923044	428	076956	3
58	807766	251	884466	176	923300	428	076700	2
59	807917	251	884360	176	923557	427	076443	1
60	808067	251	884254	177	923813	427	076187	0
	Cosine		Sine		Cotang.		Tang.	M.

50 Degrees.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.808067	251	9.884254	177	9.923813	427	10.076187	60
1	808218	251	884148	177	924070	427	075930	59
2	808368	251	884042	177	924327	427	075673	58
3	808519	250	883936	177	924583	427	075417	57
4	808669	250	883829	177	924840	427	075160	56
5	808819	250	883723	177	925096	427	074904	55
6	808969	250	883617	177	925352	427	074648	54
7	809119	250	883510	177	925609	427	074391	53
8	809269	250	883404	177	925865	427	074135	52
9	809419	249	883297	178	926122	427	073878	51
10	809569	249	883191	178	926378	427	073622	50
11	9.809718	249	9.883084	178	9.926634	427	10.073366	49
12	809868	249	882977	178	926890	427	073110	48
13	810017	249	882871	178	927147	427	072853	47
14	810167	249	882764	178	927403	427	072597	46
15	810316	248	882657	178	927659	427	072341	45
16	810465	248	882550	178	927915	427	072085	44
17	810614	248	882443	178	928171	427	071829	43
18	810763	248	882336	179	928427	427	071573	42
19	810912	248	882229	179	928683	427	071317	41
20	811061	248	882121	179	928940	427	071060	40
21	9.811210	248	9.882014	179	9.929196	427	10.070804	39
22	811358	247	881907	179	929452	427	070548	38
23	811507	247	881799	179	929708	427	070292	37
24	811655	247	881692	179	929964	426	070036	36
25	811804	247	881584	179	930220	426	069780	35
26	811952	247	881477	179	930475	426	069525	34
27	812100	247	881369	179	930731	426	069269	33
28	812248	247	881261	180	930987	426	069013	32
29	812396	246	881153	180	931243	426	068757	31
30	812544	246	881046	180	931499	426	068501	30
31	9.812692	246	9.880938	180	9.931755	426	10.068245	29
32	812840	246	880830	180	932010	426	067990	28
33	812988	246	880722	180	932266	426	067734	27
34	813135	246	880613	180	932522	426	067478	26
35	813283	246	880505	180	932778	426	067222	25
36	813430	245	880397	180	933033	426	066967	24
37	813578	245	880289	181	933289	426	066711	23
38	813725	245	880180	181	933545	426	066455	22
39	813872	245	880072	181	933800	426	066200	21
40	814019	245	879963	181	934056	426	065944	20
41	9.814166	245	9.879855	181	9.934311	426	10.065689	19
42	814313	245	879746	181	934567	426	065433	18
43	814460	244	879637	181	934823	426	065177	17
44	814607	244	879529	181	935078	426	064922	16
45	814753	244	879420	181	935333	426	064667	15
46	814900	244	879311	181	935589	426	064411	14
47	815046	244	879202	182	935844	426	064156	13
48	815193	244	879093	182	936100	426	063900	12
49	815339	244	878984	182	936355	426	063645	11
50	815485	243	878875	182	936610	426	063390	10
51	9.815631	243	9.878766	182	9.936866	425	10.063134	9
52	815778	243	878656	182	937121	425	062879	8
53	815924	243	878547	182	937376	425	062624	7
54	816069	243	878438	182	937632	425	062368	6
55	816215	243	878328	182	937887	425	062113	5
56	816361	243	878219	183	938142	425	061858	4
57	816507	242	878109	183	938398	425	061602	3
58	816652	242	877999	183	938653	425	061347	2
59	816798	242	877890	183	938908	425	061092	1
60	816943	242	877780	183	939163	425	060837	0
	Cosine		Sine		Cotang.		Tang.	M.

49 Degrees.

M.	Sine	D	Cosine	D.	Tang.	D.	Cotang.	
0	9.816943	242	9.877780	183	9.939163	425	10.060837	60
1	817088	242	877670	183	939418	425	060582	59
2	817233	242	877560	183	939673	425	060327	58
3	817379	242	877450	183	939928	425	060072	57
4	817524	241	877340	183	940183	425	059817	56
5	817668	241	877230	184	940438	425	059562	55
6	817813	241	877120	184	940694	425	059306	54
7	817958	241	877010	184	940949	425	059051	53
8	818103	241	876899	184	941204	425	058796	52
9	818247	241	876789	184	941458	425	058542	51
10	818392	241	876678	184	941714	425	058286	50
11	9.818536	240	9.876568	184	9.941968	425	10.058032	49
12	818681	240	876457	184	942223	425	057777	48
13	818825	240	876347	184	942478	425	057522	47
14	818969	240	876236	185	942733	425	057267	46
15	819113	240	876125	185	942988	425	057012	45
16	819257	240	876014	185	943243	425	056757	44
17	819401	240	875904	185	943498	425	056502	43
18	819545	239	875793	185	943752	425	056248	42
19	819689	239	875682	185	944007	425	055993	41
20	819832	239	875571	185	944262	425	055738	40
21	9.819976	239	9.875459	185	9.944517	425	10.055483	39
22	820120	239	875348	185	944771	424	055229	38
23	820263	239	875237	185	945026	424	054974	37
24	820406	239	875126	186	945281	424	054719	36
25	820550	238	875014	186	945535	424	054465	35
26	820693	238	874903	186	945790	424	054210	34
27	820836	238	874791	186	946045	424	053955	33
28	820979	238	874680	186	946299	424	053701	32
29	821122	238	874568	186	946554	424	053446	31
30	821265	238	874456	186	946808	424	053192	30
31	9.821407	238	9.874344	186	9.947063	424	10.052937	29
32	821550	238	874232	187	947318	424	052682	28
33	821693	237	874121	187	947572	424	052428	27
34	821835	237	874009	187	947826	424	052174	26
35	821977	237	873896	187	948081	424	051919	25
36	822120	237	873784	187	948336	424	051664	24
37	822262	237	873672	187	948590	424	051410	23
38	822404	237	873560	187	948844	424	051156	22
39	822546	237	873448	187	949099	424	050901	21
40	822688	236	873335	187	949353	424	050647	20
41	9.822830	236	9.873223	187	9.949607	424	10.050393	19
42	822972	236	873110	188	949862	424	050138	18
43	823114	236	872998	188	950116	424	049884	17
44	823255	236	872885	188	950370	424	049630	16
45	823397	236	872772	188	950625	424	049375	15
46	823539	236	872659	188	950879	424	049121	14
47	823680	235	872547	188	951133	424	048867	13
48	823821	335	872434	188	951388	424	048612	12
49	823963	235	872321	188	951642	424	048358	11
50	824104	235	872208	188	951896	424	048104	10
51	9.824245	235	9.872095	189	9.952150	424	10.047850	9
52	824386	235	871981	189	952405	424	047595	8
53	824527	235	871868	189	952659	424	047341	7
54	824668	234	871755	189	952913	424	047087	6
55	824808	234	871641	189	953167	423	046833	5
56	824949	234	871528	189	953421	423	046579	4
57	825090	234	871414	189	953675	423	046325	3
58	825230	234	871301	189	953929	423	046071	2
59	825371	234	871187	189	954183	423	045817	1
60	825511	234	871073	190	954437	423	045563	0
	Cosine		Sine		Cotang.		Tang.	M.

48 Degrees.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang	
0	9.825511	234	9.871073	190	9.954437	423	10.045563	60
1	825651	233	870960	190	954691	423	045309	59
2	835791	233	870846	190	954945	423	045055	58
3	825931	233	870732	190	955200	423	044800	57
4	826071	233	870618	190	955454	423	044546	56
5	826211	233	870504	190	955707	423	044293	55
6	826351	233	870390	190	955961	423	044039	54
7	826491	233	870276	190	956215	423	043785	53
8	826631	233	870161	190	956469	423	043531	52
9	826770	232	870047	191	956723	423	043277	51
10	826910	232	869933	191	956977	423	043023	50
11	9.827049	232	9.869818	191	9.957231	423	10.042769	49
12	827189	232	869704	191	957485	423	042515	48
13	827328	232	869589	191	957739	423	042261	47
14	827467	232	869474	191	957993	423	042007	46
15	827606	232	869360	191	958246	423	041754	45
16	827745	232	869245	191	958500	423	041500	44
17	827884	231	869130	191	958754	423	041246	43
18	828023	231	869015	192	959008	423	040992	42
19	828162	231	868900	192	959262	423	040738	41
20	828301	231	868785	192	959516	423	040484	40
21	9.828439	231	9.868670	192	9.959769	423	10.040231	39
22	828578	231	868555	192	960023	423	039977	38
23	828716	231	868440	192	960277	423	039723	37
24	828855	230	868324	192	960531	423	039469	36
25	828993	230	868209	192	960784	423	039216	35
26	829131	230	868093	192	961038	423	038962	34
27	829269	230	867978	193	961291	423	038709	33
28	829407	230	867862	193	961545	423	038455	32
29	829545	230	867747	193	961799	423	038201	31
30	829683	230	867631	193	962052	423	037948	30
31	9.829821	229	9.867515	193	9.962306	423	10.037694	29
32	829959	229	867399	193	962560	423	037440	28
33	830097	229	867283	193	962813	423	037187	27
34	830234	229	867167	193	963067	423	036933	26
35	830372	229	867051	193	963320	423	036680	25
36	830509	229	866935	194	963574	423	036426	24
37	830646	229	866819	194	963827	423	036173	23
38	830784	229	866703	194	964081	423	035919	22
39	830921	228	866586	194	964335	423	035665	21
40	831058	228	866470	194	964588	422	035412	20
41	9.831195	228	9.866353	194	9.964842	422	10.035158	19
42	831332	228	866237	194	965095	422	034905	18
43	831469	228	866120	194	965349	422	034651	17
44	831606	228	866004	195	965602	422	034398	16
45	831742	228	865887	195	965855	422	034145	15
46	831879	228	865770	195	966109	422	033891	14
47	832015	227	865653	195	966362	422	033638	13
48	832152	227	865536	195	966616	422	033384	12
49	832288	227	865419	195	966869	422	033131	11
50	832425	227	865302	195	967123	422	032877	10
51	9.832561	227	9.865185	195	9.967376	422	10.032624	9
52	832697	227	865068	195	967629	422	032371	8
53	832833	227	864950	195	967883	422	032117	7
54	832969	226	864833	196	968136	422	031864	6
55	833105	226	864716	196	968389	422	031611	5
56	833241	226	864598	196	968643	422	031357	4
57	833377	226	864481	196	968896	422	031104	3
58	833512	226	864363	196	969149	422	030851	2
59	833648	226	864245	196	969403	422	030597	1
60	833783	226	864127	196	969656	422	030344	0
	Cosine		Sine		Cotang.		Tang.	M.

47 Degrees.

M.	Sine	D	Cosine	D.	Tang.	D.	Cotang.	
0	9.833783	226	9.864127	196	9.969656	422	10.030344	60
1	833919	225	864010	196	969909	422	030091	59
2	834054	225	863892	197	970162	422	029838	58
3	834189	225	863774	197	970416	422	029584	57
4	834325	225	863656	197	970669	422	029331	56
5	834460	225	863538	197	970922	422	029078	55
6	834595	225	863419	197	971175	422	028825	54
7	834730	225	863301	197	971429	422	028571	53
8	834865	225	863183	197	971682	422	028318	52
9	834999	224	863064	197	971935	422	028065	51
10	835134	224	862946	198	972188	422	027812	50
11	9.835269	224	9.862827	198	9.972441	422	10.027559	49
12	835403	224	862709	198	972694	422	027306	48
13	835538	224	862590	198	972948	422	027052	47
14	835672	224	862471	198	973201	422	026799	46
15	835807	224	862353	198	973454	422	026546	45
16	835941	224	862234	198	973707	422	026293	44
17	836075	223	862115	198	973960	422	026040	43
18	836209	223	861996	198	974213	422	025787	42
19	836343	223	861877	198	974466	422	025534	41
20	836477	223	861758	199	974719	422	025281	40
21	9.836611	223	9.861638	199	9.974973	422	10.025027	39
22	836745	223	861519	199	975226	422	024774	38
23	836878	223	861400	199	975479	422	024521	37
24	837012	222	861280	199	975732	422	024268	36
25	837146	222	861161	199	975985	422	024015	35
26	837279	222	861041	199	976238	422	023762	34
27	837412	222	860922	199	976491	422	023509	33
28	837546	222	860802	199	976744	422	023256	32
29	837679	222	860682	200	976997	422	023003	31
30	837812	222	860562	200	977250	422	022750	30
31	9.837945	222	9.860442	200	9.977503	422	10.022497	29
32	838078	221	860322	200	977756	422	022244	28
33	838211	221	860202	200	978009	422	021991	27
34	838344	221	860082	200	978262	422	021738	26
35	838477	221	859962	200	978515	422	021485	25
36	838610	221	859842	200	978768	422	021232	24
37	838742	221	859721	201	979021	422	020979	23
38	838875	221	859601	201	979274	422	020726	22
39	839007	221	859480	201	979527	422	020473	21
40	839140	220	859360	201	979780	422	020220	20
41	9.839272	220	9.859239	201	9.980033	422	10.019967	19
42	839404	220	859119	201	980286	422	019714	18
43	839536	220	858998	201	980538	422	019462	17
44	839668	220	858877	201	980791	421	019209	16
45	839800	220	858756	202	981044	421	018956	15
46	839932	220	858635	202	981297	421	018703	14
47	840064	219	858514	202	981550	421	018450	13
48	840196	219	858393	202	981803	421	018197	12
49	840328	219	858272	202	982056	421	017944	11
50	840459	219	858151	202	982309	421	017691	10
51	9.840591	219	9.858029	202	9.982562	421	10.017438	9
52	840722	219	857908	202	982814	421	017186	8
53	840854	219	857786	202	983067	421	016933	7
54	840985	219	857665	203	983320	421	016680	6
55	841116	218	857543	203	983573	421	016427	5
56	841247	218	857422	203	983826	421	016174	4
57	841378	218	857300	203	984079	421	015921	3
58	841509	218	857178	203	984331	421	015669	2
59	841640	218	857056	203	984584	421	015416	1
60	841771	218	856934	203	984837	421	015163	0
	Cosine		Sine		Cotang.		Tang.	M.

46 Degrees.

M.	Sine	D.	Cosine	D.	Tang.	D.	Cotang.	
0	9.841771	218	9.856934	203	9.984837	421	10.015163	60
1	841902	218	856812	203	985090	421	014910	59
2	842033	218	856690	204	985343	421	014657	58
3	842163	217	856568	204	985596	421	014404	57
4	842294	217	856446	204	985848	421	014152	56
5	842424	217	856323	204	986101	421	013899	55
6	842555	217	856201	204	986354	421	013646	54
7	842685	217	856078	204	986607	421	013393	53
8	842815	217	855956	204	986860	421	013140	52
9	842946	217	855833	204	987112	421	012888	51
10	843076	217	855711	205	987365	421	012635	50
11	9.843206	216	9.855588	205	9.987618	421	10.012382	49
12	843336	216	855465	205	987871	421	012129	48
13	843466	216	855342	205	988123	421	011877	47
14	843595	216	855219	205	988376	421	011624	46
15	843725	216	855096	205	988629	421	011371	45
16	843855	216	854973	205	988882	421	011118	44
17	843984	216	854850	205	989134	421	010866	43
18	844114	215	854727	206	989387	421	010613	42
19	844243	215	854603	206	989640	421	010360	41
20	844372	215	854480	206	989893	421	010107	40
21	9.844502	215	9.854356	206	9.990145	421	10.009855	39
22	844631	215	854233	206	990398	421	009602	38
23	844760	215	854109	206	990651	421	009349	37
24	844889	215	853986	206	990903	421	009097	36
25	845018	215	853862	206	991156	421	008844	35
26	845147	215	853738	206	991409	421	008591	34
27	845276	214	853614	207	991662	421	008338	33
28	845405	214	853490	207	991914	421	008086	32
29	845533	214	853366	207	992167	421	007833	31
30	845662	214	853242	207	992420	421	007580	30
31	9.845790	214	9.853118	207	9.992672	421	10.007328	29
32	845919	214	852994	207	992925	421	007075	28
33	846047	214	852869	207	993178	421	006822	27
34	846175	214	852745	207	993430	421	006570	26
35	846304	214	852620	207	993683	421	006317	25
36	846432	213	852496	208	993936	421	006064	24
37	846560	213	852371	208	994189	421	005811	23
38	846688	213	852247	208	994441	421	005559	22
39	846816	213	852122	208	994694	421	005306	21
40	846944	213	851997	208	994947	421	005053	20
41	9.847071	213	9.851872	208	9.995199	421	10.004801	19
42	847199	213	851747	208	995452	421	004548	18
43	847327	213	851622	208	995705	421	004295	17
44	847454	212	851497	209	995957	421	004043	16
45	847582	212	851372	209	996210	421	003790	15
46	847709	212	851246	209	996463	421	003537	14
47	847836	212	851121	209	996715	421	003285	13
48	847964	212	850996	209	996968	421	003032	12
49	848091	212	850870	209	997221	421	002779	11
50	848218	212	850745	209	997473	421	002527	10
51	9.848345	212	9.850619	209	9.997726	421	10.002274	9
52	848472	211	850493	210	997979	421	002021	8
53	848599	211	850368	210	998231	421	001769	7
54	848726	211	850242	210	998484	421	001516	6
55	848852	211	850116	210	998737	421	001263	5
56	848979	211	849990	210	998989	421	001011	4
57	849106	211	849864	210	999242	421	000758	3
58	849232	211	849738	210	999495	421	000505	2
59	849359	211	849611	210	999748	421	000253	1
60	849485	211	849485	210	10.000000	421	000000	0
	Cosine		Sine		Cotang.		Tang.	M.

45 Degrees.

A
TABLE
OF
NATURAL SINES AND TANGENTS;
TO
EVERY TEN MINUTES OF A DEGREE.

If the given angle is less than 45°, look for the title of the column, at the *top* of the page; and for the degrees and minutes, on the *left*. But if the angle is between 45° and 90°, look for the title of the column, at the *bottom;* and for the degrees and minutes on the *right*.

The *Secants and Cosecants*, which are not inserted in this table, may be easily supplied. If 1 be divided by the cosine of an arc, the quotient will be the secant of that arc. (Art. 228.) And if 1 be divided by the sine, the quotient will be the cosecant.

	0 Deg.		1 Deg.		2 Deg.		3 Deg.		4 Deg.		
M	Nat. Sine	N. Co-Sine	Nat. Sine.	N. Co-Sine	Nat. Sine	N. Co-Sine	Nat. Sine	N. Co-Sine	Nat. Sine	N. Co-Sine	M
0	00000	Unit.	01745	99985	03490	99939	05234	99863	06976	99756	60
1	00029	00000	01774	99984	03519	99938	05263	99861	07005	99754	59
2	00058	00000	01803	99984	03548	99937	05292	99860	07034	99752	58
3	00087	00000	01832	99983	03577	99936	05321	99858	07063	99750	57
4	00116	00000	01862	99983	03606	99935	05350	99857	07092	99748	56
5	00145	00000	01891	99982	03635	99934	05379	99855	07121	99746	55
6	00175	00000	01920	99982	03664	99933	05408	99854	07150	99744	54
7	00204	00000	01949	99981	03693	99932	05437	99852	07179	99742	53
8	00233	00000	01978	99980	03723	99931	05466	99851	07208	99740	52
9	00262	00000	02007	99980	03752	99930	05495	99849	07237	99738	51
10	00291	00000	02036	99979	03781	99929	05524	99847	07266	99736	50
11	00320	99999	02065	99979	03810	99927	05553	99846	07295	99734	49
12	00349	99999	02094	99978	03839	99926	05582	99844	07324	99731	48
13	00378	99999	02123	99977	03868	99925	05611	99842	07353	99729	47
14	00407	99999	02152	99977	03897	99924	05640	99841	07382	99727	46
15	00436	99999	02181	99976	03926	99923	05669	99839	07411	99725	45
16	00465	99999	02211	99976	03955	99922	05698	99838	07440	99723	44
17	00495	99999	02240	99975	03984	99921	05727	99836	07469	99721	43
18	00524	99999	02269	99974	04013	99919	05756	99834	07498	99719	42
19	00553	99998	02298	99974	04042	99918	05785	99833	07527	99716	41
20	00582	99998	02327	99973	04071	99917	05814	99831	07556	99714	40
21	00611	99998	02356	99972	04100	99916	05844	99829	07585	99712	39
22	00640	99998	02385	99972	04129	99915	05873	99827	07614	99710	38
23	00669	99998	02414	99971	04159	99913	05902	99826	07643	99708	37
24	00698	99998	02443	99970	04188	99912	05931	99824	07672	99705	36
25	00727	99997	02472	99969	04217	99911	05960	99822	07701	99703	35
26	00756	99997	02501	99969	04246	99910	05989	99821	07730	99701	34
27	00785	99997	02530	99968	04275	99909	06018	99819	07759	99699	33
28	00814	99997	02560	99967	04304	99907	06047	99817	07788	99696	32
29	00844	99996	02589	99966	04333	99906	06076	99815	07817	99694	31
30	00873	99996	02618	99966	04362	99905	06105	99813	07846	99692	30
31	00902	99996	02647	99965	04391	99904	06134	99812	07875	99689	29
32	00931	99996	02676	99964	04420	99902	06163	99810	07904	99687	28
33	00960	99995	02705	99963	04449	99901	06192	99808	07933	99685	27
34	00989	99995	02734	99963	04478	99900	06221	99806	07962	99683	26
35	01018	99995	02763	99962	04507	99898	06250	99804	07991	99680	25
36	01047	99995	02792	99961	04536	99897	06279	99803	08020	99678	24
37	01076	99994	02821	99960	04565	99896	06308	99801	08049	99676	23
38	01105	99994	02850	99959	04594	99894	06337	99799	08078	99673	22
39	01134	99994	02879	99959	04623	99893	06366	99797	08107	99671	21
40	01164	99993	02908	99958	04653	99892	06395	99795	08136	99668	20
41	01193	99993	02938	99957	04682	99890	06424	99793	08165	99666	19
42	01222	99993	02967	99956	04711	99889	06453	99792	08194	99664	18
43	01251	99992	02996	99955	04740	99888	06482	99790	08223	99661	17
44	01280	99992	03025	99954	04769	99886	06511	99788	08252	99659	16
45	01309	99991	03054	99953	04798	99885	06540	99786	08281	99657	15
46	01338	99991	03083	99952	04827	99883	06569	99784	08310	99654	14
47	01367	99991	03112	99952	04856	99882	06598	99782	08339	99652	13
48	01396	99990	03141	99951	04885	99881	06627	99780	08368	99649	12
49	01425	99990	03170	99950	04914	99879	06656	99778	08397	99647	11
50	01454	99989	03199	99949	04943	99878	06685	99776	08426	99644	10
51	01483	99989	03228	99948	04972	99876	06714	99774	08455	99642	9
52	01513	99989	03257	99947	05001	99875	06743	99772	08484	99639	8
53	01542	99988	03286	99946	05030	99873	06773	99770	08513	99637	7
54	01571	99988	03316	99945	05059	99872	06802	99768	08542	99635	6
55	01600	99987	03345	99944	05088	99870	06831	99766	08571	99632	5
56	01629	99987	03374	99943	05117	99869	06860	99764	08600	99630	4
57	01658	99986	03403	99942	05146	99867	06889	99762	08629	99627	3
58	01687	99986	03432	99941	05175	99866	06918	99760	08658	99625	2
59	01716	99985	03461	99940	05205	99864	06947	99758	08687	99622	1
M	N.CS.	N. S.	N.CS.	N. S.	N. CS.	N. S.	N. CS.	N. S.	N. CS.	N. S.	M
	89 Deg.		88 Deg.		87 Deg.		86 Deg.		85 Deg.		

	5 Deg.		6 Deg.		7 Deg.		8 Deg.		9 Deg.		
M	N. S.	N.CS.	N. S.	N.CS.	N. S.	N.CS.	N. S.	N. CS.	N. S.	N. CS.	M
0	08716	99619	10453	99452	12187	99255	13917	99027	15643	98769	60
1	08745	99617	10482	99449	12216	99251	13946	99023	15672	98764	59
2	08774	99614	10511	99446	12245	99248	13975	99019	15701	98760	58
3	08803	99612	10540	99443	12274	99244	14004	99015	15730	98755	57
4	08831	99609	10569	99440	12302	99240	14033	99011	15758	98751	56
5	08860	99607	10597	99437	12331	99237	14061	99006	15787	98746	55
6	08889	99604	10626	99434	12360	99233	14090	99002	15816	98741	54
7	08918	99602	10655	99431	12389	99230	14119	98998	15845	98737	53
8	08947	99599	10684	99428	12418	99226	14148	98994	15873	98732	52
9	08976	99596	10713	99424	12447	99222	14177	98990	15902	98728	51
10	09005	99594	10742	99421	12476	99219	14205	98986	15931	98723	50
11	09034	99591	10771	99418	12504	99215	14234	98982	15959	98718	49
12	09063	99588	10800	99415	12533	99211	14263	98978	15988	98714	48
13	09092	99586	10829	99412	12562	99208	14292	98973	16017	98709	47
14	09121	99583	10858	99409	12591	99204	14320	98969	16046	98704	46
15	09150	99580	10887	99406	12620	99200	14349	98965	16074	98700	45
16	09179	99578	10916	99402	12649	99197	14378	98961	16103	98695	44
17	09208	99575	10945	99399	12678	99193	14407	98957	16132	98690	43
18	09237	99572	10973	99396	12706	99189	14436	98953	16160	98686	42
19	09266	99570	11002	99393	12735	99186	14464	98948	16189	98681	41
20	09295	99567	11031	99390	12764	99182	14493	98944	16218	98676	40
21	09324	99564	11060	99386	12793	99178	14522	98940	16246	98671	39
22	09353	99562	11089	99383	12822	99175	14551	98936	16275	98667	38
23	09382	99559	11118	99380	12851	99171	14580	98931	16304	98662	37
24	09411	99556	11147	99377	12880	99167	14608	98927	16333	98657	36
25	09440	99553	11176	99374	12908	99163	14637	98923	16361	98652	35
26	09469	99551	11205	99370	12937	99160	14666	98919	16390	98648	34
27	09498	99548	11234	99367	12966	99156	14695	98914	16419	98643	33
28	09527	99545	11263	99364	12995	99152	14723	98910	16447	99638	32
29	09556	99542	11291	99360	13024	99148	14752	98906	16476	98633	31
30	09585	99540	11320	99357	13053	99144	14781	98902	16505	98629	30
31	09614	99537	11349	99354	13081	99141	14810	98897	16533	98624	29
32	09642	99534	11378	99351	13110	99137	14838	98893	16562	98619	28
33	09671	99531	11407	99347	13139	99133	14867	98889	16591	98614	27
34	09700	99528	11436	99344	13168	99129	14896	98884	16620	98609	26
35	09729	99526	11465	99341	13197	99125	14925	98880	16648	98604	25
36	09758	99523	11494	99337	13226	99122	14954	98876	16677	98600	24
37	09787	99520	11523	99334	13254	99118	14982	98871	16706	98595	23
38	09816	99517	11552	99331	13283	99114	15011	98867	16734	98590	22
39	09845	99514	11580	99327	13312	99110	15040	98863	16763	98585	21
40	09874	99511	11609	99324	13341	99106	15069	98858	16792	98580	20
41	09903	99508	11638	99320	13370	99102	15097	98854	16820	98575	19
42	09932	99506	11667	99317	13399	99098	15126	98849	16849	98570	18
43	09961	99503	11696	99314	13427	99094	15155	98845	16878	98565	17
44	09990	99500	11725	99310	13456	99091	15184	98841	16906	98561	16
45	10019	99497	11754	99307	13485	99087	15212	98836	16935	98556	15
46	10048	99494	11783	99303	13514	99083	15241	98832	16964	98551	14
47	10077	99491	11812	99300	13543	99079	15270	98827	16992	98546	13
48	10106	99488	11840	99297	13572	99075	15292	98823	17021	98541	12
49	10135	99485	11869	99293	13600	99071	15327	98818	17050	98536	11
50	10164	99482	11898	99290	13629	99067	15356	98814	17078	88531	10
51	10192	99479	11927	99286	13658	99063	15385	98809	17107	98526	9
52	10221	99476	11956	99283	13687	99059	15414	98805	17136	98521	8
53	10250	99473	11985	99279	13716	99055	15442	98800	17164	98516	7
54	10279	99470	12014	99276	13744	99051	15471	98796	17193	98511	6
55	10308	99467	12043	99272	13773	99047	15500	98791	17222	98506	5
56	10337	99464	12071	99269	13802	99043	15529	98787	17250	98501	4
57	10366	99461	12100	99265	13831	99039	15557	98782	17279	98496	3
58	10395	99458	12129	99262	13860	99035	15586	98778	17308	98491	2
59	10424	99455	12158	99258	13889	99031	15615	98773	17336	98486	1
M	N.CS	N. S.	N. CS.	N. S.	N. CS.	N. S.	N.CS	N. S.	N.CS.	N. S.	M
	84 Deg.		83 Deg.		82 Deg.		81 Deg.		80 Deg.		

	10 Deg.		11 Deg.		12 Deg.		13 Deg.		14 Deg.		
M	N. S.	N.CS.	N. S.	N.CS.	N. S.	N.CS.	N. S.	N. CS.	N. S.	N. CS.	M
0	17365	98481	19081	98163	20791	97815	22495	97437	24192	97030	60
1	17393	98476	19109	98157	20820	97809	22523	97430	24220	97023	59
2	17422	98471	19138	98152	20848	97803	22552	97424	24249	97015	58
3	17451	98466	19167	98146	20877	97797	22580	97417	24277	97008	57
4	17479	98461	19195	98140	20905	97791	22608	97411	24305	97001	56
5	17508	98455	19224	98135	20933	97784	22637	97404	24333	96994	55
6	17537	98450	19252	98129	20962	97778	22665	97398	24362	96987	54
7	17565	98445	19281	98124	20990	97772	22693	97391	24390	96980	53
8	17594	98440	19309	98118	21019	97766	22722	97384	24418	96973	52
9	17623	98435	19338	98112	21047	97760	22750	97378	24446	96966	51
10	17651	98430	19366	98107	21076	97754	22778	97371	24474	96959	50
11	17680	98425	19395	98101	21104	97748	22807	97365	24503	96952	49
12	17708	98420	19423	98096	21132	97742	22835	97358	24531	96945	48
13	17737	98414	19452	98090	21161	97735	22863	97351	24559	96937	47
14	17766	98409	19481	98084	21189	97729	22892	97345	24587	96930	46
15	17794	98404	19509	98079	21218	97723	22920	97338	24615	96923	45
16	17823	98399	19538	98073	21246	97717	22948	97331	24644	96916	44
17	17852	98394	19566	98067	21275	97711	22977	97325	24672	96909	43
18	17880	98389	19595	98061	21303	97705	23005	97318	24700	96902	42
19	17909	98383	19623	98056	21331	97698	23033	97311	24728	96894	41
20	17937	98378	19652	98050	21360	97692	23062	97304	24756	96887	40
21	17966	98373	19680	98044	21388	97686	23090	97298	24784	96880	39
22	17995	98368	19709	98039	21417	97680	23118	97291	24813	96873	38
23	18023	98362	19737	98033	21445	97673	23146	97284	24841	96866	37
24	18052	98357	19766	98027	21474	97667	23175	97278	24869	96858	36
25	18081	98352	19794	98021	21502	97661	23203	97271	24897	96851	35
26	18109	98347	19823	98016	21530	97655	23231	97264	24925	96844	34
27	18138	98341	19851	98010	21559	97648	23260	97257	24953	96837	33
28	18166	98336	19880	98004	21587	97642	23288	97251	24982	96829	32
29	18195	98331	19908	97998	21616	97636	23316	97244	25010	96822	31
30	18224	98325	19937	97992	21644	97630	23345	97237	25038	96815	30
31	18252	98320	19965	97987	21672	97623	23373	97230	25066	96807	29
32	18281	98315	19994	97981	21701	97617	23401	97223	25094	96800	28
33	18309	98310	20022	97975	21729	97611	23429	97217	25122	96793	27
34	18338	98304	20051	97969	21758	97604	23458	97210	25151	96786	26
35	18367	98299	20079	97963	21786	97598	23486	97203	25179	96778	25
36	18395	98294	20108	97958	21814	97592	23514	97196	25207	96771	24
37	18424	98288	20136	97952	21843	97585	23542	97189	25235	96764	23
38	18452	98283	20165	97946	21871	97579	23571	97182	25263	96756	22
39	18481	98277	20193	97940	21899	97573	23599	97176	25291	96749	21
40	18509	98272	20222	97934	21928	97566	23627	97169	25320	96742	20
41	18538	98267	20250	97928	21956	97560	23656	97162	25348	96734	19
42	18567	98261	20279	97922	21985	97553	23684	97155	25376	96727	18
43	18595	98256	20307	97916	22013	97547	23712	97148	25404	96719	17
44	18624	98250	20336	97910	22041	97541	23740	97141	25432	96712	16
45	18652	98245	20364	97905	22070	97534	23769	97134	25460	96705	15
46	18681	98240	20393	97899	22098	97528	23797	97127	25488	96697	14
47	18710	98234	20421	97893	22126	97521	23825	97120	25516	96690	13
48	18738	98229	20450	97887	22155	97515	23853	97113	25545	96682	12
49	18767	98223	20478	97881	22183	97508	23882	97106	25573	96675	11
50	18795	98218	20507	97875	22212	97502	23910	97100	25601	96667	10
51	18824	98212	20535	97869	22240	97496	23938	97093	25629	96660	9
52	18852	98207	20563	97863	22268	97489	23966	97086	25657	96653	8
53	18881	98201	20592	97857	22297	97483	23995	97079	25685	96645	7
54	18910	98196	20620	97851	22325	97476	24023	97072	25713	96638	6
55	18938	98190	20649	97845	22353	97470	24051	97065	25741	96630	5
56	18967	98185	20677	97839	22382	97463	24079	97058	25769	96623	4
57	18995	98179	20706	97833	22410	97457	24108	97051	25798	96615	3
58	19024	98174	20734	97827	22438	97450	24136	97044	25826	96608	2
59	19052	98168	20763	97821	22467	97444	24164	97037	25854	96600	1
M	N.CS.	N. S.	N.CS.	N. S.	N.CS.	N. S.	N. CS.	N. S.	N. CS.	N. S.	M
	79 Deg.		78 Deg.		77 Deg.		76 Deg.		75 Deg.		

	15 Deg.		16 Deg.		17 Deg.		18 Deg.		19 Deg.		
M	N. S.	N. CS.	N. S.	N.CS.	N. S.	N.CS.	N. S.	N. CS.	N. S.	N. CS.	M
0	25882	96593	27564	96126	29237	95630	30902	95106	32557	94552	60
1	25910	96585	27592	96118	29265	95622	30929	95097	32584	94542	59
2	25938	96578	27620	96110	29293	95613	30957	95088	32612	94533	58
3	25966	96570	27648	96102	29321	95605	30985	95079	32639	94523	57
4	25994	96562	27676	96094	29348	95596	31012	95070	32667	94514	56
5	26022	96555	27704	96086	29376	95588	31040	95061	32694	94504	55
6	26050	96547	27731	96078	29404	95579	31068	95052	32722	94495	54
7	26079	96540	27759	96070	29432	95571	31095	95043	32749	94485	53
8	26107	96532	27787	96062	29460	95562	31123	95033	32777	94476	52
9	26135	96524	27815	96054	29487	95554	31151	95024	32804	94466	51
10	26163	96517	27843	96046	29515	95545	31178	95015	32832	94457	50
11	26191	96509	27871	96037	29543	95536	31206	95006	32859	94447	49
12	26219	96502	27899	96029	29571	95528	31233	94997	32887	94438	48
13	26247	96494	27927	96021	29599	95519	31261	94988	32914	94428	47
14	26275	96486	27955	96013	29626	95511	31289	94979	32942	94418	46
15	26303	96479	27983	96005	29654	95502	31316	94970	32969	94409	45
16	26331	96471	28011	95997	29682	95493	31344	94961	32997	94399	44
17	26359	96463	28039	95989	29710	95485	31372	94952	33024	94390	43
18	26387	96456	28067	95981	29737	95476	31399	94943	33051	94380	42
19	26415	96448	28095	95972	29765	95467	31427	94933	33079	94370	41
20	26443	96440	28123	95964	29793	95459	31454	94924	33106	94361	40
21	26471	96433	28150	95956	29821	95450	31482	94915	33134	94351	39
22	26500	96425	28178	95948	29849	95441	31510	94906	33161	94342	38
23	26528	96417	28206	95940	29876	95433	31537	94897	33189	94332	37
24	26556	96410	28234	95931	29904	95424	31565	94888	33216	94322	36
25	26584	96402	28262	95923	29932	95415	31593	94878	33244	94313	35
26	26612	96394	28290	95915	29960	95407	31620	94869	33271	94303	34
27	26640	96386	28318	95907	29987	95398	31648	94860	33298	94293	33
28	26668	96379	28346	95898	30015	95389	31675	94851	33326	94284	32
29	26696	96371	28374	95890	30043	95380	31703	94842	33353	94274	31
30	26724	96363	28402	95882	30071	95372	31730	94832	33381	94264	30
31	26752	96355	28429	95874	30098	95363	31758	94823	33408	94254	29
32	26780	96347	28457	95865	30126	95354	31786	94814	33436	94245	28
33	26808	96340	28485	95857	30154	95345	31813	94805	33463	94235	27
34	26836	96332	28513	95849	30182	95337	31841	94795	33490	94225	26
35	26864	96324	28541	95841	30209	95328	31868	94786	33518	94215	25
36	26892	96316	28569	95832	30237	95319	31896	94777	33545	94206	24
37	26920	96308	28597	95824	30265	95310	31923	94768	33573	94196	23
38	26948	96301	28625	95816	30292	95301	31951	94758	33600	94186	22
39	26976	96293	28652	95807	30320	95293	31979	94749	33627	94176	21
40	27004	96285	28680	95799	30348	95284	32006	94740	33655	94167	20
41	27032	96277	28708	95791	30376	95275	32034	94730	33682	94157	19
42	27060	96269	28736	95782	30403	95266	32061	94721	33710	94147	18
43	27088	96261	28764	95774	30431	95257	32089	94712	33737	94137	17
44	27116	96253	28792	95766	30459	95248	32116	94702	33764	94127	16
45	27144	96246	28820	95757	30486	95240	32144	94693	33792	94118	15
46	27172	96238	28847	95749	30514	95231	32171	94684	33819	94108	14
47	27200	96230	28875	95740	30542	95222	32199	94674	33846	94098	13
48	27228	96222	28903	95732	30570	95213	32227	94665	33874	94088	12
49	27256	96214	28931	95724	30597	95204	32254	94656	33901	94078	11
50	27284	96206	28959	95715	30625	95195	32282	94646	33929	94068	10
51	27312	96198	28987	95707	30653	95186	32309	94637	33956	94058	9
52	27340	96190	29015	95698	30680	95177	32337	94627	33983	94049	8
53	27368	96182	29042	95690	30708	95168	32364	94618	34011	94039	7
54	27396	96174	29070	95681	30736	95159	32392	94609	34038	94029	6
55	27424	96166	29098	95673	30763	95150	32419	94599	34065	94019	5
56	27452	96158	29126	95664	30791	95142	32447	94590	34093	94009	4
57	27480	96150	29154	95656	30819	95133	32474	94580	34120	93999	3
58	27508	96142	29182	95647	30846	95124	32502	94571	34147	93989	2
59	27536	96134	29209	95639	30874	95115	32529	94561	34175	93979	1
M	N. CS.	N. S.	N.CS.	N. S.	N. CS	N. S.	N. CS.	N. S.	N. CS.	N. S.	M
	74 Deg.		73 Deg.		72 Deg.		71 Deg.		70 Deg.		

	20 Deg.		21 Deg.		22 Deg.		23 Deg.		24 Deg.		
M	N. S.	N. CS.	N. S.	N.CS	N.S.	N.CS.	N. S.	N. CS.	N. S.	N. CS.	M
0	34202	93969	35837	93358	37461	92718	39073	92050	40674	91355	60
1	34229	93959	35864	93348	37488	92707	39100	92039	40700	91343	59
2	34257	93949	35891	93337	37515	92697	39127	92028	40727	91331	58
3	34284	93939	35918	93327	37542	92686	39153	92016	40753	91319	57
4	34311	93929	35945	93316	37569	92675	39180	92005	40780	91307	56
5	34339	93919	35973	93306	37595	92664	39207	91994	40806	91295	55
6	34366	93909	36000	93295	37622	92653	39234	91982	40833	91283	54
7	34393	93899	36027	93285	37649	92642	39260	91971	40860	91272	53
8	34421	93889	36054	93274	37676	92631	39287	91959	40886	91260	52
9	34448	93879	36081	93264	37703	92620	39314	91948	40913	91248	51
10	34475	93869	36108	93253	37730	92609	39341	91936	40939	91236	50
11	34503	93859	36135	93243	37757	92598	39367	91925	40966	91224	49
12	34530	93849	36162	93232	37784	92587	39394	91914	40992	91212	48
13	34557	93839	36190	93222	37811	92576	39421	91902	41019	91200	47
14	34584	93829	36217	93211	37838	92565	39448	91891	41045	91188	46
15	34612	93819	36244	93201	37865	92554	39474	91879	41072	91176	45
16	34639	93809	36271	93190	37892	92543	39501	91868	41098	91164	44
17	34666	93799	36298	93180	37919	92532	39528	91856	41125	91152	43
18	34694	93789	36325	93169	37946	92521	39555	91845	41151	91140	42
19	34721	93779	36352	93159	37973	92510	39581	91833	41178	91128	41
20	34748	93769	36379	93148	37999	92499	39608	91822	41204	91116	40
21	34775	93759	36406	93137	38026	92488	39635	91810	41231	91104	39
22	34803	93748	36434	93127	38053	92477	39661	91799	41257	91092	38
23	34830	93738	36461	93116	38080	92466	39688	91787	41284	91080	37
24	34857	93728	36488	93106	38107	92455	39715	91775	41310	91068	36
25	34884	93718	36515	93095	38134	92444	39741	91764	41337	91056	35
26	34912	93708	36542	93084	38161	92432	39768	91752	41363	91044	34
27	34939	93698	36569	93074	38188	92421	39795	91741	41390	91032	33
28	34966	93688	36596	93063	38215	92410	39822	91729	41416	91020	32
29	34993	93677	36623	93052	38241	92399	39848	91718	41443	91008	31
30	35021	93667	36650	93042	38268	92388	39875	91706	41469	90996	30
31	35048	93657	36677	93031	38295	92377	39902	91694	41496	90984	29
32	35075	93647	36704	93020	38322	92366	39928	91683	41522	90972	28
33	35102	93637	36731	93010	38349	92355	39955	91671	41549	90960	27
34	35130	93626	36758	92999	38376	92343	39982	91660	41575	90948	26
35	35157	93616	36785	92988	38403	92332	40008	91648	41602	90936	25
36	35183	93606	36812	92978	38430	92321	40035	91636	41628	90924	24
37	35211	93596	36839	92967	38456	92310	40062	91625	41655	90911	23
38	35239	93585	36867	92956	38483	92299	40088	91613	41681	90899	22
39	35266	93575	36894	92945	38510	92287	40115	91601	41707	90887	21
40	35293	93565	36921	92935	38537	92276	40141	91590	41734	90875	20
41	35320	93555	36948	92924	38564	92265	40168	91578	41760	90863	19
42	35347	93544	36975	82913	38591	92254	40195	91566	41787	90851	18
43	35375	93534	37002	92902	38617	92243	40221	91555	41813	90839	17
44	35402	93524	37029	92892	38644	92231	40248	91543	41840	90826	16
45	35429	93514	37056	92881	38671	92220	40275	91531	41866	90814	15
46	35456	93503	37083	92870	38698	92209	40301	91519	41892	90802	14
47	35484	93493	37110	92859	38725	92198	40328	91508	41919	90790	13
48	35511	93483	37137	92849	38752	92186	40355	91496	41945	90778	12
49	35538	93472	37164	92838	38778	92175	40381	91484	41972	90766	11
50	35565	93462	37191	92827	38805	92164	40408	91472	41998	90753	10
51	35592	93452	37218	92816	38832	92152	40434	91461	42024	90741	9
52	35619	93441	37245	92805	38859	92141	40461	91449	42051	90729	8
53	35647	93431	37272	92794	38886	92130	40488	91437	42077	90717	7
54	35674	93420	37299	92784	38912	92119	40514	91425	42104	90704	6
55	35701	93410	37326	92773	38939	92107	40541	91414	42130	90692	5
56	35728	93400	37353	92762	38966	92096	40567	91402	42156	90680	4
57	35755	93389	37380	92751	38993	92085	40594	91390	42183	90668	3
58	35782	93379	37407	92740	39020	92073	40621	91378	42209	90655	2
59	35810	93368	37434	92729	39046	92062	40647	91366	42235	90643	1
M	N. CS.	N. S.	N.CS.	N. S.	N. CS.	N. S.	N. CS.	N. S.	N. CS.	N. S.	M
	69 Deg.		68 Deg.		67 Deg.		66 Deg.		65 Deg.		

	25 Deg.		26 Deg.		27 Deg.		28 Deg.		29 Deg.		
M	N. S.	N. CS.	N. S.	N. CS.	N. S.	N. CS.	N. S.	N. CS.	N. S.	N. CS.	M
0	42262	90631	43837	89879	45399	89101	46947	88295	48481	87462	60
1	42288	90618	43863	89867	45425	89087	46973	88281	48506	87448	59
2	42315	90606	43889	89854	45451	89074	46999	88267	48532	87434	58
3	42341	90594	43916	89841	45477	89061	47024	88254	48557	87420	57
4	42367	90582	43942	89828	45503	89048	47050	88240	48583	87406	56
5	42394	90569	43968	89816	45529	89035	47076	88226	48608	87391	55
6	42420	90557	43994	89803	45554	89021	47101	88213	48634	87377	54
7	42446	90545	44020	89790	45580	89008	47127	88199	48659	87363	53
8	42473	90532	44046	89777	45606	88995	47153	88185	48684	87349	52
9	42499	90520	44072	89764	45632	88981	47178	88172	48710	87335	51
10	42525	90507	44098	89752	45658	88968	47204	88158	48735	87321	50
11	42552	90495	44124	89739	45684	88955	47229	88144	48761	87306	49
12	42578	90483	44151	89726	45710	88942	47255	88130	48786	87292	48
13	42604	90470	44177	89713	45736	88928	47281	88117	48811	87278	47
14	42631	90458	44203	89700	45762	88915	47306	88103	48837	87264	46
15	42657	90446	44229	89687	45787	88902	47332	88089	48862	87250	45
16	42683	90433	44255	89674	45813	88888	47358	88075	48888	87235	44
17	42709	90421	44281	89662	45839	88875	47383	88062	48913	87221	43
18	42736	90408	44307	89649	45865	88862	47409	88048	48938	87207	42
19	42762	90396	44333	89636	45891	88848	47434	88034	48964	87193	41
20	42788	90383	44359	89623	45917	88835	47460	88020	48989	87178	40
21	42815	90371	44385	89610	45942	88822	47486	88006	49014	87164	39
22	42841	90358	44411	89597	45968	88808	47511	87993	49040	87150	38
23	42867	90346	44437	89584	45994	88795	47537	87979	49065	87136	37
24	42894	90334	44464	89571	46020	88782	47562	87965	49090	87121	36
25	42920	90321	44490	89558	46046	88768	47588	87951	49116	87107	35
26	42946	90309	44516	89545	46072	88755	47614	87937	49141	87093	34
27	42972	90296	44542	89532	46097	88741	47639	87923	49166	87079	33
28	42999	90284	44568	89519	46123	88728	47665	87909	49192	87064	32
29	43025	90271	44594	89506	46149	88715	47690	87896	49217	87050	31
30	43051	90259	44620	89493	46175	88701	47716	87882	49242	87036	30
31	43077	90246	44646	89480	46201	88688	47741	87868	49268	87021	29
32	43104	90233	44672	89467	46226	88674	47767	87854	49293	87007	28
33	43130	90221	44698	89454	46252	88661	47793	87840	49318	86993	27
34	43156	90208	44724	89441	46278	88647	47818	87826	49344	86978	26
35	43182	90196	44750	89428	46304	88634	47844	87812	49369	86964	25
36	43209	90183	44776	89415	46330	88620	47869	87798	49394	86949	24
37	43235	90171	44802	89402	46355	88607	47895	87784	49419	86935	23
38	43261	90158	44828	89389	46381	88593	47920	87770	49445	86921	22
39	43287	90146	44854	89376	46407	88580	47946	87756	49470	86906	21
40	43313	90133	44880	89363	46433	88566	47971	87743	49495	86892	20
41	43340	90120	44906	89350	46458	88553	47997	87729	49521	86878	19
42	43366	90108	44932	89337	46484	88539	48022	87715	49546	86863	18
43	43392	90095	44958	89324	46510	88526	48048	87701	49571	86849	17
44	43418	90082	44984	89311	46536	88512	48073	87687	49596	86834	16
45	43445	90070	45010	89298	46561	88499	48099	87673	49622	86820	15
46	43471	90057	45036	89285	46587	88485	48124	87659	49647	86805	14
47	43497	90045	45062	89272	46613	88472	48150	87645	49672	86791	13
48	43523	90032	45088	89259	46639	88458	48175	87631	49697	86777	12
49	43549	90019	45114	89245	46664	88445	48201	87617	49723	86762	11
50	43575	90007	45140	89232	46690	88431	48226	87603	49748	86748	10
51	43602	89994	45166	89219	46716	88417	48252	87589	49773	86733	9
52	43628	89981	45192	89206	46742	88404	48277	87575	49798	86719	8
53	43654	89968	45218	89193	46767	88390	48303	87561	49824	86704	7
54	43680	89956	45243	89180	46793	88377	48328	87546	49849	86690	6
55	43706	89943	45269	89167	46819	88363	48354	87532	49874	86675	5
56	43733	89930	45295	89153	46844	88349	48379	87518	49899	86661	4
57	43759	89918	45321	89140	46870	88336	48405	87504	49924	86646	3
58	43785	89905	45347	89127	46896	88322	48430	87490	49950	86632	2
59	43811	89892	45373	89114	46921	88308	48456	87476	49975	86617	1
M	N. CS.	N. S.	N.CS.	N. S.	N. CS.	N. S.	N. CS.	N. S.	N. CS.	N. S.	M
	64 Deg.		63 Deg.		62 Deg.		61 Deg.		60 Deg.		

	30 Deg.		31 Deg.		32 Deg.		33 Deg.		34 Deg.		
M	N. S.	N. CS.	N. S.	N. CS.	N. S.	N. CS.	N. S.	N. CS.	N. S.	N. CS.	M
0	50000	86603	51504	85717	52992	84805	54464	83867	55919	82904	60
1	50025	86588	51529	85702	53017	84789	54488	83851	55943	82887	59
2	50050	86573	51554	85687	53041	84774	54513	83835	55968	82871	58
3	50076	86559	51579	85672	53066	84759	54537	83819	55992	82855	57
4	50101	86544	51604	85657	53091	84743	54561	83804	56016	82839	56
5	50126	86530	51628	85642	53115	84728	54586	83788	56040	82822	55
6	50151	86515	51653	85627	53140	84712	54610	83772	56064	82806	54
7	50176	86501	51678	85612	53164	84697	54635	83756	56088	82790	53
8	50201	86486	51703	85597	53189	84681	54659	83740	56112	82773	52
9	50227	86471	51728	85582	53214	84666	54683	83724	56136	82757	51
10	50252	86457	51753	85567	53238	84650	54708	83708	56160	82741	50
11	50277	86442	51778	85551	53263	84635	54732	83692	56184	82724	49
12	50302	86427	51803	85536	53288	84619	54756	83676	56208	82708	48
13	50327	86413	51828	85521	53312	84604	54781	83660	56232	82692	47
14	50352	86398	51852	85506	53337	84588	54805	83645	56256	82675	46
15	50377	86384	51877	85491	53361	84573	54829	83629	56280	82659	45
16	50403	86369	51902	85476	53386	84557	54854	83613	56305	82643	44
17	50428	86354	51927	85461	53411	84542	54878	83597	56329	82626	43
18	50453	86340	51952	85446	53435	84526	54902	83581	56353	82610	42
19	50478	86325	51977	85431	53460	84511	54927	83565	56377	82593	41
20	50503	86310	52002	85416	53484	84495	54951	83549	56401	82577	40
21	50528	86295	52026	85401	53509	84480	54975	83533	56425	82561	39
22	50553	86281	52051	85385	53534	84464	54999	83517	56449	82544	38
23	50578	86266	52076	85370	53558	84448	55024	83501	56473	82528	37
24	50603	86251	52101	85355	53583	84433	55048	83485	56497	82511	36
25	50628	86237	52126	85340	53607	84417	55072	83469	56521	82495	35
26	50654	86222	52151	85325	53632	84402	55097	83453	56545	82478	34
27	50679	86207	52175	85310	53656	84386	55121	83437	56569	82462	33
28	50704	86192	52200	85294	53681	84370	55145	83421	56593	82446	32
29	50729	86178	52225	85279	53705	84355	55169	83405	56617	82429	31
30	50754	86163	52250	85264	53730	84339	55194	83389	56641	82413	30
31	50779	86148	52275	85249	53754	84324	55218	83373	56665	82396	29
32	50804	86133	52299	85234	53779	84308	55242	83356	56689	82380	28
33	50829	86119	52324	85218	53804	84292	55266	83340	56713	82363	27
34	50854	86104	52349	85203	53828	84277	55291	83324	56736	82347	26
35	50879	86089	52374	85188	53853	84261	55315	83308	56760	82330	25
36	50904	86074	52399	85173	53877	84245	55339	83292	56784	82314	24
37	50929	86059	52423	85157	53902	84230	55363	83276	56808	82297	23
38	50954	86045	52448	85142	53926	84214	55388	83260	56832	82281	22
39	50979	86030	52473	85127	53951	84198	55412	83244	56856	82264	21
40	51004	86015	52498	85112	53975	84182	55436	83228	56880	82248	20
41	51029	86000	52522	85096	54000	84167	55460	83212	56904	82231	19
42	51054	85985	52547	85081	54024	84151	55484	83195	56928	82214	18
43	51079	85970	52572	85066	54049	84135	55509	83179	56952	82198	17
44	51104	85956	52597	85051	54073	84120	55533	83163	56976	82181	16
45	51129	85941	52621	85035	54097	84104	55557	83147	57000	82165	15
46	51154	85926	52646	85020	54122	84088	55581	83131	57024	82148	14
47	51179	85911	52671	85005	54146	84072	55605	83115	57047	82132	13
48	51204	85896	52696	84989	54171	84057	55630	83098	57071	82115	12
49	51229	85881	52720	84974	54195	84041	55654	83082	57095	82098	11
50	51254	85866	52745	84959	54220	84025	55678	83066	57119	82082	10
51	51279	85851	52770	84943	54244	84009	55702	83050	57143	82065	9
52	51304	85836	52794	84928	54269	83994	55726	83034	57167	82048	8
53	51329	85821	52819	84913	54293	83978	55750	83017	57191	82032	7
54	51354	85806	52844	84897	54317	83962	55775	83001	57215	82015	6
55	51379	85792	52869	84882	54342	83946	55799	82985	57238	81999	5
56	51404	85777	52893	84866	54366	83930	55823	82969	57262	81982	4
57	51429	85762	52918	84851	54391	83915	55847	82953	57286	81965	3
58	51454	85747	52943	84836	54415	83899	55871	82936	57310	81949	2
59	51479	85732	52967	84820	54440	83883	55895	82920	57334	81932	1
M	N. CS.	N. S.	N. CS.	N. S.	N. CS.	N. S.	N. CS.	N. S.	N. CS.	N. S.	M
	59 Deg.		58 Deg.		57 Deg.		56 Deg.		55 Deg.		

	35 Deg.		36 Deg.		37 Deg.		38 Deg.		39 Deg.		
M	N. S.	N. CS.	N. S.	N. CS.	N. S.	N. CS.	N. S.	N. CS.	N. S.	N. CS.	M
0	57358	81915	58779	80902	60182	79864	61566	78801	62932	77715	60
1	57381	81899	58802	80885	60205	79846	61589	78783	62955	77696	59
2	57405	81882	58826	80867	60228	79829	61612	78765	62977	77678	58
3	57429	81865	58849	80850	60251	79811	61635	78747	63000	77660	57
4	57453	81848	58873	80833	60274	79793	61658	78729	63022	77641	56
5	57477	81832	58896	80816	60298	79776	61681	78711	63045	77623	55
6	57501	81815	58920	80799	60321	79758	61704	78694	63068	77605	54
7	57524	81798	58943	80782	60344	79741	61726	78676	63090	77586	53
8	57548	81782	58967	80765	60367	79723	61749	78658	63113	77568	52
9	57572	81765	58990	80748	60390	79706	61772	78640	63135	77550	51
10	57596	81748	59014	80730	60414	79688	61795	78622	63158	77531	50
11	57619	81731	59037	80713	60437	79671	61818	78604	63180	77513	49
12	57643	81714	59061	80696	60460	79653	61841	78586	63203	77494	48
13	57667	81698	59084	80679	60483	79635	61864	78568	63225	77476	47
14	57691	81681	59108	80662	60506	79618	61887	78550	63248	77458	46
15	57715	81664	59131	80644	60529	79600	61909	78532	63271	77439	45
16	57738	81647	59154	80627	60553	79583	61932	78514	63293	77421	44
17	57762	81631	59178	80610	60576	79565	61955	78496	63316	77402	43
18	57786	81614	59201	80593	60599	79547	61978	78478	63338	77384	42
19	57810	81597	59225	80576	60622	79530	62001	78460	63361	77366	41
20	57833	81580	59248	80558	60645	79512	62024	78442	63383	77347	40
21	57857	81563	59272	80541	60668	79494	62046	78424	63406	77329	39
22	57881	81546	59295	80524	60691	79477	62069	78405	63428	77310	38
23	57904	81530	59318	80507	60714	79459	62092	78387	63451	77292	37
24	57928	81513	59342	80489	60738	79441	62115	78369	63473	77273	36
25	57952	81496	59365	80472	60761	79424	62138	78351	63496	77255	35
26	57976	81479	59389	80455	60784	79406	62160	78333	63518	77236	34
27	57999	81462	59412	80438	60807	79388	62183	78315	63540	77218	33
28	58023	81445	59436	80420	60830	79371	62206	78297	63563	77199	32
29	58047	81428	59459	80403	60853	79353	62229	78279	63585	77181	31
30	58070	81412	59482	80386	60876	79335	62251	78261	63608	77162	30
31	58094	81395	59506	80368	60899	79318	62274	78243	63630	77144	29
32	58118	81378	59529	80351	60922	79300	62297	78225	63653	77125	28
33	58141	81361	59552	80334	60945	79282	62320	78206	63675	77107	27
34	58165	81344	59576	80316	60968	79264	62342	78188	63698	77088	26
35	58189	81327	59599	80299	60991	79247	62365	78170	63720	77070	25
36	58212	81310	59622	80282	61015	79229	62388	78152	63742	77051	24
37	58236	81293	59646	80264	61038	79211	62411	78134	63765	77033	23
38	58260	81276	59669	80247	61061	79193	62433	78116	63787	77014	22
39	58283	81259	59693	80230	61084	79176	62456	78098	63810	76996	21
40	58307	81242	59716	80212	61107	79158	62479	78079	63832	76977	20
41	58330	81225	59739	80195	61130	79140	62502	78061	63854	76959	19
42	58354	81208	59763	80178	61153	79122	62524	78043	63877	76940	18
43	58378	81191	59786	80160	61176	79105	62547	78025	63899	76921	17
44	58401	81174	59809	80143	61199	79087	62570	78007	63922	76903	16
45	58425	81157	59832	80125	61222	79069	62592	77988	63944	76884	15
46	58449	81140	59856	80108	61245	79051	62615	77970	63966	76866	14
47	58472	81123	59879	80091	61268	79033	62638	77952	63989	76847	13
48	58496	81106	59902	80073	61291	79015	62660	77934	64011	76828	12
49	58519	81089	59926	80056	61314	78998	62683	77916	64033	76810	11
50	58543	81072	59949	80038	61337	78980	62706	77897	64056	76791	10
51	58567	81055	59972	80021	61360	78962	62728	77879	64078	76772	9
52	58590	81038	59995	80003	61383	78944	62751	77861	64100	76754	8
53	58614	81021	60019	79986	61406	78926	62774	77843	64123	76735	7
54	58637	81004	60042	79968	61429	78908	62796	77824	64145	76717	6
55	58661	80987	60065	79951	61451	78891	62819	77806	64167	76698	5
56	58684	80970	60089	79934	61474	78873	62842	77788	64190	76679	4
57	58708	80953	60112	79916	61497	78855	62864	77769	64212	76661	3
58	58731	80936	60135	79899	61520	78837	62887	77751	64234	76642	2
59	58755	80919	60158	79881	61543	78819	62909	77733	64256	76623	1
M	N. CS.	N. S.	N. CS.	N. S.	N. CS.	N. S.	N. CS.	N. S.	N. CS.	N. S.	M
	54 Deg.		53 Deg.		52 Deg.		51 Deg.		50 Deg.		

	40 Deg.		41 Deg.		42 Deg.		43 Deg.		44 Deg.		
M	N. S.	N.CS.	N. S.	N.CS.	N. S.	N.CS.	N. S.	N.CS.	N. S.	N. CS.	M
0	64279	76604	65606	75471	66913	74314	68200	73135	69466	71934	60
1	64301	76586	65628	75452	66935	74295	68221	73116	69487	71914	59
2	64323	76567	65650	75433	66956	74276	68242	73096	69508	71894	58
3	64346	76548	65672	75414	66978	74256	68264	73076	69529	71873	57
4	64368	76530	65694	75395	66999	74237	68285	73056	69549	71853	56
5	64390	76511	65716	75375	67021	74217	68306	73036	69570	71833	55
6	64412	76492	65738	75356	67043	74198	68327	73016	69591	71813	54
7	64435	76473	65759	75337	67064	74178	68349	72996	69612	71792	53
8	64457	76455	65781	75318	67086	74159	68370	72976	69633	71772	52
9	64479	76436	65803	75299	67107	74139	68391	72957	69654	71752	51
10	64501	76417	65825	75280	67129	74120	68412	72937	69675	71732	50
11	64524	76398	65847	75261	67151	74100	68433	72917	69696	71711	49
12	64546	76380	65869	75241	67172	74080	68455	72897	69717	71691	48
13	64568	76361	65891	75222	67194	74061	68476	72877	69737	71671	47
14	64590	76342	65913	75203	67215	74041	68497	72857	69758	71650	46
15	64612	76323	65935	75184	67237	74022	68518	72837	69779	71630	45
16	64635	76304	65956	75165	67258	74002	68539	72817	69800	71610	44
17	64657	76286	65978	75146	67280	73983	68561	72797	69821	71590	43
18	64679	76267	66000	75126	67301	73963	68582	72777	69842	71569	42
19	64701	76248	66022	75107	67323	73944	68603	72757	69862	71549	41
20	64723	76229	66044	75088	67344	73924	68624	72737	69883	71529	40
21	64746	76210	66066	75069	67366	73904	68645	72717	69904	71508	39
22	64768	76192	66088	75050	67387	73885	68666	72697	69925	71488	38
23	64790	76173	66109	75030	67409	73865	68688	72677	69946	71468	37
24	64812	76154	66131	75011	67430	73846	68709	72657	69966	71447	36
25	64834	76135	66153	74992	67452	73826	68730	72637	69987	71427	35
26	64856	76116	66175	74973	67473	73806	68751	72617	70008	71407	34
27	64878	76097	66197	74953	67495	73787	68772	72597	70029	71386	33
28	64901	76078	66218	74934	67516	73767	68793	72577	70049	71366	32
29	64923	76059	66240	74915	67538	73747	68814	72557	70070	71345	31
30	64945	76041	66262	74896	67559	73728	68835	72537	70091	71325	30
31	64967	76022	66284	74876	67580	73708	68857	72517	70112	71305	29
32	64989	76003	66306	74857	67602	73688	68878	72497	70132	71284	28
33	65011	75984	66327	74838	67623	73669	68899	72477	70153	71264	27
34	65033	75965	66349	74818	67645	73649	68920	72457	70174	71243	26
35	65055	75946	66371	74799	67666	73629	68941	72437	70195	71223	25
36	65077	75927	66393	74780	67688	73610	68962	72417	70215	71203	24
37	65099	75908	66414	74760	67709	73590	68983	72397	70236	71182	23
38	65122	75889	66436	74741	67730	73570	69004	72377	70257	71162	22
39	65144	75870	66458	74722	67752	73551	69025	72357	70277	71141	21
40	65166	75851	66480	74703	67773	73531	69046	72337	70298	71121	20
41	65188	75832	66501	74683	67795	73511	69067	72317	70319	71100	19
42	65210	75813	66523	74664	67816	73491	69088	72297	70339	71080	18
43	65232	75794	66545	74644	67837	73472	69109	72277	70360	71059	17
44	65254	75775	66566	74625	67859	73452	69130	72257	70381	71039	16
45	65276	75756	66588	74606	67880	73432	69151	72236	70401	71019	15
46	65298	75738	66610	74586	67901	73412	69172	72216	70422	70998	14
47	65320	75719	66632	74567	67923	73393	69193	72196	70443	70978	13
48	65342	75699	66653	74548	67944	73373	69214	72176	70463	70957	12
49	65364	75680	66675	74528	67965	73353	69235	72156	70484	70937	11
50	65386	75661	66697	74509	67987	73333	69256	72136	70505	70916	10
51	65408	75642	66718	74489	68008	73314	69277	72116	70525	70896	9
52	65430	75623	66740	74470	68029	73294	69298	72095	70546	70875	8
53	65452	75604	66762	74451	68051	73274	69319	72075	70567	70855	7
54	65474	75585	66783	74431	68072	73254	69340	72055	70587	70834	6
55	65496	75566	66805	74412	68093	73234	69361	72035	70608	70813	5
56	65518	75547	66827	74392	68115	73215	69382	72015	70628	70793	4
57	65540	75528	66848	74373	68136	73195	69403	71995	70649	70772	3
58	65562	75509	66870	74353	68157	73175	69424	71974	70670	70752	2
59	65584	75490	66891	74334	68179	73155	69445	71954	70690	70731	1
60	65606	75471	66913	74314	68200	73135	69466	71934	70711	70711	0
M	N.CS.	N. S.	N.CS.	N. S.	N. CS.	N. S.	N. CS.	N. S.	N. CS.	N. S.	M
	49 Deg.		48 Deg.		47 Deg.		46 Deg.		45 Deg.		

	0 Degrees.		1 Degree.		2 Degrees.		3 Degrees.		
M	N. Tan.	N. Cot.	N. Tan.	N. Cot.	N. Tan.	N. Cot.	N. Tan.	N. Cot.	M
0	00000	0000.00	01746	57.2900	03492	28.6353	05241	19.0811	60
1	00029	3437.75	01775	56.3506	03521	28.3994	05270	18.9755	59
2	00058	1718.87	01804	55.4415	03550	28.1664	05299	18.8711	58
3	00087	1145.92	01833	54.5613	03579	27.9372	05328	18.7678	57
4	00116	859.436	01862	53.7086	03609	27.7117	05357	18.6656	56
5	00145	687.549	01891	52.8821	03638	27.4899	05387	18.5645	55
6	00175	572.957	01920	52.0807	03667	27.2715	05416	18.4645	54
7	00204	491.106	01949	51.3032	03696	27.0566	05445	18.3655	53
8	00233	429.718	01978	50.5485	03725	26.8450	05474	18.2677	52
9	00262	381.971	02007	49.8157	03754	26.6367	05503	18.1708	51
10	00291	343.774	02036	49.1039	03783	26.4316	05533	18.0750	50
11	00320	312.521	02066	48.4121	03812	26.2296	05562	17.9801	49
12	00349	286.478	02095	47.7395	03842	26.0307	05591	17.8863	48
13	00378	264.441	02124	47.0853	03871	25.8348	05620	17.7934	47
14	00407	245.552	02153	46.4489	03900	25.6418	05649	17.7015	46
15	00436	229.182	02182	45.8294	03929	25.4517	05678	17.6106	45
16	00465	214.858	02211	45.2261	03958	25.2644	05708	17.5205	44
17	00495	202.219	02240	44.6386	03987	25.0798	05737	17.4314	43
18	00524	190.984	02269	44.0661	04016	24.8978	05766	17.3432	42
19	00553	180.932	02298	43.5081	04046	24.7185	05795	17.2558	41
20	00582	171.885	02328	42.9641	04075	24.5418	05824	17.1693	40
21	00611	163.700	02357	42.4335	04104	24.3675	05854	17.0837	39
22	00640	156.259	02386	41.9158	04133	24.1957	05883	16.9990	38
23	00669	149.465	02415	41.4106	04162	24.0263	05912	16.9150	37
24	00698	143.237	02444	40.9174	04191	23.8593	05941	16.8319	36
25	00727	137.507	02473	40.4358	04220	23.6945	05970	16.7496	35
26	00756	132.219	02502	39.9655	04250	23.5321	05999	16.6681	34
27	00785	127.321	02531	39.5059	04279	23.3718	06029	16.5874	33
28	00814	122.774	02560	39.0568	04308	23.2137	06058	16.5075	32
29	00844	118.540	02589	38.6177	04337	23.0577	06087	16.4283	31
30	00873	114.589	02619	38.1885	04366	22.9037	06116	16.3499	30
31	00902	110.892	02648	37.7686	04395	22.7518	06145	16.2722	29
32	00931	107.426	02677	37.3579	04424	22.6020	06175	16.1952	28
33	00960	104.171	02706	36.9560	04454	22.4541	06204	16.1190	27
34	00989	101.107	02735	36.5627	04483	22.3081	06233	16.0435	26
35	01018	98.2179	02764	36.1776	04512	22.1640	06262	15.9687	25
36	01047	95.4895	02793	35.8006	04541	22.0217	06291	15.8945	24
37	01076	92.9085	02822	35.4313	04570	21.8813	06321	15.8211	23
38	01105	90.4633	02851	35.0695	04599	21.7426	06350	15.7483	22
39	01135	88.1436	02881	34.7151	04628	21.6056	06379	15.6762	21
40	01164	85.9398	02910	34.3678	04658	21.4704	06408	15.6048	20
41	01193	83.8435	02939	34.0273	04687	21.3369	06437	15.5340	19
42	01222	81.8470	02968	33.6935	04716	21.2049	06467	15.4638	18
43	01251	79.9434	02997	33.3662	04745	21.0747	06496	15.3943	17
44	01280	78.1263	03026	33.0452	04774	20.9460	06525	15.3254	16
45	01309	76.3900	03055	32.7303	04803	20.8188	06554	15.2571	15
46	01338	74.7292	03084	32.4213	04832	20.6932	06584	15.1893	14
47	01367	73.1390	03114	32.1181	04862	20.5691	06613	15.1222	13
48	01396	71.6151	03143	31.8205	04891	20.4465	06642	15.0557	12
49	01425	70.1533	03172	31.5284	04920	20.3253	06671	14.9898	11
50	01455	68.7501	03201	31.2416	04949	20.2056	06700	14.9244	10
51	01484	67.4019	03230	30.9599	04978	20.0872	06730	14.8596	9
52	01513	66.1055	03259	30.6833	05007	19.9702	06759	14.7954	8
53	01542	64.8580	03288	30.4116	05037	19.8546	06788	14.7317	7
54	01571	63.6567	03317	30.1446	05066	19.7403	06817	14.6685	6
55	01600	62.4992	03346	29.8823	05095	19.6273	06847	14.6059	5
56	01629	61.3829	03376	29.6245	05124	19.5156	06876	14.5438	4
57	01658	60.3058	03405	29.3711	05153	19.4051	06905	14.4823	3
58	01687	59.2659	03434	29.1220	05182	19.2959	06934	14.4212	2
59	01716	58.2612	03463	28.8771	05212	19.1879	06963	14.3607	1
60	01746	57.2900	03492	28.6363	05241	19.0811	06993	14.3007	0
M	N. Cot.	N. Tan.	N. Cot.	N. Tan.	N. Cot.	N. Tan.	N. Cot.	N. Tan.	M
	89 Degrees.		88 Degrees.		87 Degrees.		86 Degrees.		

	4 Degrees.		5 Degrees.		6 Degrees.		7 Degrees.		
M	N. Tan.	N. Cot.	N. Tan.	N. Cot.	N. Tan.	N. Cot.	N. Tan.	N. Cot.	M
0	06993	14.3007	08749	11.4301	10510	9.51436	12278	8.14435	60
1	07022	14.2411	08778	11.3919	10540	9.48781	12308	8.12481	59
2	07051	14.1821	08807	11.3540	10569	9.46141	12338	8.10536	58
3	07080	14.1235	08837	11.3163	10599	9.43515	12367	8.08600	57
4	07110	14.0655	08866	11.2789	10628	9.40904	12397	8.06674	56
5	07139	14.0079	08895	11.2417	10657	9.38307	12426	8.04756	55
6	07168	13.9507	08925	11.2048	10687	9.35724	12456	8.02848	54
7	07197	13.8940	08954	11.1681	10716	9.33154	12485	8.00948	53
8	07227	13.8378	08983	11.1316	10746	9.30599	12515	7.99058	52
9	07256	13.7821	09013	11.0954	10775	9.28058	12544	7.97176	51
10	07285	13.7267	09042	11.0594	10805	9.25530	12574	7.95302	50
11	07314	13.6719	09071	11.0237	10834	9.23016	12603	7.93438	49
12	07344	13.6174	09101	10.9881	10863	9.20566	12633	7.91582	48
13	07373	13.5634	09130	10.9528	10893	9.18028	12662	7.89734	47
14	07402	13.5098	09159	10.9178	10922	9.15554	12692	7.87895	46
15	07431	13.4566	09189	10.8829	10952	9.13093	12722	7.86064	45
16	07461	13.4039	09218	10.8483	10981	9.10646	12751	7.84242	44
17	07490	13.3515	09247	10.8139	11011	9.08211	12781	7.82428	43
18	07519	13.2996	09277	10.7797	11040	9.05789	12810	7.80622	42
19	07548	13.2480	09306	10.7457	11070	9.03379	12840	7.78825	41
20	07578	13.1969	09335	10.7119	11099	9.00983	12869	7.77035	40
21	07607	13.1461	09365	10.6783	11128	8.98598	12899	7.75254	39
22	67636	13.0958	09394	10.6450	11158	8.96227	12929	7.73480	38
23	07665	13.0458	09423	10.6118	11187	8.93867	12958	7.71715	37
24	07695	12.9962	09453	10.5789	11217	8.91520	12988	7.69957	36
25	07724	12.9469	09482	10.5462	11246	8.89185	13017	7.68208	35
26	07753	12.8981	09511	10.5136	11276	8.86862	13047	7.66466	34
27	07782	12.8496	09541	10.4813	11305	8.84551	13076	7.64732	33
28	07812	12.8014	09570	10.4491	11335	8.82252	13106	7.63005	32
29	07841	12.7536	09600	10.4172	11364	8.79964	13136	7.61287	31
30	07870	12.7062	09629	10.3854	11394	8.77689	13165	7.59575	30
31	07899	12.6591	09658	10.3538	11423	8.75425	13195	7.57872	29
32	07929	12.6124	09688	10.3224	11452	8.73172	13224	7.56176	28
33	07958	12.5660	09717	10.2913	11482	8.70931	13254	7.54487	27
34	07987	12.5199	09746	10.2602	11511	8.68701	13284	7.52806	26
35	08017	12.4742	09776	10.2294	11541	8.66482	13313	7.51132	25
36	08046	12.4288	09805	10.1988	11570	8.64275	13343	7.49465	24
37	08075	12.3838	09834	10.1683	11600	8.62078	13372	7.47806	23
38	08104	12.3390	09864	10.1381	11629	8.59893	13402	7.46154	22
39	08134	12.2946	09893	10.1080	11659	8.57718	13432	7.44509	21
40	08163	12.2505	09923	10.0780	11688	8.55555	13461	7.42871	20
41	08192	12.2067	09952	10.0483	11718	8.53402	13491	7.41240	19
42	08221	12.1632	09981	10.0187	11747	8.51259	13521	7.39616	18
43	08251	12.1201	10011	9.98930	11777	8.49128	13550	7.37999	17
44	08280	12.0772	10040	9.96007	11806	8.47007	13580	7.36389	16
45	08309	12.0346	10069	9.93101	11836	8.44896	13609	7.34786	15
46	08339	11.9923	10099	9.90211	11865	8.42795	13639	7.33190	14
47	08368	11.9504	10128	9.87338	11895	8.40705	13669	7.31600	13
48	08397	11.9087	10158	9.84482	11924	8.38625	13698	7.30018	12
49	08427	11.8673	10187	9.81641	11954	8.36555	13728	7.28442	11
50	08456	11.8262	10216	9.78817	11983	8.34496	13758	7.26873	10
51	08485	11.7853	10246	9.76009	12013	8.32446	13787	7.25310	9
52	08514	11.7448	10275	9.73217	12042	8.30406	13817	7.23754	8
53	08544	11.7045	10305	9.70441	12072	8.28376	13846	7.22204	7
54	08573	11.6645	10334	9.67680	12101	8.26355	13876	7.20661	6
55	08602	11.6248	10363	9.64935	12131	8.24345	13906	7.19125	5
56	08632	11.5853	10393	9.62205	12160	8.22344	13935	7.17594	4
57	08661	11.5461	10422	9.59490	12190	8.20352	13965	7.16071	3
58	08690	11.5072	10452	9.56791	12219	8.18370	13995	7.14553	2
59	08720	11.4685	10481	9.54106	12249	8.16398	14024	7.13042	1
60	08749	11.4301	10510	9.51436	12278	8.14435	14054	7.11537	0
M	N Cot.	N. Tan.	N. Cot.	N. Tan.	N. Cot.	N. Tan.	N. Cot.	N. Tan.	M
	85 Degrees.		84 Degrees.		83 Degrees.		82 Degrees.		

	8 Degrees.		9 Degrees.		10 Degrees.		11 Degrees.		
M	N. Tan.	N. Cot.	N. Tan.	N. Cot.	N. Tan.	N. Cot.	N. Tan.	N. Cot.	M
0	14054	7.11537	15838	6.31375	17633	5.67128	19438	5.14455	60
1	14084	7.10038	15868	6.30189	17663	5.66165	19468	5.13658	59
2	14113	7.08546	15898	6.29007	17693	5.65205	19498	5.12862	58
3	14143	7.07059	15928	6.27829	17723	5.64248	19529	5.12069	57
4	14173	7.05579	15958	6.26655	17753	5.63295	19559	5.11279	56
5	14202	7.04105	15988	6.25486	17783	5.62344	19589	5.10490	55
6	14232	7.02637	16017	6.24321	17813	5.61397	19619	5.09704	54
7	14262	7.01174	16047	6.23160	17843	5.60452	19649	5.08921	53
8	14291	6.99718	16077	6.22003	17873	5.59511	19680	5.08139	52
9	14321	6.98268	16107	6.20851	17903	5.58573	19710	5.07360	51
10	14351	6.96823	16137	6.19703	17933	5.57638	19740	5.06584	50
11	14381	6.95385	16167	6.18559	17963	5.56706	19770	5.05809	49
12	14410	6.93952	16196	6.17419	17993	5.55777	19801	5.05037	48
13	14440	6.92525	16226	6.16283	18023	5.54851	19831	5.04267	47
14	14470	6.91104	16256	6.15151	18053	5.53927	19861	5.03499	46
15	14499	6.89688	16286	6.14023	18083	5.53007	19891	5.02734	45
16	14529	6.88278	16316	6.12899	18113	5.52090	19921	5.01971	44
17	14559	6.86874	16346	6.11779	18143	5.51176	19952	5.01210	43
18	14588	6.85475	16376	6.10664	18173	5.50264	19982	5.00451	42
19	14618	6.84082	16405	6.09552	18203	5.49356	20012	4.99695	41
20	14648	6.82694	16435	6.08444	18233	5.48451	20042	4.98940	40
21	14678	6.81312	16465	6.07340	18263	5.47548	20073	4.98188	39
22	14707	6.79936	16495	6.06240	18293	5.46648	20103	4.97438	38
23	14737	6.78564	16525	6.05143	18323	5.45751	20133	4.96690	37
24	14767	6.77199	16555	6.04051	18353	5.44857	20164	4.95945	36
25	14796	6.75838	16585	6.02962	18383	5.43966	20194	4.95201	35
26	14826	6.74483	16615	6.01878	18414	5.43077	20224	4.94460	34
27	14856	6.73133	16645	6.00797	18444	5.42192	20254	4.93721	33
28	14886	6.71789	16674	5.99720	18474	5.41309	20285	4.92984	32
29	14915	6.70450	16704	5.98646	18504	5.40429	20315	4.92249	31
30	14945	6.69116	16734	5.97576	18534	5.39552	20345	4.91516	30
31	14975	6.67787	16764	5.96510	18564	5.38677	20376	4.90785	29
32	15005	6.66463	16794	5.95448	18594	5.37805	20406	4.90056	28
33	15034	6.65144	16824	5.94390	18624	5.36936	20436	4.89330	27
34	15064	6.63831	16854	5.93335	18654	5.36070	20466	4.88605	26
35	15094	6.62523	16884	5.92283	18684	5.35206	20497	4.87882	25
36	15124	6.61219	16914	5.91235	18714	5.34345	20527	4.87162	24
37	15153	6.59921	16944	5.90191	18745	5.33487	20557	4.86444	23
38	15183	6.58627	16974	5.89151	18775	5.32631	20588	4.85727	22
39	15213	6.57339	17004	5.88114	18805	5.31778	20618	4.85013	21
40	15243	6.56055	17033	5.87080	18835	5.30928	20648	4.84300	20
41	15272	6.54777	17063	5.86051	18865	5.30080	20679	4.83590	19
42	15302	6.53503	17093	5.85024	18895	5.29235	20709	4.82882	18
43	15332	6.52234	17123	5.84001	18925	5.28393	20739	4.82175	17
44	15362	6.50970	17153	5.82982	18955	5.27553	20770	4.81471	16
45	15391	6.49710	17183	5.81966	18986	5.26715	20800	4.80769	15
46	15421	6.48456	17213	5.80953	19016	5.25880	20830	4.80068	14
47	15451	6.47206	17243	5.79944	19046	5.25048	20861	4.79370	13
48	15481	6.45961	17273	5.78938	19076	5.24218	20891	4.78673	12
49	15511	6.44720	17303	5.77936	19106	5.23391	20921	4.77978	11
50	15540	6.43484	17333	5.76937	19136	5.22566	20952	4.77286	10
51	15570	6.42253	17363	5.75941	19166	5.21744	20982	4.76595	9
52	15600	6.41026	17393	5.74949	19197	5.20925	21013	4.75906	8
53	15630	6.39804	17423	5.73960	19227	5.20107	21043	4.75219	7
54	15660	6.38587	17453	5.72974	19257	5.19293	21073	4.74534	6
55	15689	6.37374	17483	5.71992	19287	5.18480	21104	4.73851	5
56	15719	6.36165	17513	5.71013	19317	5.17671	21134	4.73170	4
57	15749	6.34961	17543	5.70037	19347	5.16863	21164	4.72490	3
58	15779	6.33761	17573	5.69064	19378	5.16058	21195	4.71813	2
59	15809	6.32566	17603	5.68094	19408	5.15256	21225	4.71137	1
60	15838	6.31375	17633	5.67128	19438	5.14455	21256	4.70463	0
M	N. Cot.	N. Tan.	N. Cot.	N. Tan.	N. Cot.	N. Tan.	N. Cot.	N. Tan.	M
	81 Degrees.		80 Degrees.		79 Degrees.		78 Degrees.		

M	12 Degrees.		13 Degrees.		14 Degrees.		15 Degrees.		M
	N. Tan.	N. Cot.	N. Tan.	N. Cot.	N. Tan.	N. Cot.	N. Tan.	N. Cot.	
0	21256	4.70463	23087	4.33148	24933	4.01078	26795	3.73205	60
1	21286	4.69791	23117	4.32573	24964	4.00582	26826	3.72771	59
2	21316	4.69121	23148	4.32001	24995	4.00086	26857	3.72338	58
3	21347	4.68452	23179	4.31430	25026	3.99592	26888	3.71907	57
4	21377	4.67786	23209	4.30860	25056	3.99099	26920	3.71476	56
5	21408	4.67121	23240	4.30291	25087	3.98607	26951	3.71046	55
6	21438	4.66458	23271	4.29724	25118	3.98117	26982	3.70616	54
7	21469	4.65797	23301	4.29159	25149	3.97627	27013	3.70188	53
8	21499	4.65138	23332	4.28595	25180	3.97139	27044	3.69761	52
9	21529	4.64480	23363	4.28032	25211	3.96651	27076	3.69335	51
10	21560	4.63825	23393	4.27471	25242	3.96165	27107	3.68909	50
11	21590	4.63171	23424	4.26911	25273	3.95680	27138	3.68485	49
12	21621	4.62518	23455	4.26352	25304	3.95196	27169	3.68061	48
13	21651	4.61868	23484	4.25795	25335	3.94713	27201	3.67638	47
14	21682	4.61219	23516	4.25239	25366	3.94232	27232	3.67217	46
15	21712	4.60572	23547	4.24685	25397	3.93751	27263	3.66796	45
16	21743	4.59927	23578	4.24132	25428	3.93271	27294	3.66376	44
17	21773	4.59283	23608	4.23580	25459	3.92793	27326	3.65957	43
18	21804	4.58641	23639	4.23030	25490	3.92316	27357	3.65538	42
19	21834	4.58001	23670	4.22481	25521	3.91839	27388	3.65121	41
20	21864	4.57363	23700	4.21933	25552	3.91364	27419	3.64705	40
21	21895	4.56726	23731	4.21387	25583	3.90890	27451	3.64289	39
22	21925	4.56091	23762	4.20842	25614	3.90417	27482	3.63874	38
23	21956	4.55458	23793	4.20298	25645	3.89945	27513	3.63461	37
24	21986	4.54826	23823	4.19756	25676	3.89474	27545	3.63048	36
25	22017	4.54196	23854	4.19215	25707	3.89004	27576	3.62636	35
26	22047	4.53568	23885	4.18675	25738	3.88536	27607	3.62224	34
27	22078	4.52941	23916	4.18137	25769	3.88068	27638	3.61814	33
28	22108	4.52316	23946	4.17600	25800	3.87601	27670	3.61405	32
29	22139	4.51693	23977	4.17064	25831	3.87136	27701	3.60996	31
30	22169	4.51071	24008	4.16530	25862	3.86671	27732	3.60588	30
31	22200	4.50451	24039	4.15997	25893	3.86208	27764	3.60181	29
32	22231	4.49832	24069	4.15465	25924	3.85745	27795	3.59775	28
33	22261	4.49215	24100	4.14934	25955	3.85284	27826	3.59370	27
34	22292	4.48600	24131	4.14405	25986	3.84824	27858	3.58966	26
35	22322	4.47986	24162	4.13877	26017	3.84364	27889	3.58562	25
36	22353	4.47374	24193	4.13350	26048	3.83904	27920	3.58160	24
37	22383	4.46764	24223	4.12825	26079	3.83449	27952	3.57758	23
38	22414	4.46155	24254	4.12301	26110	3.82992	27983	3.57357	22
39	22444	4.45548	24285	4.11778	26141	3.82537	28015	3.56957	21
40	22475	4.44942	24316	4.11256	26172	3.82083	28046	3.56557	20
41	22505	4.44338	24347	4.10736	26203	3.81630	28077	3.56159	19
42	22536	4.43735	24377	4.10216	26235	3.81177	28109	3.55761	18
43	22567	4.43134	24408	4.09699	26266	3.80726	28140	3.55364	17
44	22597	4.42534	24439	4.09182	26297	3.80276	28172	3.54968	16
45	22628	4.41936	24470	4.08666	26328	3.79827	28203	3.54573	15
46	22658	4.41340	24501	4.08152	26359	3.79378	28234	3.54179	14
47	22689	4.40745	24532	4.07639	26390	3.78931	28266	3.53785	13
48	22719	4.40152	24562	4.07127	26421	3.78485	28297	3.53393	12
49	22750	4.39560	24593	4.06616	26452	3.78040	28329	3.53001	11
50	22781	4.38969	24624	4.06107	26483	3.77595	28360	3.52609	10
51	22811	4.38381	24655	4.05599	26515	3.77152	28391	3.52219	9
52	22842	4.37793	24686	4.05092	26546	3.76709	28423	3.51829	8
53	22872	4.37207	24717	4.04586	26577	3.76268	28454	3.51441	7
54	22903	4.36623	24747	4.04081	26608	3.75828	28486	3.51053	6
55	22934	4.36040	24778	4.03578	26639	3.75388	28517	3.50666	5
56	22964	4.35459	24809	4.03075	26670	3.74950	28549	3.50279	4
57	22995	4.34879	24840	4.02574	26701	3.74512	28580	3.49894	3
58	23026	4.34300	24871	4.02074	26733	3.74075	28612	3.49509	2
59	23056	4.33723	24902	4.01576	26764	3.73644	28643	3.49125	1
60	23087	4.33148	24933	4.01078	26795	3.73205	28675	3.48741	0
M	N. Cot.	N. Tan.	N. Cot.	N. Tan.	N. Cot.	N. Tan.	N Cot.	N. Tan.	M
	77 Degrees.		76 Degrees.		75 Degrees.		74 Degrees.		

	16 Degrees.		17 Degrees.		18 Degrees.		19 Degrees.		
M	N. Tan.	N. Cot.	N. Tan.	N. Cot.	N. Tan.	N. Cot.	N. Tan.	N. Cot.	M
0	28675	3.48741	30573	3.27085	32492	3.07768	34433	2.90421	60
1	28706	3.48359	30605	3.26745	32524	3.07464	34465	2.90147	59
2	28738	3.47977	30637	3.26406	32556	3.07160	34498	2.89873	58
3	28769	3.47596	30669	3.26067	32588	3.06857	34530	2.89600	57
4	28800	3.47216	30700	3.25729	32621	3.06654	34563	2.89327	56
5	28832	3.46837	30732	3.25392	32653	3.06252	34596	2.89055	55
6	28864	3.46458	30764	3.25055	32685	3.05950	34628	2.88783	54
7	28895	3.46080	30796	3.24719	32717	3.05649	34661	2.88511	53
8	28927	3.45703	30828	3.24383	32749	3.05349	34693	2.88240	52
9	28958	3.45327	30860	3.24049	32782	3.05049	34726	2.87970	51
10	28990	3.44951	30891	3.23714	32814	3.04749	34758	2.87700	50
11	29021	3.44576	30923	3.23381	32846	3.04450	34791	2.87430	49
12	29053	3.44202	30955	3.23048	32878	3.04152	34824	2.87161	48
13	29084	3.43829	30987	3.22715	32911	3.03854	34856	2.86892	47
14	29116	3.43456	31019	3.22384	32943	3.03556	34889	2.86624	46
15	29147	3.43084	31051	3.22053	32975	3.03260	34922	2.86356	45
16	29179	3.42713	31083	3.21722	33007	3.02963	34954	2.86089	44
17	29210	3.42343	31115	3.21392	33040	3.02667	34987	2.85822	43
18	29242	3.41973	31147	3.21063	33072	3.02372	35019	2.85555	42
19	29274	3.41604	31178	3.20734	33104	3.02077	35052	2.85289	41
20	29305	3.41236	31210	3.20406	33136	3.01783	35085	2.85023	40
21	29337	3.40869	31242	3.20079	33169	3.01489	35117	2.84758	39
22	29368	3.40502	31274	3.19752	33201	3.01196	35150	2.84494	38
23	29400	3.40136	31306	3.19426	33233	3.00903	35183	2.84229	37
24	29432	3.39771	31338	3.19100	33266	3.00611	35216	2.83965	36
25	29463	3.39406	31370	3.18775	33298	3.00319	35248	2.83702	35
26	29495	3.39042	31402	3.18451	33330	3.00028	35281	2.83439	34
27	29526	3.38679	31434	3.18127	33363	2.99738	35314	2.83176	33
28	29558	3.38317	31466	3.17804	33395	2.99447	35346	2.82914	32
29	29590	3.37955	31498	3.17481	33427	2.99158	35379	2.82653	31
30	29621	3.37594	31530	3.17159	33460	2.98868	35412	2.82391	30
31	29653	3.37234	31562	3.16838	33492	2.98580	35445	2.82130	29
32	29685	3.36875	31594	3.16517	33524	2.98292	35477	2.81870	28
33	29716	3.36516	31626	3.16197	33557	2.98004	35510	2.81610	27
34	29748	3.36158	31658	3.15877	33589	2.97717	35543	2.81350	26
35	29780	3.35800	31690	3.15558	33621	2.97430	35576	2.81091	25
36	29811	3.35443	32722	3.15240	33654	2.97144	35608	2.80833	24
37	29843	3.35087	31754	3.14922	33686	2.96858	35641	2.80574	23
38	29875	3.34732	31786	3.14605	33718	2.96573	35674	2.80316	22
39	29906	3.34377	31818	3.14288	33751	2.96288	35707	2.80059	21
40	29938	3.34023	31850	3.13972	33783	2.96004	35740	2.79802	20
41	29970	3.33670	31882	3.13656	33816	2.95720	35772	2.79545	19
42	30001	3.33317	31914	3.13341	33848	2.95437	35805	2.79289	18
43	30033	3.32965	31946	3.13027	33881	2.95155	35838	2.79033	17
44	30065	3.32614	31978	3.12713	33913	2.94872	35871	2.78778	16
45	30097	3.32264	32010	3.12400	33945	2.94590	35904	2.78523	15
46	30128	3.31914	32042	3.12087	33978	2.94309	35937	2.78269	14
47	30160	3.31565	32074	3.11775	34010	2.94028	35969	2.78014	13
48	30192	3.31216	32106	3.11464	34043	2.93748	36002	2.77761	12
49	30224	3.30868	32139	3.11153	34075	2.93468	36035	2.77507	11
50	30255	3.30521	32171	3.10842	34108	2.93189	36068	2.77254	10
51	30287	3.30174	32203	3.10532	34140	2.92910	36101	2.77002	9
52	30319	3.29829	32235	3.10223	34173	2.92632	36134	2.76750	8
53	30351	3.29483	32267	3.09914	34205	2.92354	36167	2.76498	7
54	30382	3.29139	32299	3.09606	34238	2.92076	36199	2.76247	6
55	30414	3.28795	32331	3.09298	34270	2.91799	36232	2.75996	5
56	30446	3.28452	32363	3.08991	34303	2.91523	36265	2.75746	4
57	30478	3.28109	32396	3.08685	34335	2.91246	36298	2.75496	3
58	30509	3.27767	32428	3.08379	34368	2.90971	36331	2.75246	2
59	30541	3.27426	32460	3.08073	34400	2.90696	36364	2.74997	1
60	30573	3.27085	32492	3.07768	34433	2.90421	36397	2.74748	0
M	N. Cot.	N. Tan.	N. Cot.	N. Tan.	N. Cot.	N. Tan.	N. Cot.	N. Tan.	M
	73 Degrees.		72 Degrees.		71 Degrees.		70 Degrees.		

	20 Degrees.		21 Degrees.		22 Degrees.		23 Degrees.		
M	N. Tan.	N. Cot.	N. Tan.	N. Cot.	N. Tan.	N. Cot.	N. Tan.	N. Cot.	M
0	36397	2.74748	38386	2.60509	40403	2.47509	42447	2.35585	60
1	36430	2.74499	38420	2.60283	40436	2.47302	42482	2.35395	59
2	36463	2.74251	38453	2.60057	40470	2.47095	42516	2.35205	58
3	36496	2.74004	38487	2.59831	40504	2.46888	42551	2.35015	57
4	36529	2.73756	38520	2.59606	40538	2.46682	42585	2.34825	56
5	36562	2.73509	38553	2.59381	40572	2.46476	42619	2.34636	55
6	36595	2.73263	38587	2.59156	40606	2.46270	42654	2.34447	54
7	36628	2.73017	38620	2.58932	40640	2.46065	42688	2.34258	53
8	36661	2.72771	38654	2.58708	40674	2.45860	42722	2.34069	52
9	36694	2.72526	38687	2.58484	40707	2.45655	42757	2.33881	51
10	36727	2.72281	38721	2.58261	40741	2.45451	42791	2.33693	50
11	36760	2.72036	38754	2.58038	40775	2.45246	42826	2.33505	49
12	36793	2.71792	38787	2.57815	40809	2.45043	42860	2.33317	48
13	36826	2.71548	38821	2.57593	40843	2.44839	42894	2.33130	47
14	36859	2.71305	38854	2.57371	40877	2.44636	42929	2.32943	46
15	36892	2.71062	38888	2.57150	40911	2.44433	42963	2.32756	45
16	36925	2.70819	38921	2.56928	40945	2.44230	42998	2.32570	44
17	36958	2.70577	38955	2.56707	40979	2.44027	43032	2.32383	43
18	36991	2.70335	38988	2.56487	41013	2.43825	43067	2.32197	42
19	37024	2.70094	39022	2.56266	41047	2.43623	43101	2.32012	41
20	37057	2.69853	39055	2.56046	41081	2.43422	43136	2.31826	40
21	37090	2.69612	39089	2.55827	41115	2.43220	43170	2.31641	39
22	37123	2.69371	39122	2.55608	41149	2.43019	43205	2.31456	38
23	37157	2.69131	39156	2.55389	41183	2.42819	43239	2.31271	37
24	37190	2.68892	39190	2.55170	41217	2.42618	43274	2.31086	36
25	37223	2.68653	39223	2.54952	41251	2.42418	43308	2.30902	35
26	37256	2.68414	39257	2.54734	41285	2.42218	43343	2.30718	34
27	37289	2.68175	39290	2.54516	41319	2.42019	43378	2.30534	33
28	37322	2.67937	39324	2.54299	41353	2.41819	43412	2.30351	32
29	37355	2.67700	39357	2.54082	41387	2.41620	43447	2.30167	31
30	37388	2.67462	39391	2.53865	41421	2.41421	43481	2.29984	30
31	37422	2.67225	39425	2.53648	41455	2.41223	43516	2.29801	29
32	37455	2.66989	39458	2.53432	41490	2.41025	43550	2.29619	28
33	37488	2.66752	39492	2.53217	41524	2.40827	43585	2.29437	27
34	37521	2.66516	39526	2.53001	41558	2.40629	43620	2.29254	26
35	37554	2.66281	39559	2.52786	41592	2.40432	43654	2.29073	25
36	37588	2.66046	39593	2.52571	41626	2.40235	43689	2.28891	24
37	37621	2.65811	39626	2.52357	41660	2.40038	43724	2.28710	23
38	37654	2.65576	39660	2.52142	41694	2.39841	43758	2.28528	22
39	37687	2.65342	39694	2.51929	41728	2.39645	43793	2.28348	21
40	37720	2.65109	39727	2.51715	41763	2.39449	43828	2.28167	20
41	37754	2.64875	39761	2.51502	41797	2.39253	43862	2.27987	19
42	37787	2.64642	39795	2.51289	41831	2.39058	43897	2.27806	18
43	37820	2.64410	39829	2.51076	41865	2.38862	43932	2.27626	17
44	37853	2.64177	39862	2.50864	41899	2.38668	43966	2.27447	16
45	37887	2.63945	39896	2.50652	41933	2.38473	44001	2.27267	15
46	37920	2.63714	39930	2.50440	41968	2.38279	44036	2.27088	14
47	37953	2.63483	39963	2.50229	42002	2.38084	44071	2.26909	13
48	37986	2.63252	39997	2.50018	42036	2.37891	44105	2.26730	12
49	38020	2.63021	40031	2.49807	42070	2.37697	44140	2.26552	11
50	38053	2.62791	40065	2.49597	42105	2.37504	44175	2.26374	10
51	38086	2.62561	40098	2.49386	42139	2.37311	44210	2.26196	9
52	38120	2.62332	40132	2.49177	42173	2.37118	44244	2.26018	8
53	38153	2.62103	40166	2.48967	42207	2.36925	44279	2.25840	7
54	38186	2.61874	40200	2.48758	42242	2.36733	44314	2.25663	6
55	38220	2.61646	40234	2.48549	42276	2.36541	44349	2.25486	5
56	38253	2.61418	40267	2.48340	42310	2.36349	44384	2.25309	4
57	38286	2.61190	40301	2.48132	42345	2.36158	44418	2.25132	3
58	38320	2.60963	40335	2.47924	42379	2.35967	44453	2.24956	2
59	38353	2.60736	40369	2.47716	42413	2.35776	44488	2.24780	1
60	38386	2.60509	10403	2.47509	42447	2.35585	44523	2.24604	0
M	N Cot.	N. Tan.	N. Cot.	N. Tan.	N. Cot.	N. Tan.	N. Cot.	N. Tan.	M
	69 Degrees.		68 Degrees.		67 Degrees.		66 Degrees.		

	24 Degrees.		25 Degrees.		26 Degrees.		27 Degrees.		
M	N. Tan.	N. Cot.	N. Tan.	N. Cot.	N. Tan.	N. Cot.	N. Tan.	N. Cot.	M
0	44523	2.24604	46631	2.14451	48773	2.05030	50953	1.96261	60
1	44558	2.24428	46666	2.14288	48809	2.04879	50989	1.96120	59
2	44593	2.24252	46702	2.14125	48845	2.04728	51026	1.95979	58
3	44627	2.24077	46737	2.13963	48881	2.04577	51063	1.95838	57
4	44662	2.23902	46772	2.13801	48917	2.04426	51099	1.95698	56
5	44697	2.23727	46808	2.13639	48953	2.04276	51136	1.95557	55
6	44732	2.23553	46843	2.13477	48989	2.04125	51173	1.95417	54
7	44767	2.23378	46879	2.13316	49026	2.03975	51209	1.95277	53
8	44802	2.23204	46914	2.13154	49062	2.03825	51246	1.95137	52
9	44837	2.23030	46950	2.12993	49098	2.03675	51283	1.94997	51
10	44872	2.22857	46985	2.12832	49134	2.03526	51319	1.94858	50
11	44907	2.22683	47021	2.12671	49170	2.03376	51356	1.94718	49
12	44942	2.22510	47056	2.12511	49206	2.03227	51393	1.94579	48
13	44977	2.22337	47092	2.12350	49242	2.03078	51430	1.94440	47
14	45012	2.22164	47128	2.12190	49278	2.02929	51467	1.94301	46
15	45047	2.21992	47163	2.12030	49315	2.02780	51503	1.94162	45
16	45082	2.21819	47199	2.11871	49351	2.02631	51540	1.94023	44
17	45117	2.21647	47234	2.11711	49387	2.02483	51577	1.93885	43
18	45152	2.21475	47270	2.11552	49423	2.02335	51614	1.93746	42
19	45187	2.21304	47305	2.11392	49459	2.02187	51651	1.93608	41
20	45222	2.21132	47341	2.11233	49495	2.02039	51688	1.93470	40
21	45257	2.20961	47377	2.11075	49532	2.01891	51724	1.93332	39
22	45292	2.20790	47412	2.10916	49568	2.01743	51761	1.93197	38
23	45327	2.20619	47448	2.10758	49604	2.01596	51798	1.93057	37
24	45362	2.20449	47483	2.10600	49640	2.01449	51835	1.92920	36
25	45397	2.20278	47519	2.10441	49677	2.01302	51872	1.92782	35
26	45432	2.20108	47555	2.10284	49713	2.01155	51909	1.92645	34
27	45467	2.19938	47590	2.10126	49749	2.01008	51946	1.92508	33
28	45502	2.19769	47626	2.09969	49786	2.00862	51983	1.92371	32
29	45537	2.19599	47662	2.09811	49822	2.00715	52020	1.92235	31
30	45573	2.19430	47698	2.09654	49858	2.00569	52057	1.92098	30
31	45608	2.19261	47733	2.09498	49894	2.00423	52094	1.91962	29
32	45643	2.19092	47769	2.09341	49931	2.00277	52131	1.91825	28
33	45678	2.18923	47805	2.09184	49967	2.00131	52168	1.91690	27
34	45713	2.18755	47840	2.09028	50004	1.99986	52205	1.91554	26
35	45748	2.18587	47876	2.08872	50040	1.99841	52242	1.91418	25
36	45784	2.18419	47912	2.08716	50076	1.99695	52279	1.91282	24
37	45819	2.18251	47948	2.08560	50113	1.99550	52316	1.91148	23
38	45854	2.18084	47984	2.08405	50149	1.99406	52353	1.91017	22
39	45889	2.17916	48019	2.08250	50185	1.99261	52390	1.90876	21
40	45924	2.17749	48055	2.08094	50222	1.99116	52427	1.90741	20
41	45960	2.17582	48091	2.07939	50258	1.98972	52464	1.90607	19
42	45995	2.17416	48127	2.07785	50295	1.98828	52501	1.90472	18
43	46030	2.17249	48163	2.07630	50331	1.98684	52538	1.90337	17
44	46065	2.17083	48198	2.07476	50368	1.98540	52575	1.90203	16
45	46101	2.16917	48234	2.07321	50404	1.98396	52613	1.90069	15
46	46136	2.16751	48270	2.07167	50441	1.98253	52650	1.89935	14
47	46171	2.16585	48306	2.07014	50477	1.98110	52687	1.89801	13
48	46206	2.16420	48342	2.06860	50514	1.97966	52724	1.89667	12
49	46242	2.16255	48378	2.06706	50550	1.97823	52761	1.89533	11
50	46277	2.16090	48414	2.06553	50587	1.97680	52798	1.89400	10
51	46312	2.15925	48450	2.06400	50623	1.97538	52836	1.89266	9
52	46348	2.15760	48486	2.06247	50660	1.97395	52873	1.89133	8
53	46383	2.15596	48521	2.06094	50696	1.97253	52910	1.89000	7
54	46418	2.15432	48557	2.05942	50733	1.97111	52947	1.88867	6
55	46454	2.15268	48593	2.05789	50769	1.96969	52984	1.88734	5
56	46489	2.15104	48629	2.05637	50806	1.96827	53024	1.88602	4
57	46525	2.14940	48665	2.05485	50843	1.96685	53059	1.88469	3
58	46560	2.14777	48701	2.05333	50879	1.96544	53096	1.88337	2
59	46595	2.14614	48737	2.05182	50916	1.96402	53134	1.88205	1
60	46631	2.14451	48773	2.05030	50953	1.96261	53171	1.88073	0
M	N. Cot.	N. Tan.	N. Cot.	N. Tan.	N. Cot.	N. Tan.	N. Cot.	N. Tan.	M
	65 Degrees.		64 Degrees.		63 Degrees.		62 Degrees.		

	28 Degrees.		29 Degrees.		30 Degrees.		31 Degrees.		
M	N. Tan.	N. Cot.	N. Tan.	N. Cot.	N. Tan.	N. Cot.	N. Tan.	N. Cot.	M
0	53171	1.88073	55431	1.80405	57735	1.73205	60086	1.66428	60
1	53208	1.87941	55469	1.80281	57774	1.73089	60126	1.66318	59
2	53246	1.87809	55507	1.80158	57813	1.72973	60165	1.66209	58
3	53283	1.87677	55545	1.80034	57851	1.72857	60205	1.66099	57
4	53320	1.87546	55583	1.79911	57890	1.72741	60245	1.65990	56
5	53358	1.87415	55621	1.79788	57929	1.72625	60284	1.65881	55
6	53395	1.87283	55659	1.79665	57968	1.72509	60324	1.65772	54
7	53432	1.87152	55697	1.79542	58007	1.72393	60364	1.65663	53
8	53470	1.87021	55736	1.79419	58046	1.72278	60403	1.65554	52
9	53507	1.86891	55774	1.79296	58085	1.72163	60443	1.65445	51
10	53545	1.86760	55812	1.79174	58124	1.72047	60483	1.65337	50
11	53582	1.86630	55850	1.79051	58162	1.71932	60522	1.65228	49
12	53620	1.86499	55888	1.78929	58201	1.71817	60562	1·65120	48
13	53657	1.86369	55926	1.78807	58240	1.71702	60602	1.65011	47
14	53694	1.86239	56964	1.78685	58279	1.71588	60642	1.64903	46
15	53732	1.86109	56003	1.78563	58318	1.71473	60681	1.64795	45
16	53769	1.85979	56041	1.78441	58357	1.71358	60721	1.64687	44
17	53807	1.85850	56079	1.78319	58396	1.71244	60761	1.64579	43
18	53844	1.85720	56117	1.78198	58435	1.71129	60801	1.64471	42
19	53882	1.85591	56156	1.78077	58474	1.71015	60841	1.64363	41
20	53920	1.85462	56194	1.77955	58513	1.70901	60881	1.64256	40
21	53957	1.85333	56232	1.77834	58552	1.70787	60921	1.64148	39
22	53995	1.85204	56270	1.77713	58591	1.70673	60960	1.64041	38
23	54032	1.85075	56309	1.77592	58631	1.70560	61000	1.63933	37
24	54070	1.84946	56347	1.77471	58670	1.70446	61040	1.63826	36
25	54107	1.84818	56385	1.77351	58709	1.70332	61080	1.63719	35
26	54145	1.84689	56424	1.77230	58748	1.70219	61120	1.63612	34
27	54183	1.84561	56462	1.77110	58787	1.70106	61160	1.63505	33
28	54220	1.84433	56500	1.76990	58826	1.69992	61200	1.63398	32
29	54258	1.84305	56539	1.76869	58865	1.69879	61240	1.63292	31
30	54296	1.84177	56577	1.76749	58904	1.69766	61280	1.63185	30
31	54333	1.84049	56616	1.76629	58944	1.69653	61320	1.63079	29
32	54371	1.83922	56654	1.76510	58983	1.69541	61360	1.62972	28
33	54409	1.83794	56693	1.76390	59022	1.69428	61400	1.62866	27
34	54446	1.83667	56731	1.76271	59061	1.69315	61440	1.62760	26
35	54484	1.83540	56769	1.76151	59101	1.69203	61480	1.62654	25
36	54522	1.83413	56808	1.76032	59140	1.69091	61520	1.62548	24
37	54560	1.83286	56846	1.75913	59179	1.68979	61561	1.62442	23
38	54597	1.83159	56885	1.75794	59218	1.68866	61601	1.62336	22
39	54635	1.83033	56923	1.75675	59258	1.68754	61641	1.62230	21
40	54673	1.82906	57962	1.75556	59297	1.68643	61681	1.62125	20
41	54711	1.82780	57000	1.75437	59336	1.68531	61721	1.62019	19
42	54748	1.82654	57039	1.75319	59376	1.68419	61761	1.61914	18
43	54786	1.82528	57078	1.75200	59415	1.68308	61801	1.61808	17
44	54824	1.82402	57116	1.75082	59454	1.68196	61842	1.61703	16
45	54862	1.82276	57155	1.74964	59494	1.68085	61882	1.61598	15
46	54900	1.82150	57193	1.74846	59533	1.67974	61922	1.61493	14
47	54938	1.82025	57232	1.74728	59573	1.67869	61962	1.61388	13
48	54975	1.81899	57271	1.74610	59612	1.67752	62003	1.61283	12
49	55013	1.81774	57309	1.74492	59651	1.67641	62043	1.61179	11
50	55051	1.81649	57348	1.74375	59691	1.67530	62083	1.61074	10
51	55089	1.81524	57386	1.74257	59730	1.67419	62124	1.60970	9
52	55127	1.81399	57425	1.74140	59770	1.67309	62164	1.60865	8
53	55165	1.81274	57464	1.74022	59809	1.67198	62204	1.60761	7
54	55203	1.81150	57503	1.73905	59849	1.67088	62245	1.60657	6
55	55241	1.81025	57541	1.73788	59888	1.66978	62285	1.60553	5
56	55279	1.80901	57580	1.73671	59928	1.66867	62325	1.60449	4
57	55317	1.80777	57619	1.73555	59967	1.66757	62366	1.60345	3
58	55355	1.80653	57657	1.73438	60007	1.66647	62406	1.60241	2
59	55393	1.80529	57696	1.73321	60046	1.66538	62446	1.60137	1
60	55431	1.80405	57735	1.73205	60086	1.66428	62487	1.60033	0
M	N Cot.	N. Tan.	N. Cot.	N. Tan.	N. Cot.	N. Tan.	N. Cot.	N. Tan.	M
	61 Degrees.		60 Degrees.		59 Degrees.		58 Degrees.		

	32 Degrees.		33 Degrees.		34 Degrees.		35 Degrees.		
M	N. Tan.	N. Cot.	N. Tan.	N. Cot.	N. Tan.	N. Cot.	N. Tan.	N. Cot.	M
0	62487	1.60033	64941	1.53986	67451	1.48256	70021	1.42815	60
1	62527	1.59930	64982	1.53888	67493	1.48163	70064	1.42726	59
2	62568	1.59826	65023	1.53791	67536	1.48070	70107	1.42638	58
3	62608	1.59723	65065	1.53693	67578	1.47977	70151	1.42550	57
4	62649	1.59620	65106	1.53595	67620	1.47885	70194	1.42462	56
5	62689	1.59517	65148	1.53497	67663	1.47792	70238	1.42374	55
6	62730	1.59414	65189	1.53400	67705	1.47699	70281	1.42286	54
7	62770	1.59311	65231	1.53302	67748	1.47607	70325	1.42198	53
8	62811	1.59208	65272	1.53205	67790	1.47514	70368	1.42110	52
9	62852	1.59105	65314	1.53107	67832	1.47422	70412	1.42022	51
10	62892	1.59002	65355	1.53010	67875	1.47330	70455	1.41934	50
11	62933	1.58900	65397	1.52913	67917	1.47238	70499	1.41847	49
12	62973	1.58797	65438	1.52816	67960	1.47146	70542	1.41759	48
13	63014	1.58695	65480	1.52719	68002	1.47053	70586	1.41672	47
14	63055	1.58593	65521	1.52622	68045	1.46962	70629	1.41584	46
15	63095	1.58490	65563	1.52525	68088	1.46870	70673	1.41497	45
16	63136	1.58388	65604	1.52429	68130	1.46778	70717	1.41409	44
17	63177	1.58286	65646	1.52332	68173	1.46686	70760	1.41322	43
18	63217	1.58184	65688	1.52235	68215	1.46595	70804	1.41235	42
19	63258	1.58083	65729	1.52139	68258	1.46503	70848	1.41148	41
20	63299	1.57981	65771	1.52043	68301	1.46411	70891	1.41061	40
21	63340	1.57879	65813	1.51946	68343	1.46320	70935	1.40974	39
22	63380	1.57778	65854	1.51850	68386	1.46229	70979	1.40887	38
23	63421	1.57676	65896	1.51754	68429	1.46137	71023	1.40800	37
24	63462	1.57575	65938	1.51658	68471	1.46046	71066	1.40714	36
25	63503	1.57474	65980	1.51562	68514	1.45955	71110	1.40627	35
26	63544	1.57372	66021	1.51466	68557	1.45864	71154	1.40540	34
27	63584	1.57271	66063	1.51370	68600	1.45773	71198	1.40454	33
28	63625	1.57170	66105	1.51275	68642	1.45682	71242	1.40368	32
29	63666	1.57069	66147	1.51179	68685	1.45592	71285	1.40281	31
30	63707	1.56969	66189	1.51084	68728	1.45501	71329	1.40195	30
31	63748	1.56868	66230	1.50988	68771	1.45410	71373	1.40109	29
32	63789	1.56767	66272	1.50893	68814	1.45320	71417	1.40022	28
33	63830	1.56667	66314	1.50797	68857	1.45229	71461	1.39936	27
34	63871	1.56566	66356	1.50702	68900	1.45139	71505	1.39850	26
35	63912	1.56466	66398	1.50607	68942	1.45048	71549	1.39764	25
36	63953	1.56366	66440	1.50512	88985	1.44958	71593	1.39679	24
37	63994	1.56265	66482	1.50417	69028	1.44868	71637	1.39593	23
38	64035	1.56165	66524	1.50322	69071	1.44778	71681	1.39507	22
39	64076	1.56065	66566	1.50228	69114	1.44688	71725	1.39421	21
40	64117	1.55966	66608	1.50133	69157	1.44598	71769	1.39336	20
41	64158	1.55866	66650	1.50038	69200	1.44508	71813	1.39250	19
42	64199	1.55766	66692	1.49944	69243	1.44418	71857	1.39165	18
43	64240	1.55666	66734	1.49849	69286	1.44329	71901	1.39079	17
44	64281	1.55567	66776	1.49755	69329	1.44239	71946	1.38994	16
45	64322	1.55467	66818	1.49661	69372	1.44149	71990	1.38909	15
46	64363	1.55368	66860	1.49566	69416	1.44060	72034	1.38824	14
47	64404	1.55269	66902	1.49472	69459	1.43970	72078	1.38738	13
48	64446	1.55170	66944	1.49378	69502	1.43881	72122	1.38653	12
49	64487	1.55071	66986	1.49284	69545	1.43792	72166	1.38568	11
50	64528	1.54972	67028	1.49190	69588	1.43703	72211	1.38484	10
51	64569	1.54873	67071	1.49097	69631	1.43614	72255	1.38399	9
52	64610	1.54774	67113	1.49003	69675	1.43525	72299	1.38314	8
53	64652	1.54675	67155	1.48909	69718	1.43436	72344	1.38229	7
54	64693	1.54576	67197	1.48816	69761	1.43347	72388	1.38145	6
55	64734	1.54478	67239	1.48722	69804	1.43258	72432	1.38060	5
56	64775	1.54379	67282	1.48629	69847	1.43169	72477	1.37976	4
57	64817	1.54281	67324	1.48536	69891	1.43080	72521	1.37891	3
58	64858	1.54183	67366	1.48442	69934	1.42992	72565	1.37807	2
59	64899	1.54085	67409	1.48349	69977	1.42903	72610	1.37722	1
60	64941	1.53986	67451	1.48256	70021	1.42815	72654	1.37638	0
M	N Cot.	N. Tan.	N. Cot.	N. Tan.	N. Cot.	N. Tan.	N. Cot.	N. Tan.	M
	57 Degrees.		56 Degrees.		55 Degrees.		54 Degrees.		

	36 Degrees.		37 Degrees.		38 Degrees.		39 Degrees.		
M	N. Tan.	N. Cot.	N. Tan.	N. Cot.	N. Tan.	N. Cot.	N. Tan	N. Cot.	M
0	72654	1.37638	75355	1.32704	78129	1.27994	80978	1.23489	60
1	72699	1.37554	75401	1.32624	78175	1.27917	81027	1.23416	59
2	72743	1.37470	75447	1.32544	78222	1.27841	81075	1.23343	58
3	72788	1.37386	75492	1.32464	78269	1.27764	81123	1.23270	57
4	72832	1.37302	75538	1.32384	78316	1.27688	81171	1.23196	56
5	72877	1.37218	75584	1.32304	78363	1.27611	81220	1.23123	55
6	72921	1.37134	75629	1.32224	78410	1.27535	81268	1.23050	54
7	72966	1.37050	75675	1.32144	78457	1.27458	81316	1.22977	53
8	73010	1.36967	75721	1.32064	78504	1.27382	81364	1.22904	52
9	73055	1.36883	75767	1.31984	78551	1.27306	81413	1.22831	51
10	73100	1.36800	75812	1.31904	78598	1.27230	81461	1.22758	50
11	73144	1.36716	75858	1.31825	78645	1.27153	81510	1.22685	49
12	73189	1.36633	75904	1.31745	78692	1.27077	81558	1.22612	48
13	73234	1.36549	75950	1.31666	78739	1.27001	81606	1.22539	47
14	73278	1.36466	75996	1.31586	78786	1.26925	81655	1.22467	46
15	73323	1.36383	76042	1.31507	78834	1.26849	81703	1.22394	45
16	73368	1.36300	76088	1.31427	78881	1.26774	81752	1.22321	44
17	73413	1.36217	76134	1.31348	78928	1.26698	81800	1.22249	43
18	73457	1.36133	76180	1.31269	78975	1.26622	81849	1.22176	42
19	73502	1.36051	76226	1.31190	79022	1.26546	81898	1.22104	41
20	73547	1.35968	76272	1.31110	79070	1.26471	81946	1.22031	40
21	73592	1.35885	76318	1.31031	79117	1.26395	81995	1.21959	39
22	73637	1.35802	76364	1.30952	79164	1.26319	82044	1.21886	38
23	73681	1.35719	76410	1.30873	79212	1.26244	82092	1.21814	37
24	73726	1.35637	76456	1.30795	79259	1.26169	82141	1.21742	36
25	73771	1.35554	76502	1.30716	79306	1.26093	82190	1.21670	35
26	73816	1.35472	76548	1.30637	79354	1.26018	82238	1.21598	34
27	73816	1.35389	76594	1.30558	79401	1.25943	82287	1.21526	33
28	73906	1.35307	76640	1.30480	79449	1.25867	82336	1.21454	32
29	73951	1.35224	76686	1.30401	79496	1.25792	82385	1.21382	31
30	73996	1.35142	76733	1.30323	79544	1.25717	82434	1.21310	30
31	74041	1.35060	76779	1.30244	79591	1.25642	82483	1.21238	29
32	74086	1.34978	76825	1.30166	79639	1.25567	82531	1.21166	28
33	74131	1.34896	76871	1.30087	79686	1.25492	82580	1.21094	27
34	74176	1.34814	76918	1.30009	79734	1.25417	82629	1.21023	26
35	74221	1.34732	76964	1.29931	79781	1.25343	82678	1.20951	25
36	74267	1.34650	77010	1.29853	79829	1.25268	82727	1.20879	24
37	74312	1.34568	77057	1.29775	79877	1.25193	82776	1.20808	23
38	74357	1.34487	77103	1.29696	79924	1.25118	82825	1.20736	22
39	74402	1.34405	77149	1.29618	79972	1.25044	82874	1.20665	21
40	74447	1.34323	77196	1.29541	80020	1.24969	82923	1.20593	20
41	74492	1.34242	77242	1.29463	80067	1.24895	82972	1.20522	19
42	74538	1.34160	77289	1.29385	80115	1.24820	83022	1.20451	18
43	74583	1.34079	77335	1.29307	80163	1.24746	83071	1.20379	17
44	74628	1.33998	77382	1.29229	80211	1.24672	83120	1.20308	16
45	74674	1.33916	77428	1.29152	80258	1.24597	83169	1.20237	15
46	74719	1.33835	77475	1.29074	80306	1.24523	83218	1.20166	14
47	74764	1.33754	77521	1.28997	80354	1.24449	83268	1.20095	13
48	74810	1.33673	77568	1.28919	80402	1.24375	83317	1.20024	12
49	74855	1.33592	77615	1.28842	80450	1.24301	83366	1.19953	11
50	74900	1.33511	77661	1.28764	80498	1.24227	83415	1.19882	10
51	74946	1.33430	77708	1.28687	80546	1.24153	83465	1.19811	9
52	74991	1.33349	77754	1.28610	80594	1.24079	83514	1.19740	8
53	75037	1.33268	77801	1.28533	80642	1.24005	83564	1.19669	7
54	75082	1.33187	77848	1.28456	80690	1.23931	83613	1.19599	6
55	75128	1.33107	77895	1.28379	80738	1.23858	83662	1.19528	5
56	75174	1.33026	77941	1.28302	80786	1.23784	83712	1.19457	4
57	75219	1.32946	77988	1.28225	80834	1.23710	83761	1.19387	3
58	75264	1.32865	77035	1.28148	80882	1.23637	83811	1.19316	2
59	75310	1.32785	77082	1.28071	80930	1.23563	83860	1.19246	1
60	75355	1.32704	77129	1.27994	80978	1.23490	83910	1.19175	0
M	N. Cot.	N. Tan.	N. Cot.	N. Tan.	N. Cot.	N. Tan.	N. Cot.	N. Tan.	M
	53 Degrees.		52 Degrees.		51 Degrees.		50 Degrees.		

	40 Degrees.		41 Degrees.		42 Degrees.		43 Degrees.		
M	N. Tan.	N. Cot.	N. Tan.	N. Cot.	N. Tan.	N. Cot.	N. Tan.	N. Cot.	M
0	83910	1.19175	86929	1.15037	90040	1.11061	93252	1.07237	60
1	83960	1.19105	86980	1.14969	90093	1.10996	93306	1.07174	59
2	84009	1.19035	87031	1.14902	90146	1.10931	93360	1.07112	58
3	84059	1.18964	87082	1.14834	90199	1.10867	93415	1.07049	57
4	84108	1.18894	87133	1.14767	90251	1.10802	93469	1.06987	56
5	84158	1.18824	87184	1.14699	90304	1.10737	93524	1.06925	55
6	84208	1.18754	87236	1.14632	90357	1.10672	93578	1.06862	54
7	84258	1.18684	87287	1.14565	90410	1.10607	93633	1.06800	53
8	84307	1.18614	87338	1.14498	90463	1.10543	93688	1.06738	52
9	84357	1.18544	87389	1.14430	90516	1.10478	93742	1.06676	51
10	84407	1.18474	87441	1.14363	90569	1.10414	93797	1.06613	50
11	84457	1.18404	87492	1.14296	90621	1.10349	93852	1.06551	49
12	84507	1.18334	87543	1.14229	90674	1.10285	93906	1.06489	48
13	84556	1.18264	87595	1.14162	90727	1.10220	93961	1.06427	47
14	84606	1.18194	87646	1.14095	90781	1.10156	94016	1.06365	46
15	84656	1.18125	87698	1.14028	90834	1.10091	94071	1.06303	45
16	84706	1.18055	87749	1.13961	90887	1.10027	94125	1.06241	44
17	84756	1.17986	87801	1.13894	90940	1.09963	94180	1.06179	43
18	84806	1.17916	87852	1.13828	90993	1.09899	94235	1.06117	42
19	84856	1.17846	87904	1.13761	91046	1.09834	94290	1.06056	41
20	84906	1.17777	87955	1.13694	91099	1.09770	94345	1.05994	40
21	84956	1.17708	88007	1.13627	91153	1.09706	94400	1.05932	39
22	85006	1.17638	88059	1.13561	91206	1.09642	94455	1.05870	38
23	85057	1.17569	88110	1.13494	91259	1.09578	94510	1.05809	37
24	85107	1.17500	88162	1.13428	91313	1.09514	94565	1.05747	36
25	85157	1.17430	88214	1.13361	91366	1.09450	94620	1.05685	35
26	85207	1.17361	88265	1.13295	91419	1.09386	94676	1.05624	34
27	85257	1.17292	88317	1.13228	91473	1.09322	94731	1.05562	33
28	85307	1.17223	88369	1.13162	91526	1.09258	94786	1.05501	32
29	85358	1.17154	88421	1.13096	91580	1.09195	94841	1.05439	31
30	85408	1.17085	88473	1.13029	91633	1.09131	94896	1.05378	30
31	85458	1.17016	88524	1.12963	91687	1.09067	94952	1.05317	29
32	85509	1.16947	88576	1.12897	91740	1.09003	95007	1.05255	28
33	85559	1.16878	88628	1.12831	91794	1.08940	95062	1.05194	27
34	85609	1.16809	88680	1.12765	91847	1.08876	95118	1.05133	26
35	85660	1.16741	88732	1.12699	91901	1.08813	95173	1.05072	25
36	85710	1.16672	88784	1.12633	91955	1.08749	95229	1.05010	24
37	85761	1.16603	88836	1.12567	92008	1.08686	95284	1.04949	23
38	85811	1.16535	88888	1.12501	92062	1.08622	95340	1.04888	22
39	85862	1.16466	88940	1.12435	92116	1.08559	95395	1.04827	21
40	85912	1.16398	88992	1.12369	92170	1.08496	95451	1.04766	20
41	85963	1.16329	89045	1.12303	92223	1.08432	95506	1.04705	19
42	86014	1.16261	89097	1.12238	92277	1.08369	95562	1.04644	18
43	86064	1.16192	89149	1.12172	92331	1.08306	95618	1.04583	17
44	86115	1.16124	89201	1.12106	92385	1.08243	95673	1.04522	16
45	86166	1.16056	89253	1.12041	92439	1.08179	95729	1.04461	15
46	86216	1.15987	89306	1.11975	92493	1.08116	95785	1.04401	14
47	86267	1.15919	89358	1.11909	92547	1.08053	95841	1.04340	13
48	86318	1.15851	89410	1.11844	92601	1.07990	95897	1.04279	12
49	86368	1.15783	89463	1.11778	92655	1.07927	96952	1.04218	11
50	86419	1.15715	89515	1.11713	92709	1.07864	96008	1.04158	10
51	86470	1.15647	89567	1.11648	92763	1.07801	96064	1.04097	9
52	86521	1.15579	89620	1.11582	92817	1.07738	96120	1.04036	8
53	86572	1.15511	89672	1.11517	92872	1.07676	96176	1.03976	7
54	86623	1.15443	89725	1.11452	92926	1.07613	96232	1.03915	6
55	86674	1.15375	89777	1.11387	92980	1.07550	96288	1.03855	5
56	86725	1.15308	89830	1.11321	93034	1.07487	96344	1.03794	4
57	86776	1.15240	89883	1.11256	93088	1.07425	96400	1.03734	3
58	86827	1.15172	89935	1.11191	93143	1.07362	96457	1.03674	2
59	86878	1.15104	89988	1.11126	93197	1.07299	96513	1.03613	1
60	86929	1.15037	90040	1.11061	93252	1.07237	96569	1.03553	0
M	N Cot	N. Tan.	N. Cot.	N. Tan.	N. Cot.	N. Tan.	N. Cot.	N. Tan.	M
	49 Degrees.		48 Degrees.		47 Degrees.		46 Degrees.		